D2 - Arithmetic Sequences new LESSON

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MCR 3UI - U7 - D2 - Arithmetic Sequences new LESSON ...

MCR3UI **Arithmetic Sequences** **U7 D2**

What is similar about the following sequences?

a) 3, 5, 7, 9, 11 (+2, +2, +2)

b) -1, 4, 9, 14, 19 (+5, +5, +5)

c) 20, 17, 14, 11, 8 (−3, −3, −3)

All have a common difference → created by a linear equation

All of these sequences are classified as **arithmetic** sequences since each term is generated by adding a **COMMON DIFFERENCE** or 'd' to the previous term. The first term is designated as 'a'.

An arithmetic sequence looks like :

$a,\ a+d,\ a+2d,\ a+3d,\ a+4d$ . . . . . or

In general

$$t_n = a + (n-1)d$$

t_n = 'General Term' or 'the n^{th} term'

a = value of the first term (t_1)

n = 'number of terms' or 'term number'

d = common difference between terms

Examples:

1. Determine t_n and t_{50} for the following arithmetic sequences:

a) 2, 6, 10, 14 . . . . (+4, +4, +4)

$a = 2$

$d = 4$

$t_n = a + (n-1)d$

$t_n = 2 + (n-1)(4)$

$t_n = 2 + 4n - 4$

$\boxed{t_n = 4n - 2}$

$t_{50} \rightarrow n = 50$

$t_{50} = 4(50) - 2$

$t_{50} = 200 - 2$

$\boxed{t_{50} = 198}$

b) $\frac{10}{1}, \frac{19}{2}, \frac{9}{1}, \frac{17}{2} \ldots \rightarrow \frac{20}{2}, \frac{19}{2}, \frac{18}{2}, \frac{17}{2} \ldots$

★ can subtract consecutive terms to determine d

$d = \frac{19}{2} - \frac{20}{2}$

$d = -\frac{1}{2}$

$a = 10$

$d = -\frac{1}{2}$

$t_n = a + (n-1)d$

$t_n = 10 + (n-1)\left(-\frac{1}{2}\right)$

$t_n = 10 - \frac{1}{2}n + \frac{1}{2}$

$t_n = -\frac{1}{2}n + \frac{21}{2}$

or $t_n = \frac{21}{2} - \frac{1}{2}n$

$t_{50} \rightarrow$ sub $n = 50$

$t_{50} = -\frac{1}{2}(50) + \frac{21}{2}$

$= -\frac{50}{2} + \frac{21}{2}$

$t_{50} = -\frac{29}{2}$

2. Determine the number of terms in the sequence 3, 7, 11, 15 . . . . 199.

+4 +4 +4

$n = ?$

$a = 3$

$d = 4$

$t_n = 199$

$t_n = a + (n-1)d$

$199 = 3 + (n-1)(4)$

$199 = 3 + 4n - 4$

$199 = 4n - 1$

$200 = 4n$

$\frac{200}{4} = n$

$\boxed{n = 50}$

∴ the sequence has 50 terms.

3. Determine t_{50} if $t_4 = 5$ and $t_{11} = 26$ for an arithmetic sequence.

Grade 10 method

$t_n = 5$, $n = 4$

$t_n = a + (n-1)d$

$5 = a + (4-1)d$

① $5 = a + 3d$

$26 = a + (11-1)d$

② $26 = a + 10d$

★ use substitution or elimination to solve ★

$26 = a + 10d$

$\ominus$ $5 = a + 3d$

$21 = 7d$

$\frac{21}{7} = d$

$d = 3$

sub $d=3$ into ① or ②

$5 = a + 3(3)$

$5 = a + 9$

$a = 5 - 9$

$a = -4$

$\therefore t_n = -4 + (n-1)(3)$

$t_n = -4 + 3n - 3$

$t_n = 3n - 7$

sub $n = 50$

$t_{50} = 3(50) - 7$

$t_{50} = 150 - 7$

$t_{50} = 143$

Grade 9 method

$t_4 = 5$ $(4, 5)$ x_1, y_1

$t_{11} = 26$ $(11, 26)$ x_2, y_2

$m = \frac{\Delta y}{\Delta x}$

$= \frac{26-5}{11-4}$

$= \frac{21}{7}$

$m = 3$

↑ slope is the common difference (d)

sub $m = 3$ & a point into

$y = mx + b$

$5 = 3(4) + b$

$5 - 12 = b$

$b = -7$

$\therefore y = 3x - 7$

$\therefore t_n = 3n - 7$

then sub $n = 50$

⋮

4. Describe the arithmetic sequence $t_n = 3n - 2$ as a recursive sequence.

$t_1 = 3(1) - 2$ $t_2 = 3(2) - 2$ $t_3 = 3(3) - 2$

$t_1 = 1$ $t_2 = 4$ $t_3 = 7$

based on the previous terms value

$+3$ $+3$ ← take the previous term value and add 3.

$t_n = t_{n-1} + 3$, $t_1 = 1$, $n \geq 2$