D6 - Application of Sequences and Series LESSON 2018

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[W] MCR 3UI - U7 - D6 - Application of Sequences and Series...

MCR 3UI   **Sequences & Series Story Questions**   U7D6

Some hints for solving Story Questions:
1.   If possible, write out the first few terms of the series.
2.   Determine if it is Arithmetic or Geometric, and a Sequence or Series
2.   List "Given" and "Unknowns"
3.   Determine which formula(s) you need to use
4.   Solve and write a conclusion.

1. A wall of blocks is built up so that each row has 2 less blocks from the previous row. If there are 43 blocks in the first row and 11 blocks in the top row, how many rows high is the wall?

$$43, \overset{-2}{\overset{\curvearrowright}{}} 41, \overset{-2}{\overset{\curvearrowright}{}} 39, \ldots \quad 11 \qquad \leftarrow \text{common difference}$$
$$\therefore \text{arithmetic}$$

$a = 43$
$d = -2$
$t_n = 11$
$n = ?$

$t_n = a + (n-1)d$
$11 = 43 + (n-1)(-2)$
$11 - 43 = -2(n-1)$
$-32 = -2(n-1)$
$16 = n - 1$
$\boxed{n = 17}$

∴ there are 17 rows on the wall.

2. A stereo system costing $1200 depreciates by 30 % per year. Find the value of the stereo after 6 years.

$\downarrow$ ∴ 70% of cost remains

$$1200, 840, 588, \ldots$$
$$\underset{\times 0.7}{\curvearrowright} \quad \underset{\times 0.7}{\curvearrowright} \quad \leftarrow \text{repeated multiplication by 0.7} \quad \therefore \text{geometric}$$

$a = 1200$
$r = 0.7$
$n = \underset{\smile}{6 + 1}$
   $= 7$ ← depreciates 6 times by 0.7 (so simplified exponent must be 6)

$t_n = a r^{n-1}$
$t_7 = 1200(0.7)^{7-1}$
$t_7 = 1200(0.7)^6$
$t_7 = \$141.18$

∴ the stereo is worth $141.18 after 6 years

← round to two decimal places for money.

3. Suppose you researched your ancestors back ten (biological) generations (starting with your mom and dad, then their parents, then their parent's parents, etc.). How many people would you research? (total sum!)

your mom and dad, then their parents, then their parent's parents, etc.).
How many people would you research? *(total sum!)*

M D Generation 1

M D M D Gen. 2

M D M D M D M D Gen 3

etc.
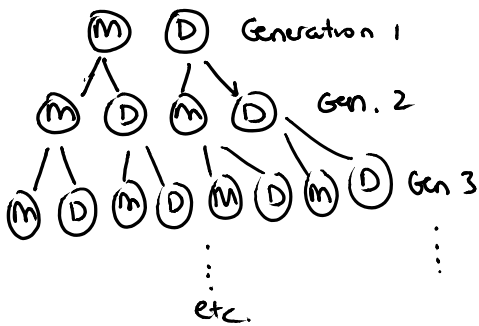
$2, 4, 8, \ldots$

$\times 2 \quad \times 2$ ← common ratio of 2 ∴ geometric

$a = 2$
$r = 2$
$n = 10$
$S_n = ?$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{2(2^{10} - 1)}{2 - 1}$$

$$S_{10} = 2(1024 - 1)$$

$$S_{10} = 2046$$

∴ you would have to research 2046 people total.

4. How many multiples of 5 are from 20 to 200?

divisible by 5 (ie. numbers that end in 0 or 5)

$20, 25, 30, 35, \ldots, 200$

$+5 \quad +5$ ← common difference ∴ arithmetic

$a = 20$
$d = 5$
$t_n = 200$
$n = ?$

$t_n = a + (n-1)d$

$200 = 20 + (n-1)(5)$

$200 - 20 = 5(n - 1)$

$180 = 5(n - 1)$

$\dfrac{180}{5} = n - 1$

$36 = n - 1$

$\boxed{n = 37}$

∴ there are 37 terms from 20 to 200 (inclusive)

5. Determine the value of x such $\overbrace{x-4}^{t_1}, \overbrace{2x+1}^{t_2}, \overbrace{5x+4}^{t_3}$, are consecutive terms of an arithmetic sequence.

↖ in a row

↑ ∴ common difference between terms

$a = ?$
$d = ?$
$x = ?$

$d = t_2 - t_1$ and $d = t_3 - t_2$

$d = (2x+1) - (x-4)$ $d = (5x+4) - (2x+1)$

① $d = x + 5$ ② $d = 3x + 3$

Sub ① into ②

$x + 5 = 3x + 3$
$5 - 3 = 3x - x$ ∴ $x = 1$
$2 = 2x$ $d = 6$
$\frac{2}{2} = x$ $t_1 = -3$
$\boxed{x = 1}$ $t_2 = 3$
                          $t_3 = 9$

6. Determine the value of x such $x-2, -2-x, x+10$, are consecutive terms of a geometric sequence.

↑ ∴ common ratio between terms

$r = \dfrac{t_2}{t_1}$ and $r = \dfrac{t_3}{t_2}$

① $r = \dfrac{-2-x}{x-2}$ ② $r = \dfrac{x+10}{-2-x}$

Substitute ① into ②

$\dfrac{-2-x}{x-2} = \dfrac{x+10}{-2-x}$ ⎫ cross multiply to solve

$(-2-x)(-2-x) = (x-2)(x+10)$ ⎫ expand
$4 + 4x + x^2 = x^2 + 8x - 20$ ⎫ collect like terms
$4 + 20 = 8x - 4x$
$24 = 4x$ ∴ $x = 6$
$x = \dfrac{24}{4}$ ⎫ isolate x $r = -2$
$\boxed{x = 6}$ $t_1 = 4$
                          $t_2 = -8$
                          $t_3 = 16$