D3 - Geometric Sequences LESSON 2018

May 27, 2018 8:09 PM

MCR 3UI - U7 - D3 - Geometric Sequences LESSON 2018

MCR 3UI **Geometric Sequences** U7D3

What is similar about the following sequences?

1. $2, \overset{\times 3}{} 6, \overset{\times 3}{} 18, 54 \ldots$
2. $2, \overset{\times 5}{} 10, \overset{\times 5}{} 50, 250 \ldots$
3. $5, \overset{\times -2}{} -10, \overset{\times -2}{} 20, -40, 80 \ldots$

multiplying by a constant value to get to the next consecutive term } each of these sequences are the result of an exponential function

All of these sequences are classified as **geometric** sequences since each term is generated by multiplying the previous term by the same amount called the Common ratio .(r)

A geometric sequence looks like :

$a, \ ar, \ ar^2, \ ar^3 \ \ldots\ldots$ or

In general

$$t_n = a r^{n-1}$$

$t_n =$ "general term" or "n^{th} term" $a =$ first term value (t_1)

$n =$ "the term number" or "the number of terms" $r =$ common ratio

Examples:

1. Determine t_n and t_{10} for the following geometric sequences:

a) $5, \overset{\times 4}{} 20, \overset{\times 4}{} 80, 320 \ldots$

$a = 5$
$r = 4$

$t_n = ar^{n-1}$

$\boxed{t_n = 5(4)^{n-1}}$

$t_{10} \rightarrow$ sub $n = 10$

$t_{10} = 5(4)^{10-1}$

$t_{10} = 5(4)^9$

$\boxed{t_{10} = 1310720}$

b) $2, -\frac{3}{2}, \frac{9}{8}, -\frac{27}{32} \ldots$

$a = 2$

$r = \frac{9}{8} \div \frac{-3}{2}$

$r = \frac{\cancel{9}^{3}}{\cancel{8}_{4}} \times \frac{-\cancel{2}^{1}}{\cancel{3}}$

$r = -\frac{3}{4}$

$\boxed{t_n = 2\left(\frac{-3}{4}\right)^{n-1}}$

$t_{10} = 2\left(\frac{-3}{4}\right)^{10-1}$

$= 2\left(\frac{-3}{4}\right)^9$

$= 2\,\frac{(-3)^9}{4^9}$

$= -\frac{39366}{262144}$

$\boxed{t_{10} = -\frac{19683}{131072}}$

2. Determine the number of terms in the sequence
3, 6, 12, 24 . . . . 96.

$a = 3$
$r = 2$
$t_n = 96$
$n = ?$

$t_n = ar^{n-1}$
$96 = 3(2)^{n-1}$
$\frac{96}{3} = (2)^{n-1}$
$32 = (2)^{n-1}$ } get common base
$2^5 = (2)^{n-1}$ } equate exponents
$\therefore 5 = n-1$
$\boxed{6 = n}$

∴ there is 6 terms in the sequence

3. Determine t_{10} if for each of the following geometric sequences:

a) if $t_3 = 15$ and $t_6 = -405$.

n

$t_n = ar^{n-1}$
$15 = ar^{3-1}$
(1) $15 = ar^2$

$-405 = ar^{6-1}$
$-405 = ar^5$ (2)

$\div$
$-405 = ar^5$
$15 = ar^2$
$-27 = r^3$

Divide equations (because can't subtract since ar^5 & ar^2 are not like terms)

★ cube rooting is the opposite of cubing

$\sqrt[3]{-27} = r$
$\boxed{r = -3}$

In calculator $\sqrt[x]{\ }$ or $y^{\frac{1}{x}}$

Sub $r = -3$ into (1) or (2) to solve for a.

$15 = ar^2$
$15 = a(-3)^2$
$15 = 9a$
$a = \frac{15}{9}$
$a = \frac{5}{3}$

$\therefore t_n = \frac{5}{3}(-3)^{n-1}$
$t_{10} = \frac{5}{3}(-3)^{10-1}$
$t_{10} = \frac{5(-19683)}{3}$
$\boxed{t_{10} = -32805}$

b) if $t_3 = 60$ and $t_7 = 960$.

(1) $60 = ar^2$ (2) $960 = ar^6$

$\div$
$960 = ar^6$
$60 = ar^2$
$16 = r^4$

any time taking an even root, both a positive & negative is

$\pm\sqrt[4]{16} = r$
$r = \pm 2$

sub $r = 2$

$60 = ar^2$
$60 = a(2)^2$
$60 = 4a$
$\frac{60}{4} = a$
$a = 15$

★ Note: The value of 'a' will remain the same regardless of the sign of r, ∴ no need to solve twice for a.

$\therefore t_n = 15(2)^{n-1}$ or $t_n = 15(-2)^{n-1}$

however possible tw...

root, both a positive & negative answer is possible.

$r = \pm 2$

$\therefore t_n = 15(2)^{n-1}$ or $t_n = 15(-2)^{n-1}$

however two possible equations result since there are two values of r.

$t_{10} = 15(2)^{10-1}$

$t_{10} = 7680$

$t_{10} = 15(-2)^9$

$t_{10} = -7680$

4. Express the geometric sequences defined by the general term $t_n = 3\left(\frac{2}{5}\right)^{n-1}$, as a recursive sequence.

based on the value of the previous term

$t_1 = 3$

$t_2 = 3\left(\frac{2}{5}\right)^{2-1}$

$t_2 = \frac{6}{5}$

$\times \frac{2}{5}$

$t_3 = 3\left(\frac{2}{5}\right)^{3-1}$

$= 3\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)$

$= \left(\frac{6}{5}\right)\left(\frac{2}{5}\right)$

$= \frac{12}{25}$

$\times \frac{2}{5}$

...

$t_n = t_{n-1}\left(\frac{2}{5}\right), \quad t_1 = 3, \quad n \geq 2$