D5 - Geometric Series LESSON 2018

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📄 MCR 3UI - U7 - D5 - Geometric Series LESSON 2018

MCR 3UI   **Geometric Series**   U7D5

A geometric series is the __Sum__ of the terms of a geometric sequence.

If the sequence is $t_1, t_2, t_3, t_4 \ldots t_n$ then the series is $s_1, s_2, s_3, s_4 \ldots s_n$ where :

$$s_1 = t_1 = a$$
$$s_2 = t_1 + t_2 = a + ar$$
$$s_3 = t_1 + t_2 + t_3 = a + ar + ar^2$$
$$s_4 = t_1 + t_2 + t_3 + t_4 = a + ar + ar^2 + ar^3$$

. . . . . .

In general

$$s_n = \frac{a(r^n - 1)}{r - 1} \ , \ r \neq 1$$

a is   first term value (t_1)

r is   common ratio

n is   number of terms
(or term number of last term in the series)

Examples:
1. Find the sum of the first 10 terms of 5, 10, 20, 40, . . .

$a = 5$
$r = 2$
$S_n = ?$
$n = 10$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{5(2^{10} - 1)}{2 - 1}$$ follow BEDMAS!

$$S_{10} = \frac{5(1024 - 1)}{1}$$

$$\boxed{S_{10} = 5115}$$

∴ the sum of the first 10 terms is 5115.

2. Find the sum of $972 - 324 + 108 - \ldots - 4$.

(annotations: a pointing to 972, t_n pointing to -4)

Determine n first. (the number of terms)

$a = 972$

$t_n = -4$

$r = \dfrac{-324}{972}$

$r = -\dfrac{1}{3}$

$t_n = ar^{n-1}$

$-4 = 972\left(\dfrac{-1}{3}\right)^{n-1}$

$\dfrac{-4}{972} = \left(-\dfrac{1}{3}\right)^{n-1}$

$-\dfrac{1}{243} = \left(-\dfrac{1}{3}\right)^{n-1}$

$\left(-\dfrac{1}{3}\right)^{5} = \left(-\dfrac{1}{3}\right)^{n-1}$

$\therefore 5 = n - 1$

$\boxed{n = 6}$

$S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_6 = \dfrac{972\left(\left(-\dfrac{1}{3}\right)^{6} - 1\right)}{-\dfrac{1}{3} - 1}$

$S_6 = \dfrac{972\left(\dfrac{1}{729} - \dfrac{729}{729}\right)}{-\dfrac{4}{3}}$

$S_6 = \dfrac{972}{1}\left(\dfrac{-728}{729}\right) \times \dfrac{-3}{4}$

$\boxed{S_6 = 728}$

(red annotation: to divide by fraction, invert & multiply)

$\therefore$ the sum of the first 6 terms is 728.

Sometimes we can reverse the order of the sequence so we don't have to deal with a rational value of r. (the sum will be the same regardless of whether we add from beginning to end of sequence or the end to the beginning)

$-4 \ldots 108 - 324 + 972$

(annotations: $\times -3$, -3)

$a = -4$

$r = -3$

$t_n = 972$

$t_n = ar^{n-1}$

$972 = -4(-3)^{n-1}$

$\dfrac{972}{-4} = (-3)^{n-1}$

$-243 = (-3)^{n-1}$

$(-3)^{5} = (-3)^{n-1}$

$\therefore 5 = n - 1$

$\boxed{n = 6}$

$S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_6 = \dfrac{-4((-3)^{6} - 1)}{-3 - 1}$

$S_6 = \dfrac{-4(729 - 1)}{-4}$

$\boxed{S_6 = 728}$

Summary : Formula List

General Term

Geometric Sequence

Arithmetic Sequence

Series

Arithmetic

Arithmetic (alternate version)

Geometric