D1 - Intro to Sequences LESSON 2018

May 21, 2018 11:55 AM

MCR 3UI - U7 - D1 - Intro to Sequences LESSON 2018

MCR 3UI **Introduction to Sequences** **U7 D1**

A function can be used to generate a sequence of numbers :

Example: $f(x) = x^2$ generates

$f(1) =$ 1 $f(2) =$ 4 $f(3) =$ 9 $f(4) =$ 16

We have the sequence 1, 4, 9, 16 . . . . .

Thus a sequence is the set of numbers generated by a function, f(x), $x \in \mathbb{N}$
if x is restricted to the Natural Numbers.

N = { 1, 2, 3, 4, . . . } (counting numbers)

Each element in a sequence is referred to as a **TERM**. We use t with a **SUBSCRIPT** to indicate a specific **TERMS**.

i.e., $t_1 =$ 1 $t_2 =$ 4 $t_3 =$ 9 $t_4 =$ 16 . . . . .

Types of Sequences

1. **Finite Sequence**:
   A set of a limited number of numbers that follow a mathematical pattern

   e.g., 1, 4, 9, 16, 25 **has exactly five terms**

2. **Infinite Sequence**:
   A set of an unlimited number of numbers that follow a mathematical pattern

   e.g., 1, 4, 9, 16, 25, 36 . . . . an **unlimited number of terms**

In general, sequences can be generated using functions that utilize individual or combined mathematical operations, or even previous numbers in the sequence.

1. Arithmetic Sequences: **sets of numbers with a common difference generated from a linear function, f(n), with $n \in \mathbb{N}$**

E.g., $t_n = n + 6$

generates the sequence: 7, 8, 9, . . .

2. Geometric Sequences: **sets of numbers with a common ratio generated from an exponential function, f(n), with $n \in \mathbb{N}$.**

E.g., $t_n = -3^n$ not $(-3)^n$

generates the sequence: −3, −9, −27, . . .

3. Recursive Sequences: **sets of numbers generated by using previous numbers in the sequence.**

E.g., $t_{k+2} = t_k + t_{k+1}$, *where* $t_1 = 1$ *and* $t_2 = 1$

Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, ...

sub $k=1$

$t_{1+2} = t_1 + t_{1+1}$

$t_3 = t_1 + t_2$

$t_3 = 1 + 1$

$t_3 = 2$

$t_4 = t_2 + t_3$
$= 1 + 2$
$= 3$

$t_5 = t_3 + t_4$
$= 2 + 3$
$= 5$

Examples:

1. Write the first 3 terms for the following sequences:

a) $t_n = n^3 - 5$

$t_1 = (1)^3 - 5$

$t_1 = -4$

$t_2 = (2)^3 - 5$

$t_2 = 3$

$t_3 = (3)^3 - 5$

$t_3 = 22$

$\boxed{-4, 3, 22}$

b) $t_n = n^2 + 2n$

$t_1 = (1)^2 + 2(1)$

$t_1 = 3$

$t_2 = (2)^2 + 2(2)$

$t_2 = 8$

$t_3 = (3)^2 + 2(3)$

$t_3 = 15$

$\boxed{3, 8, 15}$

c) $t_k = t_{k-1} + k$, *where* $\boxed{t_1 = 5}$

$t_2 = t_1 + 2$
$= 5 + 2$
$t_2 = 7$

$t_3 = t_2 + 3$
$= 7 + 3$
$= 10$

$t_4 = t_3 + 4$
$= 10 + 4$
$= 14$

$\boxed{5, 7, 10, 14}$

* To find any term in this sequence, take the previous term and add the term number.

2. Write the general term for each of the following.

$n = 1\ 2\ 3\ 4$

a) $\underbrace{5, 6,}_{1}\underbrace{\ 7,}_{1}\underbrace{\ 8}_{1}$ . . . .

"slope value" → 1 1 1 ← linear because first diff. are the same

$t_n = n + 4$

$n\ 1\ 2\ 3\ 4$

b) 2, 5, 8, 11 . . .

3 3 3 ← linear

$t_n = 3n - 1$

$n\ 1\ 2\ 3\ 4$

c) 1,3, 9, 27, . . .

$3^0\ 3^1\ 3^2\ 3^3$

exponent is always 1 less than the term number

$t_n = 3^{n-1}$

$n = 1\ 2\ 3\ 4$

d) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ . . . .

1 1 1

$t_n = \frac{n}{n+1}$

$n: 1\ 2\ 3\ 4$

e) $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}$, ...

$\frac{2}{1}$

$1^2\ 2^2\ 3^2\ 4^2$

numerator values are 1 larger than the term number or $n+1$

denominator is term number squared. or n^2

$t_n = \frac{n+1}{n^2}$

$n: 1\ 2\ 3\ 4\ 5$

f) $2, \frac{15}{8}, \frac{7}{4}, \frac{13}{8}, \frac{3}{2}$, ...

★ try a common denominator

$\frac{16}{8}, \frac{15}{8}, \frac{14}{8}, \frac{13}{8}, \frac{12}{8}$ . . .

-1 -1 -1

$t_n = \frac{17-n}{8}$

or

$t_n = \frac{-n+17}{8}$

3 3 3 ← slope 3

g) 4, 7, 10, 13 . . .

n = 1 2 3 4

$t_n = 3n + 1$

n 1 2 3 4

h) -3, 0, 5, 12, ...

3 5 7

2 2 ← 2nd diff. are constant ∴ quadratic

n^2 1 4 9 16

$t_n = n^2 - 4$

1 2 3 4 5

i) 3, 9, 19, 33, 51, ...

6 10 14 18

4 4 4 ∴ quadratic

$2n^2$ 2 8 18 32

$a = \frac{4}{2}$

$a = 2$

$ax^2 + bx + c$

$t_n = 2n^2 + 1$