Page 452 # 1-7 (oee), 9, 12, 16 → for fun

1a) 1, 4, 9, 16

$4 \quad \frac{9}{4} \quad \frac{16}{9}$ ⟸ $r = \frac{t_{n+1}}{t_n}$ not same so not geometric.

Neither. ⟸ $d \neq t_{n+1} - t_n$ → not same so not arithmetic.

1c) 7, 14, 21, 28

$2 \quad \frac{3}{2} \quad \frac{4}{3}$ ⟸ $r = \frac{t_{n+1}}{t_n}$ not same so not geometric.

Next terms, 35, 42 $\quad d = 7$ → arithmetic

1e) 20, 16, 12

$d = -4 \quad -4$ → arithmetic

Next terms are 8, 4

1g) $\frac{11}{3}, \frac{10}{3}, \frac{8}{3}, \frac{5}{3}$

Neither

2a) 1, 3, 9, 27 $\quad r = 3$

$3 \quad 3 \quad 3$

Next terms are, 81, 243, 729

2c) 2, −8, 32, −128 $\quad r = -4$

$-4 \quad -4$

Next terms are 512, −2048, 8192

2e) 0.5, 5, 50, 500 $\quad r = 10$

Next terms are, 5000, 50000, 500000

2g) $64, 32, 16, 8$ $r = \frac{1}{2}$

Next three terms, $4, 2, 1$

3a) $a = 4, r = 3$

$t_n = 4(3^{n-1})$

Sequence $4, 12, 36, 108, 324$

3c) $a = 1024$ $r = 0.5$

$t_n = 1024(0.5^{n-1})$

Sequence $1024, 512, 256, 128, 64$

3e) $a = 8$ $r = -1$

$t_n = 8(-1)^{n-1}$

Sequence $8, -8, 8, -8, 8$

4a) $t_n = 4(2)^{n-1}$

Sequence $4, 8, 16, 32, 64$

4c) $t_n = 2(-2)^{n-1}$

Sequence $2, -4, 8, -16, 32$

4e) $t_n = -3(2)^{n-1}$

Sequence $-3, -6, -12, -24, -48$

4g) ~~t_n~~ $f(n) = 0.5(4)^{n-1}$

Sequence $0.5, 2, 8, 32, 128$

Page 452 #4, 5

4i) $f(n) = 200(0.5)^{n-1}$

Sequence: 200, 100, 50, 25, 12.5

4j) $f(n) = -1000(-0.1)^{n-1}$

Sequence: −1000, 100, −10, 1, −0.1

5a) 2, 4, 8... $a = 2, r = 2$ $t_n = 2(2)^{n-1}$

$r = \frac{4}{2} = 2$

$t_7 = 2(2)^{7-1}$

$t_7 = 2(2)^6$

$\boxed{t_7 = 128}$

$t_{12} = 2(2)^{12-1}$

$t_{12} = 2(2)^{11}$

$\boxed{t_{12} = 4096}$

5c) 4, 12, 36... $a = 4, r = 3$ $t_n = 4(3)^{n-1}$

$r = \frac{12}{4} = 3$

$t_8 = 4(3)^{8-1}$

$t_8 = 4(3)^7$

$\boxed{t_8 = 8748}$

$t_{10} = 4(3)^{10-1}$

$t_{10} = 4(3)^9$

$\boxed{t_{10} = 78732}$

5e) 6, 0.6, 0.06... $a = 6$ $r = 0.1$ $t_n = 6(0.1)^{n-1}$

$r = \frac{0.6}{6} = 0.1$ or $\frac{1}{10}$

$t_6 = 6(0.1)^{6-1}$

$= 6(0.1)^5$

$\boxed{t_6 = 6 \times 10^{-5}}$

$t_8 = 6(0.1)^{8-1}$

$= 6(0.1)^7$

$\boxed{t_8 = 6 \times 10^{-7}}$

Page 452 #5, 6a

5g) $729, -243, 81$ $\quad a = 729,\ r = -\frac{1}{3} \quad t_n = 729\left(-\frac{1}{3}\right)^{n-1}$

$r = \frac{-243}{729} = \frac{-1}{3}$

$t_6 = 729\left(-\frac{1}{3}\right)^{6-1}$

$= 729\left(-\frac{1}{3}\right)^5$

$= 729\left(-\frac{1}{243}\right)$

$= -\frac{\cancel{729}^{81}}{\cancel{243}_{27}}$

$\boxed{t_6 = -3}$

$t_{10} = 729\left(-\frac{1}{3}\right)^{10-1}$

$= 729\left(-\frac{1}{3}\right)^9$

$= 729\left(\frac{1}{-19683}\right)$

$\boxed{t_{10} = -\frac{1}{27}}$

6a) $4, 12, 36, \ldots, 2916$ $\quad a = 4,\ r = 3 \quad t_n = 4(3)^{n-1}$

$r = \frac{12}{4} = 3$

Let $t_n = 2196$

$2916 = 4(3)^{n-1}$

$\frac{\cancel{2916}^{729}}{\cancel{4}_1} = \frac{\cancel{4}(3)^{n-1}}{\cancel{4}}$

$729 = 3^{n-1}$

$3(729) = 3^n(3^{-1})(3)$

$2187 = 3^n$

$3^7 = 3^n$

$\boxed{7 = n}$

Page 452 #6

6c) $2, -4, 8, \ldots, -1024$ $\quad a=2 \quad r=-2 \quad t_n=$

$r = \frac{-4}{2} = -2$

Let $t_n = -1024$

$$-1024 = 2(-2)^{n-1}$$

$$\frac{-1024}{2} = \frac{2(-2)^{n-1}}{2}$$

$$-512 = (-2)^{n-1}$$

$$(-2)^9 = (-2)^{n-1}$$

$$9 = n-1$$

$$\boxed{10 = n}$$

6e) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{1024}$ $\quad a=\frac{1}{2} \quad r=\frac{1}{2} \quad t_n = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1}$

$\Downarrow$

$t_n = \left(\frac{1}{2}\right)^n$

$$r = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \left(\frac{1}{4}\right) \times \left(\frac{2}{1}\right) = \frac{1}{2}$$

Let $t_n = \frac{1}{1024}$

$$\frac{1}{1024} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^n$$

$$\boxed{10 = n}$$

6g) $\frac{2}{81}, \frac{4}{27}, \frac{8}{9}, \ldots \underline{6912}$ $a=\frac{2}{81}$ $r=6$ $t_n=\left(\frac{2}{81}\right)(6)$

$r=\frac{\left(\frac{4}{27}\right)}{\left(\frac{2}{81}\right)}$

$r=\frac{\cancel{4}^{2}}{\cancel{27}_{1}}\times\frac{\cancel{81}^{3}}{\cancel{2}_{1}}$

$r=6$

Let $t_n=6912$

$$6912=\left(\frac{2}{81}\right)(6)^{n-1}$$

$$6912\left(\frac{81}{2}\right)=6^{n-1}$$

$$279936=6^{n-1}$$

$$6^7=6^{n-1}$$

$$7=n-1$$

$$\boxed{8=n}$$

7a) General Term $t_n=ar^{n-1}$

Since $t_3=36$ $36=ar^2 \Rightarrow ① \; a=\frac{36}{r^2}$

Since $t_4=108$ ② $108=ar^3$

Sub ① into ②

$$108=\left(\frac{36}{r^2}\right)r^3$$

$$108=36r$$

$$\boxed{3=r}$$

Sub $r=3$ into ① $a=\frac{36}{(3)^2}=4$

$\therefore t_n=4(3)^{n-1}$

Page 452 #7c, e

7c) $t_4 = 64$, $t_5 = 32$

$$r = \frac{t_5}{t_4}$$

$$r = \frac{32}{64}$$

$$r = \frac{1}{2}$$

Since $t_4 = 64$ and $t_n = a\left(\frac{1}{2}\right)^{n-1}$

$$\therefore \quad t_4 = a\left(\frac{1}{2}\right)^3$$

$$64 = a\left(\frac{1}{2}\right)^3$$

$$64 = a\left(\frac{1}{8}\right)$$

$$64(8) = a$$

$$\boxed{512 = a}$$

$$\therefore \quad t_n = 512\left(\frac{1}{2}\right)^{n-1}$$

7e) $t_5 = 80$ $\quad t_7 = 320$

General Term: $t_n = ar^{n-1}$

$t_5 = ar^4$ $\qquad t_7 = ar^6$

$80 = ar^4$ $\qquad$ ② $320 = ar^6$

Solve for a.

① $a = \frac{80}{r^4}$

Sub ① into ② $\Longrightarrow$ $320 = \left(\frac{80}{r^4}\right) r^6$

$$4 = r^2$$

$$\boxed{\pm 2 = r}$$

two choices for r but it makes no difference for cal'c of a since

$$a = \frac{80}{r^4} \quad \text{if } r = \pm 2 \Rightarrow r^4 = 16$$

$$a = \frac{80}{(-2)^4}$$

$$\boxed{a = 5}$$

$\therefore$ $t_n = 5(-2)^{n-1}$ $\quad t_n = 5(2)^{n-1}$

$r = 1 - \frac{1}{5} \Rightarrow r = \frac{4}{5}$

9a) 20000cm^3, 16000cm^3, 12800cm^3, 10240cm^3, 8192cm^3

$\leftarrow 20000 \times \frac{4}{5}$

b) $r = \frac{4}{5}$ $\Leftarrow$ since $\frac{4}{5}$ of every day is left over.

c) $t_n = 20000\left(\frac{4}{5}\right)^{n-1}$ $\quad t_7 = 20000\left(\frac{4}{5}\right)^{7-1}$

$t_6 = 20000\left(\frac{4}{5}\right)^{6-1}$

$\boxed{t_6 = 6553.6\text{cm}^3}$ $\quad \boxed{t_7 = 5242.9\text{cm}^3}$

$\therefore$ There will be 6553.6cm^3 after 6 days

" " " 5242.9cm^3 " 7 days.

12 $a = 100$ $\quad r = 0.64$ $\quad t_n = 100(0.64)^{n-1}$

Need to know when $t_n < 10$

Trial and error

$$10 = 100(0.64)^{n-1}$$

$$\frac{10}{100} = \frac{100(0.64)^{n-1}}{100}$$

$$\frac{1}{10} = (0.64)^{n-1}$$

after 6 copies!

16. a) Let be ___ to start with.

$a = 100 \quad r = \frac{1}{2}$

$t_n = 100\left(\frac{1}{2}\right)^{n-1}$

b) Let $a = 80 \quad r = \frac{1}{2}$ in $t_n = 80\left(\frac{1}{2}\right)^{n-1}$

$t_{10} = 80\left(\frac{1}{2}\right)^{10-1}$

$= 80\left(\frac{1}{2}\right)^9$

$= 80\left(\frac{1}{512}\right)$

$= \frac{\cancel{80}^{20}}{\cancel{512}_{128}}$

$= \frac{20}{128}$

$= \frac{10}{64}$

$\boxed{t_{10} = \frac{5}{32}}$

After 10 minutes $\frac{5}{32}$ g left over.