

Name: MarkerMark: 15/52

For FULL MARKS, you must provide complete solutions using the methods learned this unit. For example, writing out all terms in a series in order to find the sum will not receive full marks.

Formulas: $t_n = a + (n-1)d$ $t_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_n = \frac{n}{2}(a + t_n)$ $S_n = \frac{n}{2}[2a + (n-1)d]$

[4] 1. Write the first five terms of the sequence defined by:

a) $t_n = \frac{(n+1)^3}{n^2}$

b) $f(1) = 3, f(2) = -5,$
for $n > 2, f(n) = 2f(n-2)$

$$\frac{2^3}{1^2}, \frac{3^3}{2^2}, \frac{4^3}{3^2}, \frac{5^3}{4^2}, \frac{6^3}{5^2}$$

$$= 8, \frac{27}{4}, \frac{64}{9}, \frac{125}{16}, \frac{216}{25}$$

$$= 1, -5, 2, -10, 4$$

[11] 2. Classify each sequence as arithmetic, geometric or neither, and find an explicit formula for the general term, t_n . Please, simplify the general term of any arithmetic sequences.

(a) $27, 18, 12, 8, \dots$

$$\underbrace{-9 \quad -6 \quad -4}_{+3} + 2$$

$$\frac{18}{27} = \frac{2}{3}$$

$$\frac{12}{18} = \frac{2}{3}$$

$$\frac{8}{12} = \frac{2}{3}$$

Type: Geometric
General Term: $t_n = 27\left(\frac{2}{3}\right)^{n-1}$

(c) $-4, -1, 4, 11, 20, \dots$

$$\underbrace{3 \quad 5 \quad 7 \quad 9}_{+2} + 2$$

$$n^2 - 5n + 4$$

$$= 1 - 5 + 4 = 0$$

$$= 4 - 5 + 4 = 3$$

Type: neither
General Term: $t_n = n^2 - 5n$

Type: Arithmetic
General Term: $t_n = 0.4n - 5.7$

(d) $0, 1, 8, 27, 64, \dots$

$$0^3, 1^3, 2^3, 3^3, 4^3$$

Type: neither
General Term: $t_n = (n-1)^3$

[4] 3. Classify the following sequence as arithmetic or geometric and determine how many terms are in the sequence. $8, 12, 16, 20, \dots, 400$

$$a = 8 \quad d = 4 \quad n = ?$$

$$8 + 4(n-1) = 400$$

$$4(n-1) = 392$$

$$n-1 = 98$$

$$n = 99$$

Type: Arithmetic
Number of Terms: 99

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- [7] 4. Classify the following series as arithmetic or geometric, and determine the sum of the series.
 $\frac{2}{3} + 2 + 6 + \dots + 486$ Leave your answer as a fraction in lowest terms.

$a = \frac{2}{3}$ $r = 3$ $n = ?$ $t_n = 486$ $S_n = \frac{a(r^n - 1)}{r - 1}$

$\sqrt{\frac{2}{3}} (3)^{n-1} = 486$ ✓
 $3^{n-1} = 486 \times \frac{3}{2}$ ✓
 $3^{n-1} = 729$ ✓
 $3^{n-1} = 3^6$ ✓
 $n = 7$

Type: Geometric ✓ Sum: $\frac{2186}{3}$ ✓ or $728\frac{2}{3}$

- [6] 5. In an arithmetic sequence, $t_{35} = 163$ and $t_{20} = 88$. Find an expression for the n th term. Simplify that expression.

$a + 34d = 163$ ✓ $a + 19(5) = 88$
 $a + 19d = 88$ ✓ $a = 88 - 95$
 $15d = 75$ $a = -7$
 $d = 5$

$a = \frac{-7}{1}$ ✓
 $d = \frac{5}{1}$ ✓
 $t_n = \frac{5n - 12}{1}$ ✓

$t_n = -7 + (n-1)(5)$ ✓
 $= -7 - 5 + 5n$
 $= 5n - 12$

- [5] 6. Mr. Math suffers from allergies. When allergy season arrives, his doctor recommends that he take 300 mg of his medication the first day and decrease the dosage by 17 mg each day for a week.

a) Develop a formula for the amount of medication Mr. Math will take on the n th day.

$t_n = 300 + (n-1)(-17)$ ✓

$t_n = 317 - 17n$ ✓

b) What is the amount of medication that Mr. Math will take on the 7th day?

$t_7 = 317 - 17(7)$
 $= 198$ ✓

Therefore, Mr. Math will take 198 mg on the seventh day.

c) To the nearest milligram, what is the total amount of medication that Mr. Math consumed during the seven days.

$S_n = \frac{n}{2} [a + t_n]$ ✓

$S_7 = \frac{7}{2} (300 + 198)$ ✓

$= 7 \left(\frac{498}{2} \right)$

$= 7 (249)$

$= 1743$ ✓

Therefore, during the seven days, Mr. Math will consume 1743 mg of allergy medication.

[6] 7. You visit the Grand Canyon and drop a rock off the edge of a cliff. The distance the rock will fall in the first second, 48 feet in the second second, 80 feet in the third second, and so on.

a) Write a RECURSIVE formula for this pattern.

$$t_1 = 16 \quad t_n = t_{n-1} + 32 \quad \text{for } n > 1$$

b) Write an EXPLICIT formula for this pattern.

$$t_n = a + (n-1)d \quad \rightarrow = 32n - 16$$

$$t_n = 16 + 32(n-1) \quad \checkmark$$

c) Assuming this pattern continues, find the total distance the object will fall in six seconds.

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad a = 16 \quad d = 32 \quad n = 6 \quad S_n = ?$$

$$S_6 = \frac{6}{2} [2(16) + 5(32)]$$

$$= 3(32 + 160)$$

$$= 3(192) = 576$$

Therefore, the object will fall 576 feet in the first six seconds.

[4] 8. You complain that the hot tub in your hotel is too hot. The hotel tells you they will decrease the temperature by 2% every thirty minutes. If the current temperature of the tub is 105° F, what will be the temperature of the hot tub after 2 hours, to the nearest tenth of a degree?

$$a = 105 \quad \text{note: 1 hour is the 3rd term} \quad n = 2 \times 2 + 1 = 5$$

note: 1 hour is the 3rd term

$$t_5 = 105(0.98)^4$$

$$= 96.848$$

$$\approx 96.8$$

Therefore, the temperature of the hot tub would be 96.8 ° after 2 hours.

[9] 9. After working at the same company for 4 years, Alicia received a salary of \$26620. After 6 years, she received \$32210.20 per year. If she received the same percentage pay increase every year, what was her starting salary?

$$t_4 = 26620 \quad t_6 = 32210.20 \quad a = ?$$

$$\frac{a r^5}{a r^3} = \frac{32210.20}{26620} \quad a(1.1)^3 = 26620$$

$$a = \frac{26620}{1.1^3}$$

$$r^2 = 1.21$$

$$r = 1.1 \quad \checkmark$$

r inadmissible.

$$a = 20000$$

Therefore, her starting salary was \$ 20000.

BONUS: A 4 by 4 "anti-magic" square is an arrangement of the numbers 1 to 16 inclusive in a square, so that the totals of each of the four rows and four columns and two main diagonals are ten consecutive numbers in some order. The diagram shows an incomplete "anti-magic" square. When it is completed, what number will replace the letter A?

4	5	7	14	total 30
6	13	3	A = 15	
11	12	9	1	
10	8	16	8	

total 39 total 31