D7 - Binomial Expansion and Pascal's Triangle LESSON 2018

May 31, 2018 9:20 AM

MCR 3UI - U7 - D7 - Binomial Expansion and Pascals Tria...

MCR 3UI **Pascal's Triangle and Binomial Theorem** U7D7

Preamble A binomial is an algebraic expression containing two terms.

Ex. $3x+1$, $1-x^2$, $3x-7y$

Today we will learn how to expand binomials raised to any power without the use of tedious and lengthy calculations.

Ex. $(x-3)^2 = (x-3)(x-3) = x^2 - 3x - 3x + 9 \Rightarrow x^2 - 6x + 9$

$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

$(3x-2y)^{10} = ???$

Part A Expand and simplify the following. Place your final answer below.

Keep in mind that $(x+y)^n = (x+y)(x+y)^{n-1}$, in other words you may use your previous answer to proceed.

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = (x+y)(x^2+2xy+y^2)$

$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 \Rightarrow x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = (x+y)(x^3+3x^2y+3xy^2+y^3)$

$= x^4 + 3x^3y + 3x^2y^2 + xy^3 + x^3y + 3x^2y^2 + 3xy^3 + y^4$

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)(x^4+4x^3y+6x^2y^2+4xy^3+y^4)$

$(x+y)^5 = x^5 + 4x^4y + 6x^3y^2 + 4x^2y^3 + xy^4 + x^4y + 4x^3y^2 + 6x^2y^3 + 4xy^4 + y^5$

$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Part B List all of the patterns you see in the final answers in the expansions. (done the long way). If n is the exponent on $(x+y)$

1. The sum of exponents on each term are equal to n
2. Coefficients of the 1st & last terms are always 1
3. The total number of terms is always $n+1$
4. The 2nd & 2nd last terms have a coefficient of n
5. x-exponents : start at 'n', then decrease by 1 each term $n, n-1, n-2 \ldots, 1, 0$ (first term of binomial)
6. y-exponents : Start at 0, then increase by 1 each term until they get to n (2nd term of binomial) $0, 1, \ldots, n-1, n$

Part C Write the coefficients of the expansions below centering each row in the space.
Then add the coefficients in each row.

row sum $= 2^n$ ← exponent on the binomial

n		row sum
0	1	$1 = 2^0$
1	1 1	$2 = 2^1$
2	1 2 1	$4 = 2^2$
3	1 3 3 1	$8 = 2^3$
4	1 4 6 4 1	$16 = 2^4$
5	1 5 10 10 5 1	$32 = 2^5$
6	1 6 15 20 15 6 1	$64 = 2^6$
7	1 7 21 35 35 21 7 1	$128 = 2^7$
8	1 8 28 56 70 56 28 8 1	$256 = 2^8$
	⋮	

This is Pascal's Triangle.

Part D List some of the characteristics of the numbers in Pascal's triangle.

1. The row sum is 2^n
2. Each term is sum of the two terms above
3. The number of terms in each row increases by 1
4. Each row begins and ends with 1 / It's also Symmetric

Part E Using Pascal's triangle and the patterns you have discovered today, expand :

$(x+y)^8 = x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$

Part F How is the expansion of $(x-y)^2$ different from $(x+y)^2$?

$(x-y)(x-y)$

$= \underbrace{x^2 - 2xy + y^2}_{\text{every other term is negative}}$

Part G Expand the following. $x^3 + 3x^2y + 3xy^2 + y^3 = (x+y)^3$

$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x-y)^4 = x^4 + 4x^3(-y)^1 + 6x^2(-y)^2 + 4x(-y)^3 + (-y)^4$

$(x-y)^3 =$ $x^3 - 3x^2y + 3xy^2 - y^3$

$(x-y)^4 =$ $x^4 + 4x^3(-y)^1 + 6x^2(-y)^2 + 4x(-y)^3 + (-y)^4$

$(x+(-y))^4$ $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

$(x-y)^5 =$ $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

$(2x+3)^5 =$ $(2x)^5(3)^0 + 5(2x)^4(3)^1 + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)(3)^4 + 3^5$

$= 32x^5 + 5(16x^4)(3) + 10(8x^3)(9) + 10(4x^2)(27) + 5(2x)(81) + 243$

$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$

$(x^2-1)^6 = 1(x^2)^6(-1)^0 + 6(x^2)^5(-1)^1 + 15(x^2)^4(-1)^2 + 20(x^2)^3(-1)^3 + 15(x^2)^2(-1)^4 + 6(x^2)^1(-1)^5 + 1(x^2)^0(-1)^6$

$= x^{12} - 6x^{10} + 15x^8 - 20x^6 + 15x^4 - 6x^2 + 1$

$(x^2-\sqrt{x})^4 =$ $1(x^2)^4(-\sqrt{x})^0 + 4(x^2)^3(-\sqrt{x})^1 + 6(x^2)^2(-\sqrt{x})^2 + 4(x^2)^1(-\sqrt{x})^3 + 1(x^2)^0(-\sqrt{x})^4$

$= x^8 - 4x^6x^{\frac{1}{2}} + 6x^4x^{2/2} - 4x^2x^{3/2} + x^{4/2}$

$= x^8 - 4x^{13/2} + 6x^5 - 4x^{7/2} + x^2$

Part H We can generate the coefficients on your calculator if you have a special button.

${}_nC_r = \binom{n}{r}$ **Binomial Theorem**

$(a+b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 \dots \binom{n}{n-1}a^1b^{n-1} + \binom{n}{n}a^0b^n$

$(x+y)^4 = \binom{4}{0}x^4y^0 + \binom{4}{1}x^3y^1 + \binom{4}{2}x^2y^2 + \binom{4}{3}xy^3 + \binom{4}{4}x^0y^4$

1 4 6 4 1

$\boxed{4}\boxed{nCr}\boxed{2}$ in calculator

Combinations will tell you the coefficients from the corresponding row of Pascal's Triangle. (without having to build Pascal's Triangle)

Part H Using binomial Theorem expand the following:

$(2x+1)^5 =$ $\binom{5}{0}(2x)^5(1)^0 + \binom{5}{1}(2x)^4(1) + \binom{5}{2}(2x)^3(1)^2 + \binom{5}{3}(2x)^2(1)^3 + \binom{5}{4}(2x)(1)^4 + \binom{5}{5}(2x)^0(1)^5$

$= 1(32x^5) + 5(16x^4) + 10(8x^3) + (10)(4x^2) + 5(2x) + 1(1)$

$= 32x^5 + 80x^4 + 80x^3 + 40x^2 + 10x + 1$

$(x^2-3)^6 =$ $\binom{6}{0}(x^2)^6(-3)^0 + \binom{6}{1}(x^2)^5(-3)^1 + \binom{6}{2}(x^2)^4(-3)^2 + \binom{6}{3}(x^2)^3(-3)^3 + \binom{6}{4}(x^2)^2(-3)^4 + \binom{6}{5}(x^2)^1(-3)^5 + \binom{6}{6}(x^2)^0(-3)^6$

$= 1(x^8)(1) + 6(x^{10})(-3) + 15(x^8)(9) + 20(x^6)(-27) + 15(x^4)(81) + 6(x^2)(-243) + 1(1)(729)$

$= x^8 - 18x^{10} + 135x^8 - 540x^6 + 1215x^4 - 1458x^2 + 729$