Binomial Theorem and Pascal's Triangle WORKSHEET

1. Find the binomial expansion of each expression in simplified for.

   a) $(x+2)^3$ b) $(y-5)^4$ c) $(4x+1)^5$

   d) $(2x-1)^5$ e) $(2x+y)^4$ f) $(x-7)^5$

2. a) Explain how the term x^5y^3 is formed in the expansion of $(x+y)^8$.
   b) What is the coefficient of x^5y^3?

3. In the binomial of $(x+y)^n$, a term involving x^3y^4 occurs.
   a) What is the value *n*?
   b) What is the coefficient of x^3y^4?
   c) Which term in the expansion is this?

4. In the expansion of $(x+y)^6$, explain why the coefficient of x^4y^2 is the same coefficient of x^2y^4.

5. a) How many terms are there in the expansions of $(x+y)^{15}$ and $(x+y)^{16}$?
   b) Which expansion in part a has a middle term? Which has two middle terms?
   c) When does the expansion of $(x+y)^n$ have one middle term? When does it have two middle terms?

6. Expand and simplify using the binomial theorem.

   a) $(x+2)^6$ b) $(x-3)^4$ c) $(2-x)^5$ d) $(1+x^2)^6$

   e) $(x-2y)^4$ f) $(2x+3y)^3$ g) $\left(x+\frac{1}{x}\right)^5$ h) $(3x+2y^2)^5$

7. Write the first four terms in each expansion. Simplify each term.

   a) $\left(\sqrt{x}+1\right)^{10}$ b) $(x+2)^{12}$ c) $(x-2)^8$ d) $(1-2x)^{10}$

8. Write the first three terms and the 7^{th} term in the expansion of $(x+2y)^{12}$. Simplify.

Answers:

1a) $x^3+6x^2+12x+8$

1b) $y^4-20y^3+150y^2-500y+625$

1c) $1024x^5+1280x^4+640x^3+160x^2+20x+1$

1d) $32x^5-80x^4+80x^3-40x^2+20x+1$

1e) $16x^4+32x^3y+24x^2y^2+8xy^3+y^4$

1f) $x^5-35x^4+490x^3-3430x^2+12005x-16807$

2a) $\binom{8}{3}x^5y^3$

2b) 56

3a) 7

3b) 35

3c) 5^{th} term

4) 15

5a) 16, 17

5b) $(x+y)^{16}$ has a middle term since it has 17 terms, but $(x+y)^{15}$ has 2 middle terms

5c) a single middle term when n is even and 2 middle terms when n is odd

6a) $x^6+12x^5+60x^4+160x^3+240x^2+192x+64$

6b) $x^4-12x^3+54x^2-108x+81$

6c) $-(x^5-10x^4+40x^3-80x^2+80x-32)$

6d) $x^{12}+6x^{10}+15x^8+20x^6+15x^4+6x^2+1$

6e) $x^4-8x^3y+24x^2y^2-32xy^3+16y^4$

6f) $8x^3+36x^2y+54xy^2+27y^3$

6g) $\dfrac{x^{10}+5x^8+10x^6+10x^4+5x^2+1}{x^5}$ note final answer has a common denominator

6h) $243x^5+810x^4y^2+1080x^3y^4+720x^2y^6+240xy^8+32y^{10}$

7a) $x^5+10x^4\sqrt{x}+45x^4+120x^3\sqrt{x}\ldots$

7b) $x^{12}+24x^{11}+264x^{10}+1760x^9+\ldots.$

7c) $x^8-16x^7+112x^6-448x^5\ldots$

7d) $1024x^{10}-5120x^9+11520x^8-15360x^7$ ...

8) $x^{12}+24x^{11}y+264x^{10}y^2+\ldots+59136x^6y^6\ldots$