D4 - Arithmetic Series LESSON 2018

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MCR 3UI - U7 - D4 - Arithmetic Series LESSON 2018

MCR 3UI **Arithmetic Series** U7D4

An arithmetic series is the Sum of the terms of an arithmetic sequence.

If the sequence is $t_1, t_2, t_3, t_4, \dots, t_n$,
then the series is $S_1, S_2, S_3, S_4, \dots, S_n$
where :

$$S_1 = t_1$$
$$S_2 = t_1 + t_2$$
$$S_3 = t_1 + t_2 + t_3$$
$$S_4 = t_1 + t_2 + t_3 + t_4$$
$$\dots$$

In general,

Sum of 'n' terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

a is first term value (t_1)

d is common difference

n is number of terms (term number of the last term in the series)

Or the formula can be written as :

① $$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[a + \underbrace{a + (n-1)d}_{t_n}\right]$$

② $$S_n = \frac{n}{2}\left[t_1 + t_n\right]$$

or $S_n = \frac{n}{2}\left[a + t_n\right]$

And so, we have two different versions of the same formula.

Examples:

1. Find the $\boxed{\text{sum}}$ of the first 100 terms of $8 \overset{+3}{+} 11 \overset{+3}{+} 14 + \ldots$

$a = 8$

$d = 3$

$n = 100$

$S_n = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{100} = \frac{100}{2}[2(8) + (100-1)(3)]$$

$$S_{100} = 50[16 + 297]$$

$$S_{100} = 50[313]$$

$$\boxed{S_{100} = 15650}$$

$\therefore$ the sum of the first 100 terms is 15650.

2. Find the $\boxed{\text{sum}}$ of 1.1 + 1.2 + 1.3 + 1.4 + . . . + 8.9

$a = 1.1$

$d = 0.1$

$t_n = 8.9$

$n = ?$

$S_n = ?$

recall:

$$t_n = a + (n-1)d$$

$$8.9 = 1.1 + (n-1)(0.1)$$

$$8.9 - 1.1 = 0.1(n-1)$$

$$7.8 = 0.1(n-1)$$

$$\frac{7.8}{0.1} = n-1$$

$$78 = n-1$$

$$\boxed{n = 79}$$

$$S_n = \frac{n}{2}[t_1 + t_n]$$

$$S_{79} = \frac{79}{2}[1.1 + 8.9]$$

$$S_{79} = \frac{79}{2}[10.0]$$

$$\boxed{S_{79} = 395}$$

$\therefore$ the sum of the sequence would be 395.

3. The sum of an arithmetic series is 18200. If $t_1 = 1024$ and $t_6 = 964$, determine how many terms are in the series.

$S_n = 18\,200$
$t_1 = a = 1024$
$n = ?$
$t_6 = 964$

$t_n = a + (n-1)d$
$964 = 1024 + (6-1)d$
$964 - 1024 = 5d$
$-60 = 5d$
$\frac{-60}{5} = d$
$\boxed{d = -12}$

$S_n = \frac{n}{2}[2a + (n-1)d]$
$18200 = \frac{n}{2}[2(1024) + (n-1)(-12)]$
$18200 = \frac{n}{2}[2048 - 12n + 12]$
$18\,200 = \frac{n}{2}[2060 - 12n]$
$18\,200 \times 2 = n[2060 - 12n]$
$36400 = n[2060 - 12n]$
$36400 = 2060n - 12n^2$
$12n^2 - 2060n + 36400 = 0$
$a = 12 \quad b = -2060 \quad c = 36400$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$n = \frac{2060 \pm \sqrt{(-2060)^2 - 4(12)(36400)}}{2(12)}$
$n = \frac{2060 \pm \sqrt{2496400}}{24}$
$n = \frac{2060 \pm 1580}{24}$

$n = \frac{2060 + 1580}{24}$ or $n = \frac{2060 - 1580}{24}$

$n = 151.\overline{6}$
inadmissable
(n must be a natural number)

$\boxed{n = 20}$
$\therefore$ there are 20 terms in this sequence.

4. If the sum of n terms of a arithmetic sequence is given by $S_n = n^2 + n$, find t_{11}.

$S_{11} = (11)^2 + 11$
$S_{11} = 121 + 11$
$S_{11} = 132$

$S_1 = (1)^2 + 1$
$S_1 = 2$
↳ since S_1 is the sum of only the first term, $\therefore t_1 = a = 2$

$S_n = \frac{n}{2}[t_1 + t_n]$
$S_{11} = \frac{11}{2}[t_1 + t_{11}]$
$132 = \frac{11}{2}[2 + t_{11}]$
$264 = 11[2 + t_{11}]$
$\frac{264}{11} = 2 + t_{11}$
$24 = 2 + t_{11}$
$24 - 2 = t_{11}$
$\boxed{t_{11} = 22}$

$S_2 = (2)^2 + 2$
$S_2 = 6$

$t_1 \quad t_2$
$2, 4, \ldots$

$S_2 = t_1 + t_2$
$6 = 2 + t_2$
$t_2 = 4$

$\therefore d = 4 - 2$
$\boxed{d = 2}$

$t_n = a + (n-1)d$
$t_{11} = 2 + (11-1)(2)$
$t_{11} = 2 + 20$
$\boxed{t_{11} = 22}$