7 Annuities and Mortgages

What You'll Learn

To use technology to solve problems involving annuities and mortgages and to gather and interpret information about annuities and mortgages

And Why

Annuities are used to save and pay for expenses such as a car, a home, education, and retirement. Understanding the mathematics of annuities and mortgages will help you manage your money more effectively so that you can achieve your financial goals.

Key Words

- simple interest
- compound interest
- annuity
- ordinary simple annuity
- amount of an annuity
- present value of an annuity
- mortgage
- amortization period
- amortization table

CHAPTER 7

Activate Prior Knowledge

Simple and Compound Interest

Prior Knowledge for 7.1

Simple interest is money earned on a starting principal.

- $I = Prt$, where I dollars is the interest earned, P dollars is the principal, r is the annual interest rate, and t is the time in years.

Compound interest is money earned on both the principal and previous interest.

- In compound interest problems, $A = P(1 + i)^n$, where A dollars is the amount, P dollars is the principal, i is the interest rate per compounding period as a decimal, and n is the number of compounding periods.

Example

Materials

- scientific calculator

The quarterly interest rate is $\frac{1}{4}$ of the annual interest rate.

Tezer invests $3000 for 3 years in a bond that earns 6% per year compounded quarterly. How much interest does the bond earn?

Solution

Substitute $P = 3000$, $i = \frac{0.06}{4} = 0.015$, and $n = 3 \times 4 = 12$ in the formula:

$A = P(1 + i)^n$

$A = 3000(1 + 0.015)^{12} \doteq 3586.85$

Press: 3000 (1 + 0.015) ^ 12 ENTER

The amount after 3 years is $3586.85.

To determine the interest earned, subtract the principal from the amount.

$3586.85 - 3000 = 586.85$

The bond earns $586.85 in interest.

CHECK ✓

1. Vishnu borrows $4500 for 3 months at an annual rate of 7.5%. What amount will Vishnu repay at the end of the 3 months?

2. Nancy invests $10 000 at 5% per year compounded semi-annually. Determine the amount after 3 years.

Semi-annually means 2 times a year.

3. Refer to question 2. Will the amount double in each case?

a) The principal invested is twice as great, $20 000.

b) The interest rate is twice as great, 10%.

c) The term is twice as long, 6 years.

Justify your answers.

Present Value

The principal that must be invested today to obtain a given amount in the future is the **present value** of the amount.
To calculate the present value in a situation involving compound interest, rearrange the formula $A = P(1 + i)^n$ to solve for P.

Example

Materials
- scientific calculator

What principal should Yvette invest today at 4.6% per year compounded semi-annually to have $6500 three years from now?

Solution

The semi-annual interest rate is $\frac{1}{2}$ of the annual interest rate,
so $i = \frac{0.046}{2} = 0.023$.
Interest is compounded 2 times a year for 3 years,
so $n = 3 \times 2 = 6$.
Isolate P, then substitute $A = 6500$, $i = 0.023$, and $n = 6$ in the formula:

You could also substitute, then solve for P.

$$A = P(1 + i)^n$$

$$\frac{A}{(1 + i)^n} = P$$

Since $\frac{1}{x} = x^{-1}$, $\frac{1}{(1 + i)^n} = (1 + i)^{-n}$

$$P = A(1 + i)^{-n}$$

$$P = 6500(1 + 0.023)^{-6}$$

$$\doteq 5671.00$$

Press: 6500 [×] [(] 1 [+] 0.023 [)] [^] [(–)] 6 [ENTER]

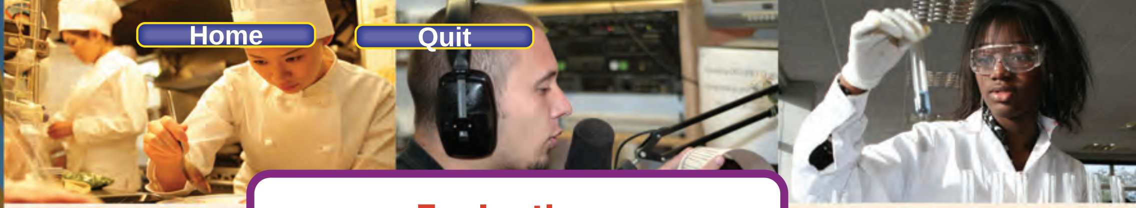
Yvette should invest $5671.00.

CHECK ✓

1. Jeremy plans to go on a cruise 4 years from now. He will need $7500 at that time. What principal should Jeremy invest now at 8.4% per year compounded monthly to obtain the required amount?

2. Rami invested money at 3.6% per year compounded semi-annually. He received $12 387.21 at the end of a 6-year term. How much interest did Rami earn?

3. Suppose you are solving a problem involving compound interest. How do you know whether to determine the amount or present value of a given sum of money? Explain.



Evaluations

Evaluations tell you and others how well you have learned and performed at school or in the workplace. They can be used to determine whether a person has the skills needed for an opportunity, such as acceptance to a college or apprenticeship program, or to assess how well a person is doing in a course or on the job.

The Workplace

In the workplace, there are many evaluation methods. For example, some employers use performance appraisals to evaluate employees' progress. A supervisor keeps a detailed record of how the employee performs tasks at work and writes an appraisal or report. Then, the supervisor and employee meet to discuss the report. Learn more about workplace evaluation.

1. Choose several jobs such as a factory worker, police officer, hairdresser in a large salon, or bank clerk. Interview people in these fields and use the Internet to answer these questions about the jobs.

- How are the employees evaluated?
- How often are they evaluated?
- What are the criteria for evaluation?
- What are the rewards for a good evaluation? What happens to those who get negative comments on their work?

College

Most college math courses have quizzes, tests, and projects. Quizzes and tests are often multiple choice. Although you may need to complete several steps to figure out your answer, in most cases, there are no part marks. Making a study sheet can help you prepare for these evaluations.

2. Create a study sheet for this chapter.

- Make it fit on one side of a sheet of paper.
- You might use the computer to create symbols and diagrams.
- Use graphic organizers such as a Venn diagram or Frayer model.
- Include notes and examples so that your study guide can help you. Refer to the instructions for *Collecting Important Ideas* in Chapter 1.
- Don't just copy from the text!

Home Quit

7.1 The Amount of an Annuity

Hiroshi plans to buy a motorcycle in 5 years. He saves for the down payment by making regular deposits into his investment account.

Investigate

Materials
- scientific calculator

Determining the Accumulated Value of Regular Deposits

Work with a partner.

Suppose Hiroshi deposits $1000 at the end of each year for 5 years. His account earns at 6% per year compounded annually. How much has Hiroshi saved at the end of 5 years?

- Determine the balance in the account at the end of each year. Organize your calculations in a table like the one below.

The interest is compounded before the deposit is made.

Year	Starting balance	Interest earned (6%)	Deposit	Ending balance
1	$0.00	$0.00	$1000.00	$1000.00
2	$1000.00	$60.00	$1000.00	$2060.00

Reflect

- Why does the interest earned increase each year?
- What is an advantage and a disadvantage of using a table to determine Hiroshi's savings after 5 years?

Connect the Ideas

Ordinary simple annuities

An **annuity** is a series of equal payments made at regular intervals. In an **ordinary simple annuity**, payments are made at the end of each compounding period. The **amount of an annuity** is the sum of the regular deposits plus interest.

Example 1 — Using a Table

Materials
- scientific calculator

Suppose \$450 is deposited at the end of each quarter for 1.5 years in an investment account that earns 10% per year compounded quarterly.

a) What is the amount of the annuity?

b) How much interest does the annuity earn?

This annuity is an ordinary simple annuity because a deposit is made at the end of each quarter and the interest is compounded quarterly.

Solution

The annual interest rate is 10%, so the quarterly rate is:

$\frac{10\%}{4} = 2.5\%$

The number of quarters in 1.5 years is:

$1.5 \times 4 = 6$

Use a table to organize the calculations.

Interest earned = Starting balance × 0.025

Ending balance = Starting balance + Interest earned + Deposit

Quarter	Starting balance	Interest earned (2.5%)	Deposit	Ending balance
1	\$0.00	\$0.00	\$450.00	\$450.00
2	\$450.00	\$11.25	\$450.00	\$911.25
3	\$911.25	\$22.78	\$450.00	\$1384.03
4	\$1384.03	\$34.60	\$450.00	\$1868.63
5	\$1868.63	\$46.72	\$450.00	\$2365.35
6	\$2365.35	\$59.13	\$450.00	\$2874.48
	Total	\$174.48	\$2700.00	

a) The amount of the annuity is \$2874.48.

b) The interest earned is \$174.48.

It is time-consuming to create a table to determine the amount of an annuity. A simpler alternative is to use a formula.

Amount of an ordinary simple annuity

$A = \dfrac{R[(1+i)^n - 1]}{i}$, where:

- A is the amount in dollars
- R is the regular payment in dollars
- i is the interest rate per compounding period as a decimal
- n is the number of compounding periods

The amount formula can only be used when:

- The payment interval is the same as the compounding period.
- A payment is made at the end of each compounding period.
- The first payment is made at the end of the first compounding period.

Example 2 Using the Amount Formula

Materials

- scientific calculator

In the annuity in *Example 1*, \$450 is deposited at the end of each quarter for 1.5 years at 10% per year compounded quarterly.

a) What is the amount of the annuity?

b) How much interest does the annuity earn?

Solution

a) The regular payment is \$450, so $R = 450$.

$i = \frac{0.10}{4} = 0.025$; $n = 1.5 \times 4 = 6$

Substitute $R = 450$, $i = 0.025$, and $n = 6$ into the amount formula.

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$A = \frac{450[(1+0.025)^6 - 1]}{0.025}$$

$$\doteq 2874.48$$

Press: 450 ((1 + 0.025) ^ 6 − 1) ÷ 0.025 ENTER

The amount is \$2874.48.

b) The amount, \$2874.48, is the total of the deposits, plus interest.

6 deposits of \$450: $6 \times \$450 = \$2700$

So, the interest earned is: $\$2874.48 - \$2700 = \$174.48$

Annity calculations can also be performed on a graphing calculator. On a TI-83 or TI-84 graphing calculator, we can use a financial application called the TVM (Time Value of Money) Solver.

- Set the calculator to 2 decimal places.
  Press: [MODE] [▼] [▶] [▶] [▶] [ENTER]
- Open the TVM Solver by pressing: [APPS] 1 1
  The variables represent the following quantities.

N	Total number of payments
I%	Annual interest rate as a percent
PV	Principal or present value
PMT	Regular payment
FV	Amount or future value
P/Y	Number of payments per year
C/Y	Number of compounding periods per year
PMT:	Indicates whether payments are made at the beginning or end of the payment period

- The calculator displays either positive or negative values for PV, PMT, and FV. Negative values indicate that money is *paid out*, while positive values mean that money is *received*.
- In annuity calculations, only one of the amount (FV) or present value (PV) is used. Enter 0 for the variable not used.

Example 3 — Using the TVM Solver

Materials

- TI-83 or TI-84 graphing calculator

In the annuity in *Example 1*, \$450 is deposited at the end of each quarter for 1.5 years at 10% per year compounded quarterly.

a) What is the amount of the annuity?

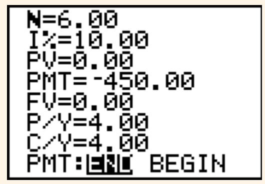
b) How much interest does the annuity earn?

Solution

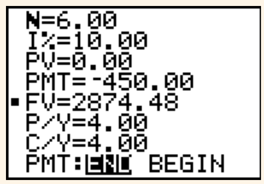
a)

To enter −450, press the negative key [(−)], not the subtract key [−].

Enter the known values.
Set PMT to END because payments are made at the end of each period in an ordinary simple annuity.



Solve for the amount.
Move the cursor to FV.
Press: [ALPHA] [ENTER]

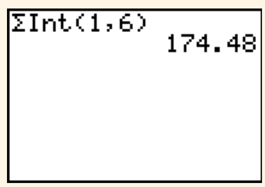


The amount after 1.5 years is \$2874.48.

b)

ΣInt(1,6) means the sum of the interest for payments 1 to 6.

Press [2nd] [MODE] to exit the TVM Solver. Use the ΣInt command to determine the total interest earned.
Press [APPS] 1 [ALPHA] [MATH] to show ΣInt(.
Press: 1 [,] 6 [)] [ENTER]



The interest earned is \$174.48.

Annuities and regular savings

Annuities are often used to save money for expenses such as a car, a down payment on a house, or a vacation. They are also used to save for education and retirement. Relatively small, regular deposits of money can accumulate to large sums of money over time.

Example 4

Materials
- TI-83 or TI-84 graphing calculator

Comparing Retirement Plans

Tom and Beth are twins. They save for retirement as follows.

- Starting at age 25, Tom deposits $1000 at the end of each year for 40 years.
- Starting at age 45, Beth deposits $2000 at the end of each year for 20 years.

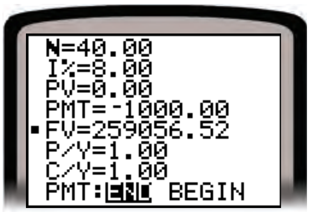
Suppose each annuity earns 8% per year compounded annually. Who will have the greater amount at retirement?

Solution

Determine the amount of each annuity.

Tom's retirement fund

- Enter: N = 40, I% = 8, PV = 0, PMT = –1000, FV = 0, P/Y = 1, and C/Y = 1
- Move the cursor to FV.
  Press: ALPHA ENTER



Tom will have $259 056.52.

Beth's retirement fund

- Enter: N = 20, I% = 8, PV = 0, PMT = –2000, FV = 0, P/Y = 1, and C/Y = 1
- Move the cursor to FV.
  Press: ALPHA ENTER

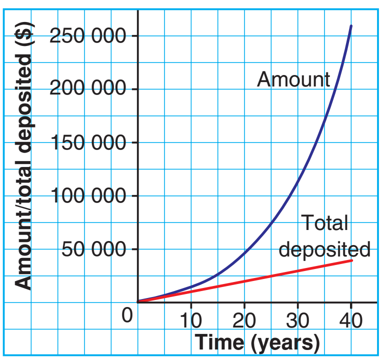
Beth will have $91 523.93.

Tom will have the greater amount saved at retirement.

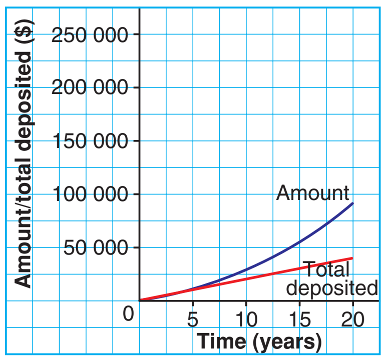
We could have used the annuity formula instead of the TVM Solver.

Example 4 illustrates the power of time on the value of money and the advantage of starting to save early. This advantage is also illustrated in these graphs.

Tom's Retirement Fund



Beth's Retirement Fund



Both Tom and Beth deposit a total of $40 000. But, by starting earlier, Tom earns an additional $167 532.59 in interest.

Practice

A

- For help with question 1, see Example 1.

1. Complete each table. What is the amount of each annuity?

a) $1000 deposited at the end of each year at 8% per year compounded annually

Year	Starting balance	Interest earned	Deposit	Ending balance
1	$0.00	$0.00	$1000.00	$1000.00
2	$1000.00	$80.00	$1000.00	$2080.00
3			$1000.00	
4			$1000.00	

b) $100 deposited at the end of each month at 6% per year compounded monthly

Month	Starting balance	Interest earned	Deposit	Ending balance
1	$0.00	$0.00	$100.00	$100.00
2	$100.00	$0.50	$100.00	$200.50
3			$100.00	
4			$100.00	

- For help with questions 2 and 3, see Example 2.

2. Determine i, the interest rate per compounding period as a decimal, and n, the number of compounding periods for each annuity.

	Time of payment	Length of annuity	Interest rate per year	Frequency of compounding
a)	end of each year	7 years	3%	annually
b)	end of every 6 months	12 years	9%	semi-annually
c)	end of each quarter	8 years	2.4%	quarterly
d)	end of each month	5 years	18%	monthly

3. Use the formula: $A = \dfrac{R[(1 + i)^n - 1]}{i}$

Calculate A for each set of values.

a) $R = \$200, i = 0.05, n = 3$

b) $R = \$1000, i = 0.08, n = 7$

c) $R = \$700, i = 0.02, n = 12$

■ For help with questions 4 and 5, see Example 3.

4. For each annuity in question 2, what values would you enter into the TVM Solver for N, I%, P/Y, and C/Y?

5. Bill entered these values into the TVM Solver to determine the amount of an annuity.

a) What is the regular payment?
b) How often are the regular payments made?
c) How many payments are made?
d) What is the annual interest rate?
e) How often is the interest compounded?
f) What is the amount of the annuity?

B

6. Determine the amount of each ordinary simple annuity.
a) \$3000 deposited every year for 10 years at 7% per year compounded annually
b) \$650 deposited every 6 months for 8 years at 9% per year compounded semi-annually
c) \$1450 deposited every quarter for 9 years at 6.25% per year compounded quarterly
d) \$375 deposited every month for 6 years at 5.9% per year compounded monthly

7. Determine the interest earned by each annuity in question 6.

8. Use a different method to verify your answers to questions 6 and 7. Which method do you prefer? Explain.

9. Shen Wei wants to save \$10 000 for his first year of college. He deposits \$300 at the end of each month in an account that earns 5.6% per year compounded monthly. Will Shen Wei have enough money saved at the end of 2.5 years? Justify your answer.

10. Geneva's parents saved for her college education by depositing $1200 at the end of each year in a *Registered Education Savings Plan* (RESP) that earns 6% per year compounded annually.

a) What is the amount of the RESP at the end of 18 years?

b) How much interest is earned?

c) How much extra interest would have been earned at an interest rate of 7% per year compounded annually?

11. Verena is saving for a new computer. She deposits $100 at the end of each month into an account that earns 4% per year compounded monthly.

a) Determine the amount in the account after 3 years.

b) Does the amount in part a double with each of these changes?

i) The deposits are twice as great, $200.

ii) The interest rate is twice as great, 8%.

iii) The time period is twice as long, 6 years.

Justify your answers.

c) Which scenario in part b produced the greatest amount? Explain.

For help with question 12, see Example 4.

12. Jackson and Abina save money for retirement.

Investment Plan	Investment Plan
Name: Jackson	Name: Abina
Monthly investment: $40	Monthly investment: $80
Start: Now	Start: 15 years from now
Time period: 30 years	Time period: 15 years
Annual interest rate: 6%	Annual interest rate: 6%
Compounding period: monthly	Compounding period: monthly

a) Compare the amount of each annuity with the total investment.

b) Determine the interest earned by each annuity.

c) Use the results of parts a and b to explain why financial planners recommend saving for retirement from an early age.

13. **Assessment Focus** Consider these two annuities.

Annuity 1: $100 deposited at the end of each month for 5 years at 4% per year compounded monthly

Annuity 2: $300 deposited at the end of each quarter for 5 years at 4% per year compounded quarterly

a) Calculate the total deposit and the amount of each annuity.

b) Why are the amounts different even though the total deposit is the same?

14. Consider an annuity of $1000 deposited at the end of each year for 5 years at 3.5% per year compounded annually.

a) Predict which of the following changes to the annuity would produce the greatest amount.

i) Doubling the regular deposit

ii) Doubling the interest rate

iii) Doubling the time period

iv) Doubling the frequency of the deposit and the compounding

b) Calculate the total deposit and the amount of each annuity. Compare your results. Was your prediction correct?

15. **Literacy in Math** Use a graphic organizer to explain how to determine the amount of an annuity. Include an example in your explanation.

C **16.** Suppose you deposit $250 at the end of every 6 months in an investment account that earns 8% per year compounded semi-annually.

a) Make a graph to illustrate the growth in the amount over a 30-year period.

b) Gareth says the growth appears to be exponential. Is he correct? Justify your answer.

17. Kishore and Giselle save for their retirement in an investment account that earns 10% per year compounded annually.

- Kishore starts saving at age 20. He invests $2000 at the end of each year for 10 years. Then he leaves the money to earn interest for the next 35 years.
- Giselle starts saving at age 35. She invests $2000 at the end of each year for 30 years.

a) Who do you think will have the greater amount at retirement? Explain.

b) Calculate the total investment and the amount of each annuity. Compare your results.

c) Are you surprised by the results? Explain.

In Your Own Words

Many young people delay saving for retirement because they think they can make up the difference by investing more money later. Explain the benefits of saving early. Use examples to illustrate your explanation.

Home Quit

7.2 The Present Value of an Annuity

Lottery winners are often given the choice of receiving their winnings over time in an annuity or as an immediate cash payment.

Investigate

Calculating the Cash Payment for Winning a Lottery

Materials
- TI-83 or TI-84 graphing calculator

Work with a partner.

Top prize winners of the PayDay lottery in British Columbia can receive their winnings as an annuity of $2000 every 2 weeks for 20 years.

Bi-weekly means every 2 weeks.

What cash payment received today is equivalent to receiving $2000 every 2 weeks for 20 years? Assume money can be invested at an annual interest rate of 5.6% compounded bi-weekly.

Reflect

- Compare your strategies with another group.
- The actual cash payment for the PayDay lottery is $625 000. Compare this value to the one you calculated. What might account for the difference?

Connect the Ideas

Present value of an annuity

The **present value of an annuity** is the principal that must be invested today to provide the regular payments of an annuity.

Present value of an ordinary simple annuity

$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$, where:

- PV is the present value in dollars
- R is the regular payment in dollars
- i is the interest rate per compounding period, as a decimal
- n is the number of compounding periods

The present value formula can only be used when:

- The payment interval is the same as the compounding period.
- A payment is made at the end of each compounding period.
- The first payment is made at the end of the first compounding period.

Example 1 — Providing for an Annuity

Materials

- scientific calculator
- TI-83 or TI-84 graphing calculator

Victor wants to withdraw $700 at the end of each month for 8 months, starting 1 month from now. His bank account earns 5.4% per year compounded monthly. How much must Victor deposit in his bank account today to pay for the withdrawals?

Solution

The principal that Victor must deposit today is the present value of an annuity of $700 per month for 8 months at 5.4% per year compounded monthly.

Method 1: Use the present value formula

Substitute $R = 700$, $i = \frac{0.054}{12} = 0.0045$, and $n = 8$ into the present value formula.

$$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$$

$$PV = \frac{700[1 - (1 + 0.0045)^{-8}]}{0.0045}$$

$$\doteq 5488.28$$

Press: 700 (1 − (1 + 0.0045) ^ (−) 8) ÷ 0.0045 ENTER

Victor must deposit $5488.28.

Method 2: Use the TVM Solver

- Enter: N = 8, I% = 5.4, PV = 0, PMT = −700, FV = 0, P/Y = 12, and C/Y = 12
- Move the cursor to PV.
  Press: ALPHA ENTER

```
N=8.00
I%=5.40
▪PV=5488.28
PMT=-700.00
FV=0.00
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
```

Victor must deposit $5488.28.

Example 2 Calculating the Amount Needed at Retirement

Materials

- scientific calculator
- TI-83 or TI-84 graphing calculator

Payments every 3 months are quarterly payments.

Azadeh plans to retire at age 60. She would like to have enough money saved in her retirement account so she can withdraw $7500 every 3 months for 25 years, starting 3 months after she retires. How much must Azadeh deposit at retirement at 9% per year compounded quarterly to provide for the annuity?

Solution

The principal that Azadeh must deposit at retirement is the present value of the annuity payments.

Method 1: Use the present value formula

Substitute $R = 7500$, $i = \frac{0.09}{4} = 0.0225$, and $n = 25 \times 4 = 100$ into the present value formula.

$$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$$

$$PV = \frac{7500[1 - (1 + 0.0225)^{-100}]}{0.0225}$$

$$\doteq 297\ 313.05$$

Press: 7500 (1 − (1 + 0.0225) ^ (−) 100) ÷ 0.0225 ENTER

Azadeh needs $297 313.05 at retirement to pay for the annuity.

Method 2: Use the TVM Solver

- Enter: N = 25 × 4, I% = 9, PV = 0, PMT = −7500, FV = 0, P/Y = 4, and C/Y = 4
- Move the cursor to PV.
  Press: ALPHA ENTER

```
N=100.00
I%=9.00
▪PV=297313.05
PMT=-7500.00
FV=0.00
P/Y=4.00
C/Y=4.00
PMT:END BEGIN
```

Azadeh needs $297 313.05 at retirement to pay for the annuity.

Repaying loans

Most loans are repaid by making equal monthly payments over a fixed period of time. These payments form an annuity whose present value is the principal borrowed. When all of the payments are made, both the principal borrowed and the interest due will have been paid.

Example 3 — Calculating the Principal Borrowed for a Loan

Materials

- TI-83 or TI-84 graphing calculator

Seema plans to buy a used car. She can afford monthly car loan payments of $300. The car dealer offers Seema a loan at 6.9% per year compounded monthly, for 3 years. The first payment will be made 1 month from the date she buys the car.

a) How much can Seema afford to borrow?

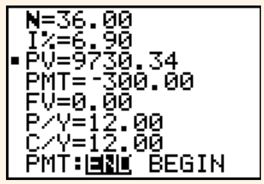
b) How much interest will Seema pay on the loan?

Solution

We could have used the formula for the present value of an annuity instead of the TVM Solver.

a) Use the TVM Solver to determine the present value of a loan with $300 monthly payments.

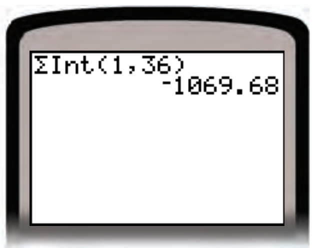
- Enter: $N = 3 \times 12$, $I\% = 6.9$, $PV = 0$, $PMT = -300$, $FV = 0$, $P/Y = 12$ and $C/Y = 12$
- Move the cursor to PV.
  Press: [ALPHA] [ENTER]



Seema can afford to borrow $9730.34.

b) Seema pays a total of $36 \times \$300 = \$10\,800$.
The original loan is $9730.34.
So, the interest paid is $\$10\,800 - \$9730.34 = \$1069.66$.
This result can be verified with the TVM Solver.

The 2¢ difference in answers is due to rounding.



Press [2nd] [MODE] to exit the TVM Solver. To determine the interest earned, press: [APPS] 1 [ALPHA] [MATH] 1 [,] 36 [)] [ENTER]

Practice

A

For help with questions 1 to 5, see Examples 1 and 2.

1. Evaluate each expression. Write each answer to 2 decimal places.

a) $\dfrac{45[1-(1+0.02)^{-24}]}{0.02}$

b) $\dfrac{575[1-(1+0.003)^{-48}]}{0.003}$

c) $\dfrac{2000[1-(1+0.0065)^{-14}]}{0.0065}$

d) $\dfrac{95[1-(1+0.12)^{-8}]}{0.12}$

2. Use the formula: $PV = \dfrac{R[1-(1+i)^{-n}]}{i}$
Calculate the value of PV for each set of values.

a) $R = \$200$, $i = 0.05$, $n = 3$

b) $R = \$1000$, $i = 0.08$, $n = 7$

c) $R = \$750$, $i = 0.02$, $n = 12$

3. Maeve wants to set up an annuity to help with her college expenses. She uses the TVM Solver to explore a possible plan.

a) What regular withdrawal does Maeve plan to make?

b) How often will she make these withdrawals?

c) What is the total number of withdrawals Maeve will make?

d) How much will Maeve have to deposit to provide for the withdrawals?

4. Use the present value formula to determine the present value of each annuity.

	Payment	Interest rate	Frequency of compounding	Length of annuity
a)	\$300	12%	monthly	2 years
b)	\$4500	4%	annually	6 years
c)	\$900	9%	semi-annually	4 years
d)	\$800	8%	quarterly	5 years

5. Use the TVM Solver to verify your answers to question 4.

B

6. Determine the present value of each ordinary simple annuity.

a) Payments of $75 for 10 years at 9.6% per year compounded annually

b) Payments of $240 for 15 years at 7.25% per year compounded semi-annually

c) Payments of $8500 for 25 years at 6.3% per year compounded annually

d) Payments of $50 for 4.5 years at 4.8% per year compounded quarterly

7. Determine the interest earned by each annuity in question 6.

8. A contest offers a prize of $1000 every month for 1 year. The first payment will be made 1 month from now. If money can be invested at 8% per year compounded monthly, what cash payment received immediately is equivalent to the annuity?

9. Tam is setting up an income fund for her retirement. She wishes to receive $1500 every month for the next 20 years, starting 1 month from now. The income fund pays 6.25% per year compounded monthly. How much must Tam deposit now to pay for the annuity?

10. **Assessment Focus** Isabel receives a disability settlement. She must choose one of these payment plans.

- A single cash payment of $80 000 to be received immediately
- Monthly disability payments of $1200 for 10 years

Assume that money can be invested at 4.8% per year compounded monthly. Which settlement do you think Isabel should accept? Justify your answer.

11. Terence's parents want to set up an annuity to help him with his college expenses. The annuity will allow Terence to withdraw $300 every month for 4 years. The first withdrawal will be 1 month from now. The annuity earns 3.5% per year compounded monthly.

a) What principal should Terence's parents invest now to pay for the annuity?

b) In which of these scenarios will Terence's parents deposit the least principal?

i) Terence's withdrawals are twice as great, $600.

ii) The interest rate is twice as great, 7%.

iii) The time period is twice as long, 8 years.

Justify your answers.

For help with question 12, see Example 3.

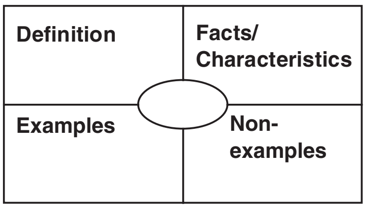
12. Jeongsoo borrows money to buy a computer. She will repay the loan by making monthly payments of $112.78 per month for the next 2 years at an interest rate of 7.75% per year compounded monthly.

a) How much did Jeongsoo borrow?

b) How much interest does Jeongsoo pay?

13. Angela's annuity pays $600 per month for 5 years at 9% per year compounded monthly. Becky's annuity pays $300 per month for 10 years at 9% per year compounded monthly.
The total of the regular payments is the same for each annuity.
Do both annuities have the same present value? Justify your answer.

14. **Literacy in Math** Create Frayer models for the amount and present value of an annuity.



C

15. Piers wins a talent contest. His prize is an annuity that pays $1000 at the end of each month for 2 years, and then $500 at the end of each month for the next 3 years. How much must the contest organizers deposit in a bank account today to provide the annuity? Assume that money can be invested at 8% per year compounded monthly.

16. Chloe borrowed money from the bank to renovate her home. She will repay the loan by making 24 monthly payments of $64.17 at 12.5% per year compounded monthly.

a) How much did Chloe borrow?

b) How much would it cost Chloe to pay off the loan after the 12th payment?

c) How much interest does Chloe save by paying off the loan early?

In Your Own Words

How are problems involving the present value of an annuity similar to problems involving the amount of an annuity? How are they different? Include examples in your explanation.

7.3 The Regular Payment of an Annuity

Many financial experts caution that young people today cannot count on government or employer pension plans to provide a comfortable retirement. They recommend that young people plan for their retirement by saving early and regularly.

Investigate

Million Dollar Retirement

Materials

- TI-83 or TI-84 graphing calculator

Work with a partner or in a small group.

Some financial experts suggest that a comfortable retirement requires savings of $1 000 000.

- What monthly payment would you have to make at ages 20, 30, 40, 50, or 60 to accumulate a $1 000 000 retirement fund at age 65? Assume that the fund earns 9% per year compounded monthly.

  You may want to organize your work in a table like this.

Age	Years until retirement	Number of monthly payments	Monthly payment
20			

- Suppose you have $1 000 000 saved in a retirement fund. What regular withdrawal can you make from the fund at the end of each year for 25 years if the fund earns 8% per year compounded annually?

Reflect

- How does the monthly payment to accumulate $1 000 000 change as the years to retirement decrease?
- At what age do you think it becomes unrealistic to expect to save $1 000 000 for retirement? Explain.
- What would you consider a “comfortable retirement”? What annual income do you think you will need when you retire to have a comfortable retirement?

Connect the Ideas

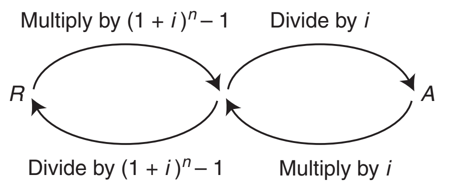
In Lessons 7.1 and 7.2, we used two different formulas to solve problems involving annuities.

- We used the amount formula to determine the accumulated value of the regular payments at the *end* of an annuity.
- We used the present value formula to determine the money needed at the *beginning* of an annuity to provide regular annuity payments.

When we know the amount or the present value, we can solve for the regular payment. To do this, we rearrange the appropriate formula to isolate *R*.

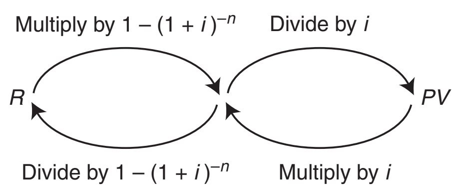
Amount formula

$$A = \frac{R[(1 + i)^n - 1]}{i}$$



Present value formula

$$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$$



Example 1 — Determining Payments Given the Amount

Materials
- scientific calculator

Payments every 6 months are semi-annual payments.

Brianne wants to save $6000 for a trip she plans to take in 5 years. What regular deposit should she make at the end of every 6 months in an account that earns 6% per year compounded semi-annually?

Solution

The $6000 represents the money to be accumulated by the regular deposits. So, the $6000 is the amount of the annuity.

We could also isolate *R*, then substitute.

Substitute, then solve for *R*.
Substitute $A = 6000$, $i = \frac{0.06}{2} = 0.03$, and $n = 5 \times 2 = 10$ into the amount formula.

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$6000 = \frac{R[(1+0.03)^{10} - 1]}{0.03}$$ Multiply each side by 0.03.

$$6000 \times 0.03 = R(1.03^{10} - 1)$$ Divide each side by $1.03^{10} - 1$

$$\frac{6000 \times 0.03}{1.03^{10} - 1} = R$$

$$R \doteq 523.38$$

Press: (6000 × 0.03) ÷ (1.03 ^ 10 − 1) ENTER

Brianne will have to make semi-annual deposits of $523.38.

Example 2 — Determining Payments Given the Present Value

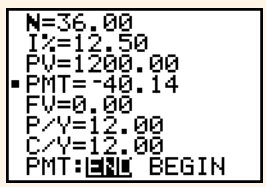
Materials
- TI-83 or TI-84 graphing calculator

Donald borrows $1200 from an electronics store to buy a computer. He will repay the loan in equal monthly payments over 3 years, starting 1 month from now. He is charged interest at 12.5% per year compounded monthly. How much is Donald's monthly payment?

Solution

The equal monthly payments Donald makes form an annuity whose present value is $1200.
Use the TVM Solver to determine his monthly payment.

- Enter: N = 3 × 12, I% = 12.5, PV = 1200, PMT = 0, FV = 0, P/Y = 12, and C/Y = 12
- Move the cursor to PMT.
  Press: ALPHA ENTER



Donald's monthly payment is $40.14.

Example 3 Choosing between Two Loan Options

Materials
- TI-83 or TI-84 graphing calculator

Sheri borrows $9500 to buy a car. She can repay her loan in 2 ways. The interest is compounded monthly.

- **Option A:** 36 monthly payments at 6.9% per year
- **Option B:** 60 monthly payments at 8.9% per year

a) What is Sheri's monthly payment under each option?

b) How much interest does Sheri pay under each option?

c) Give a reason why Sheri might choose each option.

Solution

We could have used the formula for the present value of an annuity instead of the TVM Solver.

a) ■ Enter the known values into the TVM Solver.

■ Solve for PMT.

Option A

Sheri's monthly payment is $292.90.

Option B

Sheri's monthly payment is $196.74.

b) The interest paid is the difference between the total amount paid and the principal borrowed.

Option A

36 payments of $292.90: $36 \times \$292.90 = \$10\,544.40$

The principal borrowed is $9500.

So, the total interest paid is: $\$10\,544.40 - \$9500.00 = \$1044.40$

Option B

60 payments of $196.74: $60 \times \$196.74 = \$11\,804.40$

The principal borrowed is $9500.

The total interest paid is: $\$11\,804.40 - \$9500.00 = \$2304.40$

c) Sheri might choose Option A because she will pay less total interest. She might choose Option B because the monthly payments are smaller.

Example 3 illustrates a relationship that is true in general. Extending the time taken to repay a loan reduces the regular payment, but increases the total interest paid.

Practice

A

For help with questions 1 to 5, see Examples 1 and 2.

1. Rearrange each formula to isolate R.

a) $A = \dfrac{R[(1+i)^n - 1]}{i}$ b) $PV = \dfrac{R[1-(1+i)^{-n}]}{i}$

2. Evaluate each expression. Write each answer to 2 decimal places.

a) $\dfrac{7800 \times 0.03}{1.03^{14} - 1}$ b) $\dfrac{35\ 500 \times 0.025}{1.025^{72} - 1}$

3. Imran plans to finance a new home entertainment system. He uses the TVM Solver to determine his monthly payment.

a) How much will Imran borrow?
b) What interest rate will he be charged?
c) What is the monthly payment?
d) How many payments will Imran make?
e) How many years will it take Imran to repay the loan?

4. Determine the regular payment of each annuity. Each payment is made at the end of the compounding period.

	Amount	Present value	Interest rate	Frequency of compounding	Length of annuity
a)	$4500	–	8%	semi-annually	6 years
b)	$25 000	–	6%	annually	12 years
c)	–	$4000	8%	quarterly	5 years
d)	–	$3500	24%	monthly	2 years

5. Use a different method to verify your answers to question 4.

B

6. Determine whether each situation involves the amount or present value of an annuity. Explain your reasoning.

a) Steven plans to repay his student loan of $15 000 by making equal annual payments.
b) Winnie saves $5000 by making equal weekly payments at her bank.
c) Sergio plans to retire a millionaire by making equal monthly deposits into his retirement savings plan.
d) Veronika plans to make equal quarterly withdrawals from her $300 000 retirement income fund.

7. Carolyn gets a small business loan for $75 000 to start her hair salon. She will repay the loan with equal monthly payments over 5 years at 8.4% per year compounded monthly.
   a) What is Carolyn's monthly loan payment?
   b) What is the total amount Carolyn repays?
   c) How much of the amount repaid is interest?

8. Shahrzad starts a savings program to have $23 000 in 10 years. She makes deposits at the end of each quarter in an investment account that earns 6.2% per year compounded quarterly.
   a) Determine Shahrzad's quarterly deposit.
   b) Does Shahrzad's deposit double under each change? Justify your answers.
      i) The amount is twice as great, $46 000.
      ii) The interest rate is twice as great, 12.4%.
      iii) The time period is twice as long, 20 years.

9. Boza will need $35 000 in 5 years to start his own business. He plans to save the money by making semi-annual deposits in an account earning 7.8% per year compounded semi-annually.
   a) What semi-annual deposit must Boza make?
   b) How much interest does Boza earn?

For help with question 10, see Example 3.

10. Chandra finances a car loan of $18 000 at 9.9% per year compounded monthly. She can repay the loan in 36 months or 48 months. The first payment will be made 1 month after the car is purchased.
   a) What is Chandra's monthly payment for each loan?
   b) How much interest does Chandra save by repaying the loan in 36 months instead of 48 months?

11. David and Ulani each arrange a 3-year car loan for $20 000.
   - David is charged interest at 9.3% per year compounded monthly
   - Ulani is charged interest at 12.5% per year compounded monthly
   a) Determine the monthly payment for each loan.
   b) How much extra interest does Ulani pay? Explain.

12. Create a problem involving the regular payment of an annuity whose solution is given by the TVM Solver screen.

13. **Assessment Focus** Lincoln wants to have \$10 000 in 6 years by making equal regular deposits into a bank account. He can:

- Make a deposit at the end of each month in an account that earns 7.8% per year compounded monthly
- Make a deposit at the end of each quarter in an account that earns 8.0% per year compounded quarterly

Which option should Lincoln choose? Justify your answer.

14. Megan and Nancy each want to save \$250 000 for their retirement in 40 years.

a) Nancy begins her regular deposits immediately. How much must she deposit at the end of each year at 12% per year compounded annually to achieve her goal?

b) Megan decides to wait 10 years before she starts her regular deposits. What annual deposit does she need to make?

c) Compare Megan and Nancy's total deposits. How much less does Nancy deposit by starting early?

15. **Literacy in Math** Use a concept map to summarize what you have learned about annuities. Add to the concept map as you work through the chapter.

C

16. Chukwuma deposits \$1500 in a retirement savings plan at the end of every 6 months for 20 years. The money earns 11% per year compounded semi-annually. After 20 years, Chukwuma converts the retirement savings plan into an income fund that earns 7% per year compounded monthly. He plans to make equal withdrawals at the end of every month for 15 years. What regular withdrawals can Chukwuma make?

In Your Own Words

Suppose you are asked to determine the regular payment of an annuity. How do you know which formula to use, or whether to enter a value for FV or PV in the TVM Solver? Use examples in your explanation.

7.4 Using a Spreadsheet to Investigate Annuities

Spreadsheets are an important tool in business and personal finance. We can use a spreadsheet to change the features of an annuity and analyse the effect of the change.

Inquire

Analysing Annuities with a Spreadsheet

Materials
- *Microsoft Excel*
- Amount.xls
- Loan.xls

Work with a partner.

Part A: Analysing the Amount of an Annuity

Kiran deposits $500 every 6 months into an investment account that earns 7% per year compounded semi-annually. What is the amount in the account after 3 years?

- If you are using the file *Amount.xls*, open it and begin at question 2.
- If you are not using the file *Amount.xls*, start at question 1.

1. Creating an investment template

a) • Open a new spreadsheet document.
 • Copy the headings, values, and formulas shown below.

	A	B	C	D	E
1	**Amount of an Annuity**				
2	Regular payments		500		
3	Annual interest rate		0.07		
4	Compounding periods per year		2		
5	Number of years		3		
6	Interest rate per period		=C3/C4		
7	Number of periods		=C4*C5		
8					
9	**Period**	**Starting balance**	**Interest**	**Deposit**	**Ending balance**
10	1	0	0	=$C$2	=B10+C10+D10
11	=A10+1	=E10	=ROUND($C$6*B11,2)	=$C$2	=B11+C11+D11

 • Format cells C2 and B10 to E11 as currency.
 • Format cells C3 and C6 as percents to 2 decimal places.

b) Refer to the formulas in the spreadsheet in part a.
- Explain the formulas in cells C6 and C7.
- Why is the interest 0 in the first period?
- The formula =**$C$2** appears in cells D10 and D11. The $ sign indicates that cell address C2 should not change when we copy the formula to other cells in column D. Explain why this makes sense.
- Explain the formulas for the ending and starting balances.

c)
- Select cells A11 to E11.
- **Fill Down** to copy the formulas in row 11 through row 15. Your spreadsheet should look like this.

	A	B	C	D	E
1	**Amount of an Annuity**				
2	Regular payments		$500.00		
3	Annual interest rate		7.00%		
4	Compounding periods per year		2		
5	Number of years		3		
6	Interest rate per period		3.50%		
7	Number of periods		6		
8					
9	**Period**	**Starting balance**	**Interest**	**Deposit**	**Ending balance**
10	1	$0.00	$0.00	$500.00	$500.00
11	2	$500.00	$17.50	$500.00	$1017.50
12	3	$1017.50	$35.61	$500.00	$1553.11
13	4	$1553.11	$54.36	$500.00	$2107.47
14	5	$2107.47	$73.76	$500.00	$2681.23
15	6	$2681.23	$93.84	$500.00	$3275.07

The amount of the annuity is $3275.07.

Record the amount under each change in questions 2 to 5. You will compare these amounts in question 6.

2. Changing the payment

a) Suppose the regular payment is doubled from $500 to $1000. Predict how the amount will change. Explain your reasoning.

Be as specific as you can in your prediction.

b) Check your prediction. Were you correct? Explain.

c) Repeat parts a and b when the regular payment is halved from $500 to $250.

Change the regular deposit back to $500.00.

3. Changing the interest rate

a) Suppose the annual interest rate is doubled from 7% to 14%. Predict how the amount will change. Explain your reasoning.

b) Check your prediction. Were you correct? Explain.

c) Repeat parts a and b when the annual interest rate is halved from 7% to 3.5%.

Change the interest rate back to 7%.

4. Changing the term

a) Suppose the term is doubled from 3 years to 6 years. Predict how the amount will change. Explain your reasoning.

b) Check your prediction. Were you correct? Explain.

c) Repeat parts a and b when the term is halved from 3 years to 1.5 years.

Change the term back to 3 years.

Use the value in cell C7 to determine the number of periods to display.

5. Changing the payment frequency and the compounding

a) Suppose the payment frequency and compounding are doubled from semi-annually to quarterly. Predict how the amount will change. Explain your reasoning.

b) Check your prediction. Were you correct? Explain.

c) Repeat parts a and b when the payment frequency and the compounding is halved from semi-annually to annually.

d) Compare the amount under each payment and compounding frequency. Which produces the greatest amount? Why does this result make sense?

Change the number of compounding periods back to 2.

Use the value in cell C7 to determine the number of periods to display.

6. Comparing the effect of each change

Refer to your answers in questions 2 to 5.
Which of the following changes produced the greatest amount?

- Doubling the payments
- Doubling the interest rate
- Doubling the term
- Doubling the payment frequency and compounding

Part B: Repaying a Loan

Devon plans to borrow $500 for 12 months at 18% per year compounded monthly. He will repay the loan with equal monthly payments.

Devon uses an online loan calculator to determine his monthly payment.

Calculate: ◉ **Payment** ○ **Loan Amount**

Monthly payment:	$45.84
Loan amount:	$500
Term in months:	12
Interest rate:	18.000%

Loan amount:	**$500**
Total of payments:	**$550.08**
Total interest paid:	**$50.08**

We can use a spreadsheet to analyse how Devon's loan is repaid over the 12 months.

- If you are using the file *Loan.xls*, open it and begin at question 8.
- If you are not using the file *Loan.xls*, start at question 7.

7. Creating a loan repayment template

a) • Open a new spreadsheet document.

• Copy the headings, values, and formulas shown below.

	A	B	C	D	E
1	**Loan Repayment Schedule**				
2	Principal borrowed		500		
3	Annual interest rate		0.18		
4	Compounding periods per year		12		
5	Interest rate per period		=C3/C4		
6	Number of payments		12		
7	Monthly payment		=PMT(C5, C6, -C2)		
8					
9	**Payment**	**Payment**	**Interest**	**Principal**	**Outstanding**
10	**number**		**paid**	**paid**	**balance**
11	0				=C2
12	1	=ROUND($C$7,2)	=ROUND(E11*$C$5,2)	=B12-C12	=E11-D12

In cell C7, enter -C2 to produce a positive number for the payment.

• Format cells C2, C7, E11, and B12 to E12 as currency.

• Format cells C3 and C5 as percents to 2 decimal places.

b) Refer to the formulas in the spreadsheet in part a.

• The **PMT** function in cell C7 calculates the regular payment of the loan. What do the numbers in the brackets represent?

• Explain the remaining formulas in the table.

c) • Select cells A12 to E12.

• **Fill Down** to copy the formulas in row 12 through row 23. Your spreadsheet should look like this.

	A	B	C	D	E
1	**Loan Repayment Schedule**				
2	Principal borrowed		$500.00		
3	Annual interest rate		18.00%		
4	Compounding periods per year		12		
5	Interest rate per period		1.50%		
6	Number of payments		12		
7	Monthly payment		$45.84		
8					
9	**Payment**	**Payment**	**Interest**	**Principal**	**Outstanding**
10	**number**		**paid**	**paid**	**balance**
11	0				$500.00
12	1	$45.84	$7.50	$38.34	$461.66
13	2	$45.84	$6.92	$38.92	$422.74
14	3	$45.84	$6.34	$39.50	$383.24
15	4	$45.84	$5.75	$40.09	$343.15
16	5	$45.84	$5.15	$40.69	$302.46
17	6	$45.84	$4.54	$41.30	$261.16
18	7	$45.84	$3.92	$41.92	$219.24
19	8	$45.84	$3.29	$42.55	$176.69
20	9	$45.84	$2.65	$43.19	$133.50
21	10	$45.84	$2.00	$43.84	$89.66
22	11	$45.84	$1.34	$44.50	$45.16
23	12	$45.84	$0.68	$45.16	$0.00

8. Explaining how the interest paid and the outstanding balance are calculated

In a loan, the principal borrowed plus interest must be repaid. So, part of each loan payment is interest, and the rest reduces the principal.

Payment 1

- When payment 1 is made, a month's interest is owed on the outstanding balance, $500.
  From cell C5, the monthly rate is 1.50%.
  So, the monthly interest charge is: $\$500 \times 0.015 = \$7.50$
- The monthly payment is $45.84. Since $7.50 is interest, the part that repays principal is: $\$45.84 - \$7.50 = \$38.34$
- The outstanding balance at the end of the first month is: $\$500 - \$38.34 = \$461.66$

Repeat these calculations for two other payments in the table. Explain your work.

9. Analysing the repayment schedule

a) Why do you think that the interest is paid before the principal is reduced?

b) As the outstanding balance decreases, the interest paid decreases and the principal repaid increases. Explain why this happens.

c) Has the principal of the loan been reduced by one-half after 6 of the 12 payments? Explain.

d) We can calculate the total interest paid over the life of the loan by adding the values in cells C12 to C23.
   - In cell C24, type: **=SUM(**
   - Select cells C12 to C23.
     The formula should now read: **=SUM(C12:C23**
   - Complete the formula by typing: **)**

   How does this value compare with the value Devon obtained on the online calculator?

e) How does the total interest paid compare to the original principal of the loan?

10. Making changes to the loan

a) Predict how each of the following affect the regular payment and total interest paid over the life of the loan.
   - Changing the principal borrowed to $1000
   - Changing the interest rate to 9%
   - Changing the term of the loan to 6 months
   - Changing the term of the loan to 24 months

b) Check each prediction. Remember to change the spreadsheet back to its original form after each change.

c) Suppose Devon doubles his monthly payment. Will the time taken to repay the loan decrease by one-half? Justify your answer.

d) Summarize your results. What features of the loan could Devon change to accomplish each of these goals?
   - Decrease the monthly payment
   - Reduce the total interest paid
   - Reduce the time taken to repay the loan

   Can Devon accomplish all three goals at the same time? Explain.

Reflect

- What is an advantage and disadvantage of using a spreadsheet?
- Which do you prefer to use: formulas, TVM Solver, or a spreadsheet? Explain.

7.5 Saving for Education and Retirement

Jim is a financial planner. He encourages his clients to save regularly to obtain the money needed for future expenses such as their retirement or their children's education. Most people find it easier to save by setting aside a portion of each paycheque rather than coming up with large sums of money to invest.

Inquire

Researching Savings Plans for Education and Retirement

Materials
- computer with Internet access
- print materials about registered plans or a financial planner

Work in a small group.

Post-secondary education and retirement typically involve large sums of money. The Canadian government offers *Registered Education Savings Plans* (RESPs) and *Registered Retirement Savings Plans* (RRSPs) to encourage Canadians to save for these goals.

1. Planning the research
- What sources will you use to find information about RESPs and RRSPs?
- If you use the Internet, what other search words might you use?
- How will you record your research? Which graphic organizers may be helpful?
- How will you share the work among group members?

2. Gathering information about RESPs

Use your own words to answer these questions.

- Research background information
  - What is an RESP?
  - What are the benefits of saving money in an RESP?
  - Where can you open an RESP?
  - What is a qualifying educational program?
  - What tax rules apply to an RESP?
- Research contribution rules
  - Who can contribute to an RESP?
  - What is the maximum lifetime contribution allowed?
  - How much money does the government contribute to an RESP?
- Research withdrawal rules
  - How is money withdrawn from an RESP when the student starts post-secondary education?
  - What happens to the money in the RESP if the student decides not to pursue post-secondary education?
- Record any other information you discovered in your research that you think is important. Why do you think it is important?

3. Gathering information about RRSPs and RIFs

Use your own words to answer these questions.

- Research background information
  - What is an RRSP?
  - What are the benefits of saving money in an RRSP?
  - Where can you open an RRSP?
  - What tax rules apply to an RRSP?

- Research contribution rules
  - Who can contribute to an RRSP?
  - How often and how much can a person contribute to an RRSP?
  - For how long can a person contribute to an RRSP?
- Research withdrawal rules
  - What happens when a person needs some of the money saved in an RRSP for an emergency?
  - What do you do with the money in an RRSP when you retire?
  - What is the difference between an RRSP and a *Retirement Income Fund* (RIF)?
- Record any other information you discovered in your research that you think is important. Why do you think it is important?

4. Using online financial calculators

Many financial institutions offer online RESP, RRSP, and RIF calculators.

- Find an RESP calculator.
  - What information does the calculator require?
    Try some sample calculations.
    Record or print the calculator screen.
- Find an RRSP calculator.
  - Think of the largest contribution you can afford to make every month from age 18 to age 65. What will be the amount of these contributions when you retire? Use realistic interest rates.
  - Investigate how the amount at retirement changes if you:
    - Double your contributions
    - Wait 10 years until you start contributing

    Record or print the calculator screen.

Reflect

- Why is it important to start contributing early to any savings plan?
- What was the most important fact you learned about RESPs and RRSPs? Why is it important?
- How easy was it to find and use an online calculator? Explain.

Mid-Chapter Review

7.1 **1.** Babette spends $225 a year on lottery tickets. After 15 years, her total winnings are $1200. Suppose Babette had invested the money she spends on lottery tickets in an account that earns 6% per year compounded annually. How much would Babette have accumulated after 15 years?

2. Harvey deposits $2500 at the end of each year into an RRSP that earns 9.6% per year compounded annually.

a) Determine the amount in the RRSP at the end of each number of years.
   i) 10 years ii) 20 years iii) 40 years

b) Determine the interest earned after each time period in part a.

c) Use your answers in parts a and b to explain the advantages of saving early.

7.2 **3.** Allison wins a lottery. She can receive $25 000 at the end of every 6 months for 20 years or an equivalent cash payment immediately. Determine the value of the cash payment if money can be invested at 8.5% per year compounded semi-annually.

4. Create an example to show how the present value of an annuity changes in each situation.

a) The regular payment is doubled.

b) The interest rate is doubled.

c) The number of years is doubled.

d) The compounding period is doubled.

7.3 **5.** Pilar needs $2500 three years from now. How much should she deposit at the end of each quarter at 4% per year compounded quarterly to obtain the required amount?

6. Florine borrowed $25 000 at 9.6% per year compounded monthly to buy a new houseboat. She can repay the money by making equal monthly payments for 7 years or 10 years.

a) Determine the monthly payment for each time period.

b) How much would Florine save in interest by choosing the 7-year loan?

c) Why might Florine choose the 10-year loan even though the interest costs are greater?

7.4 **7.** Malcolm plans to invest $500 at the end of every 6 months in a savings account that earns 5% per year compounded semi-annually.

a) Use a spreadsheet to determine the amount in the account after 2 years.

b) How much more would Malcolm have at the end of the 2 years under each change?
   i) The monthly deposits are $600.
   ii) The interest rate is 6% per year.

8. Elyse borrows $8000 at 12% per year compounded monthly. She will repay the loan by making monthly payments of $177.96 for the next 5 years.

a) Use a spreadsheet to create a payment schedule for Elyse's loan.

b) How much does Elyse have left to repay after 3 years?

7.5 **9.** What do you think are the two main benefits of using an RESP?

7.6 What Is a Mortgage?

*Kyle and Tea want to arrange a **mortgage** to buy a house. They talk to the mortgage specialist at their bank and research mortgages on the Internet so they can make an informed decision about the mortgage best suited to their personal and financial goals.*

Inquire

Materials

- computer with Internet access

Researching Mortgages

Work in small groups.

1. Planning the research

- What sources can you use to find information about Canadian mortgages?
- What search words might you use to research mortgage vocabulary? What search words might you use to learn about the various types of mortgages available?
- How will you record your research? Which graphic organizers may be helpful?
- How will you share the work among group members?

2. Gathering information about mortgages

- Research general information about mortgages
  - What is a mortgage?
  - Where can a mortgage be obtained?
  - What financial requirements must be met to qualify for a mortgage?
  - How is a mortgage generally repaid?
- Research down payments
  - What is the minimum down payment required for a mortgage?
  - What are the advantages and disadvantages of a large down payment instead of a small down payment?
  - What are the advantages and disadvantages of a large or small down payment?
- Research features of a mortgage
  - What are the current interest rates on mortgages?
  - How often is the interest compounded?
  - What is the difference between the *amortization period* and *term* of a mortgage? What amortization periods and terms are commonly available?
  - How often can mortgage payments be made?
  - Some financial institutions allow accelerated payments. What does this mean? What is the benefit of repaying a mortgage with accelerated payments?

3. Comparing different types of mortgages

Explain the difference between each type of mortgage. Why might a homeowner choose one type of mortgage over the other?

- Conventional mortgage or high-ratio mortgage
- Open or closed mortgage
- Fixed-rate or variable-rate mortgage
- Short-term or long-term mortgage

4. Completing the research

Record any other terms or information you came across in your research that you think are important.

Practice

You may need to do additional research to answer these questions.

A

1. Kyle and Tea purchase a house for a selling price of $155 000. They plan to make a down payment of 25% and arrange a mortgage for the rest.
   a) How much is their down payment?
   b) How much will they borrow for the mortgage?

B

2. Suppose Kyle and Tea cannot afford a 25% down payment.
   a) What additional costs will Kyle and Tea pay by making the lesser down payment? Explain.
   b) How would a greater initial down payment end up saving money over the life of the mortgage?

3. Kyle and Tea may have to pay a number of other costs when they purchase their house. These costs are usually given as a percent of the selling price.
   Land-transfer tax: 0.75% *Mortgage loan insurance premium*: 2.75%
   Building inspection: 0.25% *Legal fees*: 1%
   Determine the total of these costs for a house with selling price $155 000.

4. Kyle and Tea's bank offers them a 3% cash back incentive on their mortgage. They can use this money towards their down payment.
   a) How much cash back would they receive on a $155 000 mortgage?
   b) Why do you think banks offer a cash back option to customers?

5. Kyle and Tea obtain a mortgage for $139 500 with an amortization period of 25 years and a 5-year term.
   a) What does this mean in everyday language?
   b) Use an online mortgage calculator to determine Kyle and Tea's monthly payment. Use current interest rates.
   c) What other payment frequencies are available to repay a mortgage?

Reflect

- Are mortgages annuities? Explain.
- Which sources of information did you find most helpful? Why were they helpful?
- Which of the costs associated with buying a house surprised you the most? Explain.

Home Quit

7.7 Amortizing a Mortgage

Geri is a mortgage specialist at a bank. When a customer arranges a mortgage, she provides a payment schedule that gives a detailed breakdown of how the mortgage will be repaid.

Investigate

Analysing the Repayment of a Mortgage

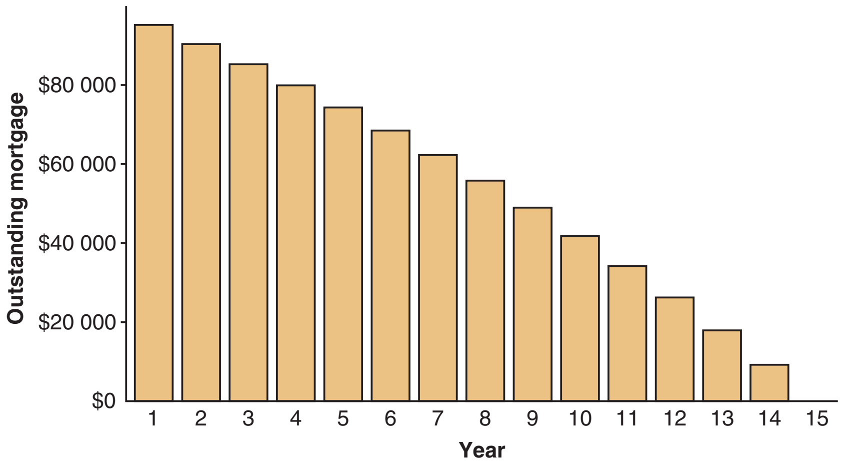
Materials

- scientific calculator

Work with a partner.

The Babiaks will repay a $100 000 mortgage loan, plus interest, over 15 years by making equal monthly payments. The mortgage calculator on their bank's Web site displays a table and graph that show how the mortgage is repaid year by year over the 15 years.

Year	Total of payments	Principal paid	Interest paid	Ending principal balance
				$100 000.00
1	$9457.44	$4612.42	$4845.02	$95 387.58
2	$9457.44	$4845.92	$4611.52	$90 541.66
3	$9457.44	$5091.26	$4366.18	$85 450.40
4	$9457.44	$5348.99	$4108.45	$80 101.41
5	$9457.44	$5619.80	$3837.64	$74 481.61
6	$9457.44	$5904.29	$3553.15	$68 577.32
7	$9457.44	$6203.20	$3254.24	$62 374.12
8	$9457.44	$6517.23	$2940.21	$55 856.89
9	$9457.44	$6847.17	$2610.27	$49 009.72
10	$9457.44	$7193.81	$2263.63	$41 815.91
11	$9457.44	$7557.98	$1899.46	$34 257.93
12	$9457.44	$7940.61	$1516.83	$26 317.32
13	$9457.44	$8342.61	$1114.83	$17 974.71
14	$9457.44	$8764.94	$692.50	$9209.77
15	$9458.52	$9209.77	$248.75	$0.00



- How do the total paid, interest, principal, and balance change over the life of the mortgage?
- What is the total amount paid and the total interest paid over the lifetime of the mortgage? How do these values compare with the principal originally borrowed?

Reflect

- What patterns do you see in the table or the graph?
- Why do you think these patterns occur?

Connect the Ideas

Mortgage interest rates

Under Canadian law, interest on mortgages can be compounded at most semi-annually. However, mortgage payments are often made monthly. These monthly payments form an annuity whose present value is the principal originally borrowed.

Since the payment period and compounding period are different, we cannot calculate the monthly payment on a mortgage by using the formula for the present value of an ordinary simple annuity. We use the TVM Solver instead. To represent monthly payments and semi-annual compounding, we set P/Y = 12 and C/Y = 2.

Example 1 — Determining the Monthly Mortgage Payment

Materials
- TI-83 or TI-84 graphing calculator

The Cafirmas take out a mortgage of \$210 000 at 5% per year compounded semi-annually for 25 years.

a) What is their monthly payment?

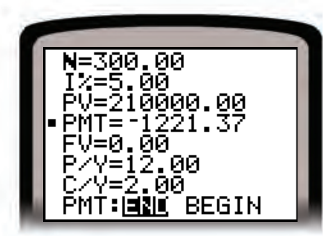
b) What is the total interest paid over the 25 years?

Solution

N is the total number of payments.

a) Use the TVM Solver.
- Enter the known values.
- Solve for PMT.

  The Cafirmas' monthly payment is \$1221.37.



b) 300 payments of \$1221.37: $300 \times \$1221.37 = \$366\,411.00$
The principal borrowed is \$210 000.
So, the total interest paid over the 25 years is:
$\$366\,411.00 - \$210\,000 = \$156\,411.00$
This result can be verified with a graphing calculator.

The 25¢ difference is due to rounding.

Amortizing a mortgage

A mortgage is *amortized* when both the principal and interest are paid off with a series of equal, regular payments. For example, the mortgage in *Example 1* was amortized by making monthly payments of \$1221.37 over an **amortization period** of 25 years.

To simplify the math, we assumed that the interest rate is fixed for the entire amortization period.

In reality, mortgage interest rates are fixed for a shorter length of time called the *term* of the mortgage. The term normally ranges from 6 months to 10 years. At the end of the term, the mortgage must be paid off or renewed at the current rate of interest.

Amortization table

We can use an **amortization table** to analyse how a mortgage is repaid. The amortization table gives a detailed breakdown of the interest and principal paid by each payment and the loan balance after the payment.

Example 2

Reading and Interpreting an Amortization Table

Here is a partial amortization table for the Cafirmas' mortgage.

Payment number	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$210 000.00
1	$1221.37	$866.02	$355.35	$209 644.65
2	$1221.37	$864.56	$356.81	$209 287.84
3	$1221.37	$863.09	$358.28	$208 929.56
4	$1221.37	$861.61	$359.76	$208 569.80
5	$1221.37	$860.12	$361.25	$208 208.55
6	$1221.37	$858.63	$362.74	$207 845.81
:	:	:	:	:
295	$1221.37	$29.79	$1191.58	$6032.26
296	$1221.37	$24.88	$1196.49	$4835.77
297	$1221.37	$19.94	$1201.43	$3634.34
298	$1221.37	$14.99	$1206.38	$2427.96
299	$1221.37	$10.01	$1211.36	$1216.60
300	$1221.62	$5.02	$1216.60	$0.00
Total	$366 411.25	$156 411.25	$210 000.00	

a) How much interest and principal is paid in the 5th payment? How much do the Cafirmas still owe after this payment?

b) What is the outstanding balance after 6 months?

c) Compare the interest and principal paid in the first 6 months of the mortgage with the interest and principal paid in the last 6 months of the mortgage. What do you notice?

d) Why is the monthly payment increased for the 300th payment?

e) What percent of the total amount paid is interest?

Solution

a) The interest paid in the 5th payment is $860.12.
The principal paid is $361.25.
The Cafirmas still owe $208 208.55 after this payment.

b) The outstanding balance after 6 months is $207 845.81.

c) In the first 6 months, the payments mostly cover interest, while in the last 6 months, the payments mostly cover principal.

d) The mortgage is paid off in the 300th payment, so the outstanding balance should be \$0.00. The outstanding principal after the 299th payment is \$1216.60 and the interest charge for this payment is \$5.02. So, the 300th payment increases to \$1216.60 + \$5.02 = \$1221.62.

e) The total paid over the life of the mortgage is \$366 411.25.
The total interest paid is \$156 411.25.
The interest as a percent of the total amount paid is:
$\frac{\$156\,411.25}{\$366\,411.25} \times 100\% \doteq 43\%$

Example 2 illustrates some key points about the amortization of a mortgage.

- Although the monthly payments are equal, the split between interest and principal changes with each payment.
- With each payment, the outstanding balance on the mortgage decreases. So, the part of each payment that covers interest decreases.
- As the portion of each payment that covers interest decreases, the part that repays principal increases.

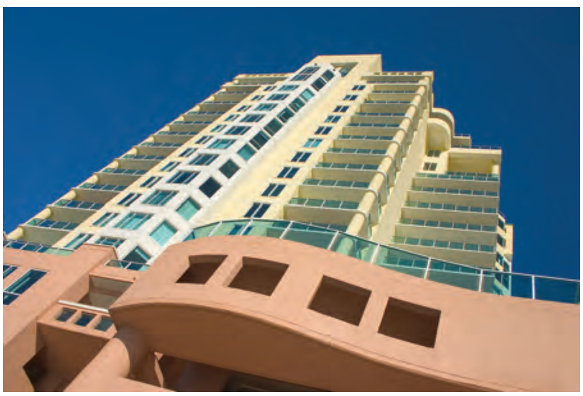
Practice

A

For help with questions 1 to 4, see Example 1.

1. Nadia uses the TVM Solver to estimate the monthly payment for her mortgage.

a) How much does Nadia plan to borrow?
b) What interest rate is Nadia charged?
c) What is Nadia's monthly payment?
d) What is the total number of payments Nadia will make?
e) Why is P/Y = 12 and C/Y = 2?



2. For each TVM Solver screen shown:

i) What was the principal borrowed?

ii) How many payments will it take to repay the mortgage?

iii) What is the total of the monthly payments over the life of the mortgage?

iv) What is the total interest paid?

a)

b)

3. Determine the monthly payment for each mortgage. The interest is compounded semi-annually.

	Principal borrowed	Interest rate	Length of mortgage
a)	$65 000	4%	15 years
b)	$150 000	5%	25 years
c)	$190 000	7.5%	20 years
d)	$289 000	6.25%	30 years

4. Determine the total interest paid over the life of each mortgage in question 3.

■ For help with question 5, see Example 2.

5. This amortization table shows the first 3 payments on the Parks' mortgage.

Payment number	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$125 000.00
1	$799.76	$617.33	$182.43	$124 817.57
2	$799.76	$616.43	$183.33	$124 634.24
3	$799.76	$615.52	$184.24	$124 450.00
Total	$2399.28	$1849.28	$550.00	

a) How much money did the Parks borrow?

b) What is their monthly payment?

c) How much of the 1st payment is interest?

d) How much of the 2nd payment is principal?

e) What is the outstanding balance after the 3rd payment?

f) Compare the total interest and total principal paid in the first 3 payments. What do you notice?

B This amortization table shows the first 3 payments and last 3 payments on a mortgage. Use the table to answer questions 6 and 7.

Payment number	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$120 000.00
1	$1104.62	$738.54	$366.08	$119 633.92
2	$1104.62	$736.29	$368.33	$119 265.59
3	$1104.62	$734.02	$370.60	$118 894.99
⋮	⋮	⋮	⋮	⋮
178	$1104.62	$20.14	$1084.48	$2187.86
179	$1104.62	$13.47	$1091.15	$1096.71
180	$1103.46	$6.75	$1096.71	$0.00
Total	$198 830.44	$78 830.44	$120 000.00	

For help with question 6, see Example 3.

6. **a)** What is the principal borrowed and the monthly payment?

b) What is the amortization period? Justify your answer.

c) How much of the 1st payment is interest and how much repays principal?

d) What is the total interest paid over the life of the mortgage? How does this compare to the principal originally borrowed?

e) What is the outstanding balance after the first 3 payments?

7. **a)** How does the interest paid in the first 3 months compare with the principal paid? Explain.

b) How does the interest paid in the last 3 months compare with the principal paid? Explain.

c) What percent of the first 3 payments pays interest and repays principal? Show your calculations.

8. The Smiths would like to buy a new cottage. They have negotiated a selling price of $175 000. They will make a down payment of 15% and arrange a mortgage at 4.5% per year compounded semi-annually over 15 years to finance the rest.

a) Determine the down payment and the principal of the mortgage loan.

b) What is the Smiths' monthly payment?

c) What is the total amount the Smiths pay over the life of the mortgage? How does this compare to the principal originally borrowed? Explain.

9. **Literacy in Math** Create a flowchart that shows the steps used to calculate a monthly mortgage payment on the TVM Solver. Identify the steps in which the values entered are always the same.

10. **Assessment Focus** Joseph has arranged a mortgage to purchase his first home. The mortgage will be repaid over 25 years at 6% per year compounded semi-annually. The amortization table shows Joseph's first 12 payments.

Payment number	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$185 000.00
1	$1183.64	$913.65	$269.99	$184 730.01
2	$1183.64	$912.31	$271.33	$184 458.68
3	$1183.64	$910.97	$272.67	$184 186.01
4	$1183.64	$909.63	$274.01	$183 912.00
5	$1183.64	$908.27	$275.37	$183 636.63
6	$1183.64	$906.91	$276.73	$183 359.90
7	$1183.64	$905.55	$278.09	$183 081.81
8	$1183.64	$904.17	$279.47	$182 802.34
9	$1183.64	$902.79	$280.85	$182 521.49
10	$1183.64	$901.40	$282.24	$182 239.25
11	$1183.64	$900.01	$283.63	$181 955.62
12	$1183.64	$898.61	$285.03	$181 670.59

a) Use the TVM Solver to verify the monthly mortgage payment. Record the screen. Justify the values you enter for the variables.

b) How much of the 6th payment is interest? How much repays principal?

c) What is the outstanding balance after 8 payments?

d) What percent of the mortgage has been repaid at the end of the first 12 payments? Explain.

11. A mortgage for $225 000 at 5.25% per year compounded semi-annually will be repaid with equal monthly payments.

a) Deanna thinks that the monthly payment for a 15-year amortization period should be double the monthly payment for a 30-year amortization period since the amortization period is one-half as long.
Do you agree? Explain your reasoning.

b) Calculate the monthly payment for each amortization period.
Were you correct? Explain.

c) These amortization tables show the first 6 payments for each mortgage.

15-year amortization

Payment number	Interest paid	Principal paid	Outstanding balance
0			$225 000.00
1	$973.78	$828.27	$224 171.73
2	$970.19	$831.86	$223 339.87
3	$966.59	$835.46	$222 504.41
4	$962.98	$839.07	$221 665.34
5	$959.35	$842.70	$220 822.64
6	$955.70	$846.35	$219 976.29

30-year amortization

Payment number	Interest paid	Principal paid	Outstanding balance
0			$225 000.00
1	$973.78	$260.81	$224 739.19
2	$972.65	$261.94	$224 477.25
3	$971.52	$263.07	$224 214.18
4	$970.38	$264.21	$223 949.97
5	$969.23	$265.36	$223 684.61
6	$968.09	$266.50	$223 418.11

Explain why the 15-year loan is paid off in half the time of a 30-year loan even though the monthly payment for the 15-year loan is only $567.46 more than the monthly payment for the 30-year loan.

C **12.** a) The TVM Solver can be used to create an amortization table. Read the user manual or research on the Internet to learn how to do this.

b) Use the TVM Solver to create an amortization table for the first 6 payments of a $200 000 mortgage at 5.3% per year compounded semi-annually for 25 years.

13. In the United States, mortgage payments can be compounded monthly. Consider a mortgage of $145 000 at 6.5% per year with an amortization period of 25 years.

a) These amortization tables show the first 6 payments for each mortgage.

US mortgage

Payment number	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$145 000.00
1	$979.05	$785.42	$193.63	$144 806.37
2	$979.05	$784.37	$194.68	$144 611.69
3	$979.05	$783.31	$195.74	$144 415.95
4	$979.05	$782.25	$196.80	$144 219.15
5	$979.05	$781.19	$197.86	$144 021.29
6	$979.05	$780.12	$198.93	$143 822.36

Canadian mortgage

Payment number	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$145 000.00
1	$971.24	$774.99	$196.25	$144 803.75
2	$971.24	$773.94	$197.30	$144 606.45
3	$971.24	$772.88	$198.36	$144 408.09
4	$971.24	$771.82	$199.42	$144 208.67
5	$971.24	$770.76	$200.48	$144 008.19
6	$971.24	$769.69	$201.55	$143 806.64

Compare the total interest paid in the first 6 months under each compounding period. Why do you think that mortgage interest rates are compounded semi-annually in Canada?

b) Use the TVM Solver to verify the monthly payment for each payment.

c) How much interest is saved over the life of the mortgage with semi-annual compounding instead of monthly compounding?

In Your Own Words

What information about a mortgage can you learn from an amortization table?
Why is this information important?

Mortgage Tic-Tac-Toe

GAME

This game is for 2 players. One player is X and the other player is O.

- Draw a 3 by 3 tic-tac-toe board.
- Players take turns rolling 3 dice.
- Both players have 15 s to use the table to secretly choose the principal, interest rate, and amortization period for a mortgage.
  The goal is to obtain the lower monthly payment.

Number rolled	Amortization period	Annual interest rate	Principal borrowed
1 or 6	20 years	5%	$150 000
2 or 5	25 years	6%	$175 000
3 or 4	30 years	7%	$200 000

The interest is compounded semi-annually.

Materials

- 3 dice
- TI-83 or TI-84 graphing calculator
- timer

Players then have 1 min to determine their monthly payments.
For example, suppose 3, 1, and 2 are rolled.

Player X's choices

- Interest rate of 7%
- Principal of $150 000
- Amortization period of 25 years

Player O's choices

- Principal of $200 000
- Amortization period of 20 years
- Interest rate of 6%

- Player X has the lower monthly payment and used the TVM Solver correctly, so she writes an X on the tic-tac-toe board.
- The first player to mark 3 Xs or 3 Os in a vertical, horizontal, or diagonal line wins.

If both players have the same monthly payment, they roll again.

Reflect

- Explain how you decided which number to use for the principal borrowed, interest rate, and amortization period.

7.8 Using Technology to Generate an Amortization Table

An amortization table can be generated quickly and efficiently with a spreadsheet. Spreadsheets also allow us to change the features of a mortgage and see the immediate effect of the change in the amortization table.

Inquire

Creating an Amortization Table

Materials
- computer with Internet access or a TI-83 or TI-84 graphing calculator
- *Microsoft Excel*
- Amortization.xls

Work with a partner.

The Lees are finalizing the purchase of their home. They arrange a mortgage of $175 000 at 6.25% per year compounded semi-annually to be repaid monthly over 25 years.

1. Determining the monthly payment

Use an online mortgage calculator to determine the monthly payment. If you have access to the TVM Solver, you may wish to use it instead.

Mortgage amount: $ 175 000
Amortization period: 25 years
Interest rate: 6.25 %
Calculate
Monthly mortgage payment: $1145.80

The Lees' monthly payment is $1145.80.

2. Creating an amortization table

- If you are using the file *Amortization.xls*, open it and begin at part b.
- If you are not using the file *Amortization.xls*, start at part a.

a) • Open a new spreadsheet document.

• Copy the headings, values, and formulas shown.

	A	B	C	D	E
1	**Amortization Table**				
2	Principal		175000		
3	Annual interest rate		0.0625		
4	Equivalent monthly rate		=(1+C3/2)^(1/6)-1		
5	Amortization period in years		25		
6	Number of payments		=C5*12		
7	Monthly payment		=PMT(C4, C6, -C2)		
8					
9	**Payment**	**Monthly**	**Interest**	**Principal**	**Outstanding**
10	**number**	**payment**	**paid**	**paid**	**balance**
11	0				=C2
12	=A11+1	=ROUND($C$7,2)	=ROUND($C$4*E11,2)	=B12-C12	=E11-D12

• Format cells C2, C7, E11, and B12 to E12 as currency.

• Format cell C3 as a percent to 2 decimal places and cell C4 as a percent to 7 decimal places.

To view the formulas in the spreadsheet, hold on **Ctrl** and press **`**.

b) Refer to the formulas in the spreadsheet in part a.

• The formula in cell C4 converts the annual interest rate compounded semi-annually into an equivalent monthly rate. This monthly rate is used to calculate the values in the *Interest paid* column in the amortization table.

• The **PMT** function in cell C7 calculates the regular payment of the loan. What do the numbers in the brackets represent?

• Explain the remaining formulas in the table.

c) • Select cells A12 to E12.

• **Fill Down** to copy the formulas in row 12 through row 311. The first 10 rows of your spreadsheet should look like this.

	A	B	C	D	E
1	**Amortization Table**				
2	Principal		$175 000.00		
3	Annual interest rate		6.25%		
4	Equivalent monthly rate		0.5141784%		
5	Amortization period in years		25		
6	Number of payments		300		
7	Monthly payment		$1145.80		
8					
9	**Payment**	**Monthly**	**Interest**	**Principal**	**Outstanding**
10	**number**	**payment**	**paid**	**paid**	**balance**
11	0				$175 000.00
12	1	$1145.80	$899.81	$245.99	$174 754.01
13	2	$1145.80	$898.55	$247.25	$174 506.76
14	3	$1145.80	$897.28	$248.52	$174 258.24
15	4	$1145.80	$896.00	$249.80	$174 008.44
16	5	$1145.80	$894.71	$251.09	$173 757.35
17	6	$1145.80	$893.42	$252.38	$173 504.97
18	7	$1145.80	$892.13	$253.67	$173 251.30
19	8	$1145.80	$890.82	$254.98	$172 996.32
20	9	$1145.80	$889.51	$256.29	$172 740.03
21	10	$1145.80	$888.19	$257.61	$172 482.42

3. Explaining how the interest paid and the outstanding balance are calculated

Part of each monthly payment is interest and the rest is principal.

Payment 1

- When payment 1 is made, a month's interest is owed on the outstanding balance, \$175 000.
  From cell C4, the monthly rate, 0.5141784%, corresponds to an annual rate of 6.25% per year compounded semi-annually.
  So, the monthly interest charge is:
  $\$175\ 000 \times 0.005141784 \doteq \$899.81$
- The monthly payment is \$1145.80. Since \$899.81 is interest, the part that repays principal is: $\$1145.80 - \$899.81 = \$245.99$
- The outstanding balance at the end of the first month is:
  $\$175\ 000 - \$245.99 = \$174\ 754.01$

Repeat these calculations for two other payments in the table.

4. Adjusting the last payment

The mortgage is paid off in the 300th payment, so the outstanding balance should be \$0.00. The outstanding balance after the 299th payment is \$1138.53 and the interest charge for this payment is \$5.85. So, the 300th payment should be:

$\$1138.53 + \$5.85 = \$1144.38$

- Enter \$1144.38 as the 300th payment to obtain an outstanding balance of \$0.00 after the 300th payment.

5. Calculating the total repaid and the total interest paid

a) We can calculate the total paid over the life of the mortgage by adding the values in cells B12 to B311.
  - In cell B312, type: **=SUM(**
  - Select cells B12 to B311.
    The formula should now read: **=SUM(B12:B311**
  - Complete the formula by typing: **)**

b) Repeat part a to determine the total interest paid and the total principal paid over the life of the mortgage.

6. Interpreting the amortization table

a) How much of the 4th payment is interest? the 8th payment?

b) What is the outstanding balance after half of the payments have been made? Is the outstanding balance also reduced by one-half? Explain.

c) What cash payment will pay off the mortgage after the 200th payment?

d) What is the total interest paid over the life of the mortgage? How does this compare with the principal originally borrowed?

Practice

A

1. Use an online Canadian mortgage calculator to determine the monthly payment on each mortgage.

	Principal borrowed	Interest rate	Amortization period
a)	$100 000	5.25%	15 years
b)	$156 000	6.75%	25 years
c)	$230 000	8.5%	20 years

B

2. Anita and Kenny are arranging a mortgage for their new home. They will be taking out a $145 000 mortgage at 5% per year compounded semi-annually for 25 years. They will repay the mortgage with monthly payments.

a) Determine the monthly payment.

b) What is the equivalent monthly interest rate?

c) Create an amortization table for the mortgage.

d) What is the total interest paid over the life of the mortgage?

3. Refer to the mortgage in question 2. Choose any two payments. Explain how the interest paid, principal paid, and outstanding balance for these payments are calculated.

4. Claude purchases a home for $225 000 and applies a $25 000 down payment. He arranges a mortgage for the outstanding balance with monthly payments for 25 years at 6.5% per year compounded semi-annually.

a) Determine the monthly payment.

b) Create an amortization schedule for Claude's first six monthly payments.

c) How much principal is paid down in the first six months?

d) What is the total interest paid in the first six months?

5. The Ugars arrange a mortgage for a new condominium for $245 000. They decide on a mortgage for 5.75% per year compounded semi-annually to be repaid with monthly payments over 30 years.

a) Determine the regular monthly payment.

b) Create an amortization schedule showing the first 6 payments.

c) What is the total interest paid during the 6 months?

6. Paul and Kaori arrange a mortgage for $168 000 to be paid back monthly over 20 years at 4.5% per year compounded semi-annually.

a) Determine their regular monthly payment.

b) Create an amortization schedule for the first 2 years of the mortgage.

c) Use the amortization table in part b to answer these questions.

i) Determine the total interest paid and principal repaid after 2 years.

ii) What is the outstanding balance after 2 years?

iii) Determine the percent of the original mortgage that has been repaid after 2 years. How long will it take to repay the mortgage completely if this rate of payment remains constant? Explain why, in reality, it does not take this long.

Reflect

- Why is it necessary to change the semi-annual interest rate into an equivalent monthly rate before completing the mortgage amortization table?
- How can you check that the values in the amortization table are correct?
- Suppose you have to determine the total interest paid on a mortgage. Do you find it easier to use a spreadsheet or the TVM Solver? Explain.

7.9 Reducing the Interest Costs of a Mortgage

The total interest paid over the lifetime of a mortgage is a considerable sum of money, often in the hundreds of thousands of dollars. There are a number of strategies a homeowner can use to reduce the interest costs of a mortgage.

Inquire

Materials

- TI-83 or TI-84 graphing calculator

Analysing Interest-Saving Strategies

Work with a partner or in a small group.

- Use the TVM Solver to determine the monthly payment. Use the ΣInt command to determine the total interest paid.
- Discuss the questions below each table as a group and record your answers.

1. Changing the amortization period

Most homeowners choose an amortization period of 25 years, but amortization periods of 15, 20, and 30 years are also allowed.

a) Copy and complete this table.

Use a mortgage of $100 000 at 5% per year compounded semi-annually.

Amortization period	Number of payments (N)	Monthly payment (PMT)	Total interest paid	Interest saved
25 years				
15 years				
20 years				
30 years				–

b) How does the monthly payment change as the amortization period increases? Explain.

c) How does the total interest paid change as the amortization period increases? Explain.

d) Why might a homeowner choose a shorter amortization period? Why might a homeowner choose a longer amortization period?

e) Compare the difference in the monthly payments with the difference in the interest saved for different pairs of amortization periods. Does the interest saved justify paying more each month? Explain.

2. Changing the interest rate

Mortgage interest rates are largely determined by economic conditions. They change frequently over time.

a) Copy and complete this table.

Use a mortgage of $100 000 amortized over 25 years.

The interest rate is an annual rate compounded semi-annually.

Interest rate	Year	Monthly payment (PMT)	Total interest paid	Interest saved
5%	1951			–
6%	2007			
10%	1969			
14%	1990			
21.5%	1982			

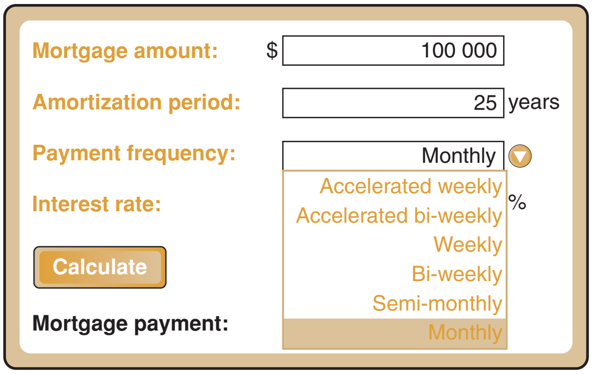
b) How do the monthly payment and the total interest paid change as the interest rate increases? Explain.

c) Does an increase of 4% in the interest rate result in a 4% increase in the total interest paid? Explain.

3. Changing the payment frequency

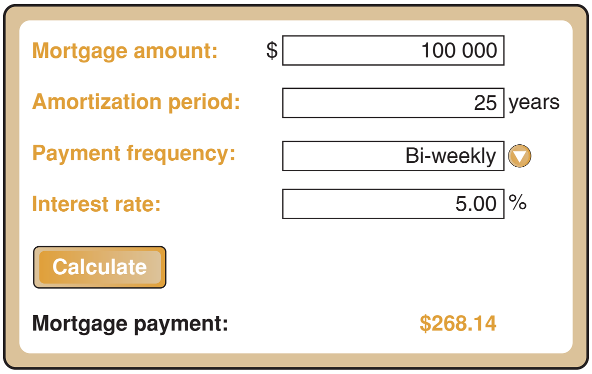
Many mortgages are repaid with monthly payments, but more frequent payments are also allowed. This online mortgage calculator screen shows other commonly used payment periods.

We will examine accelerated payments in question 4.



- semi-monthly (twice a month; 24 payments a year)
- bi-weekly (every 2 weeks; 26 payments a year)
- weekly (every week; 52 payments a year)

a) This online calculator screen shows the bi-weekly payment on a mortgage of $100 000 amortized over 25 years at 5% per year compounded semi-annually.



- Use the TVM Solver to verify the payment.
- Explain the values you entered for the variables.

b) Copy and complete this table.

Use the mortgage from part a.

Payment frequency	Payments per year (P/Y)	Number of payments (N)	Payment (PMT)	Total interest	Interest saved
Monthly					
Semi-monthly					
Bi-weekly					
Weekly					–

c) Is the interest saved significant when payments are made more often? Explain.

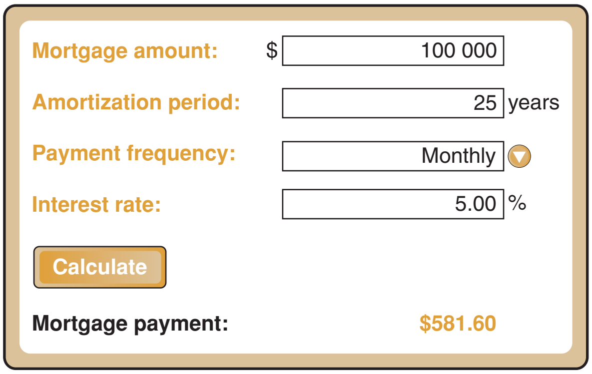
d) Why might a homeowner choose to make semi-monthly, bi-weekly, or weekly payments instead of monthly payments?

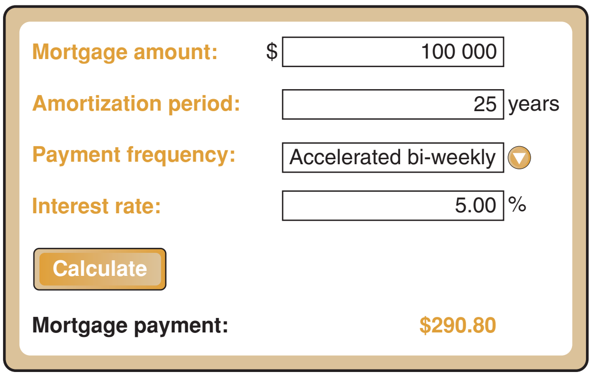
4. Making accelerated payments

Most financial institutions allow "accelerated" weekly and bi-weekly payments.

With this option, the weekly payment is one-quarter of the monthly payment, while the bi-weekly payment is one-half of the monthly payment.

a) The online calculator screens below show the monthly payment and accelerated bi-weekly payment for a mortgage of $100 000 amortized over 25 years at 5% per year compounded semi-annually.





Verify the bi-weekly payment.

b) Copy this table.
Complete the row for monthly payments and the second and third columns of the accelerated bi-weekly and accelerated weekly payments.

Payment frequency	Payments per year (P/Y)	Payment (PMT)	Number of payments (N)	Total interest	Interest saved
Monthly					–
Accelerated bi-weekly					
Accelerated weekly					

c) Compare the accelerated weekly and accelerated bi-weekly payments with the regular weekly and bi-weekly payments in question 3a. Why will the mortgage be paid off more quickly with accelerated payments?

d) Use the TVM Solver to determine the number of payments it takes to pay off the mortgage with accelerated bi-weekly payments.
- Open the TVM Solver.
- Enter the known values for I%, PV, PMT, FV, P/Y, and C/Y.
- Move the cursor to N, and press ALPHA ENTER.
  Round this result to the nearest whole number and record it in the table.

e) Use the ΣInt command to determine the total interest paid with accelerated bi-weekly payments. Use the value of N you determined in part c.

f) Repeat parts c and d for the accelerated weekly payments.

5. Comparing regular and accelerated payments
Compare the tables in questions 3 and 4.

a) Why are the interest savings much greater with accelerated payments than with regular payments?

b) How many payments are saved by making accelerated payments?
How much time does this represent in years and months?

c) In Canada, the most popular payment frequency is the accelerated bi-weekly option. Why do you think this is the most popular option?

Practice

A

1. Calculate each regular payment for a mortgage of $130 000 amortized over 20 years at 8.5% per year compounded semi-annually.

a) Monthly payment

b) Accelerated bi-weekly payment

c) Accelerated weekly payment

2. What effect do each of the following have on the regular payment and the total interest paid on a mortgage? Explain.

a) Increasing the amortization period

b) Making more frequent payments

c) Making accelerated payments

B

3. The Thompsons borrow $179 000 for their new home. They plan to repay the mortgage by making monthly payments for 25 years at 6% per year compounded semi-annually. Calculate the Thompsons' monthly payment and the total interest they will pay over the life of the mortgage.

4. Refer to the mortgage in question 3. Calculate the Thompsons' new regular payment and the total interest saved under each scenario.

a) They arrange a 20-year mortgage instead of a 25-year mortgage.

b) They receive an interest rate of 5.75% by applying for their mortgage over the Internet.

c) They make weekly payments instead of monthly payments.

d) They make accelerated bi-weekly payments instead of monthly payments.

5. Compare your answers to question 4.
Which change resulted in the greatest interest saved? Explain.

Reflect

- What are some strategies a homeowner can use to reduce the total interest paid on a mortgage? Why will these strategies reduce the interest costs?
- How do age, family circumstances, income, and lifestyle factors affect the strategies used to reduce the interest costs of a mortgage? Explain.

Home Quit

Study Guide

Ordinary Simple Annuities

- An annuity is a series of equal, regular payments. In an ordinary simple annuity, payments are made at the end of each compounding period. The interest is compounded just before the payment is made.
- The amount of an annuity is the sum of the regular payments plus the interest.
- The present value of an annuity is the money that must be deposited today to provide regular payments in the future.

Amount of an ordinary simple annuity

$A = \frac{R[(1 + i)^n - 1]}{i}$, where

- A is the amount
- R is the regular payment
- i is the interest rate per compounding period as a decimal
- n is the number of compounding periods

Present value of an ordinary simple annuity

$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$, where

- PV is the present value
- R is the regular payment
- i is the interest rate per compounding period as a decimal
- n is the number of compounding periods

- The interest earned on an annuity is the difference between the total of the regular payments and the amount or present value of the annuity.

Mortgages

- A mortgage is a loan that is used to buy property.
- In Canada, the interest rate on mortgages can be compounded at most semi-annually. However, mortgage payments are usually made monthly or bi-weekly.
- Since the payment period is different from the compounding period, mortgages are not ordinary simple annuities.

In the TVM Solver:
- Solve for FV to determine the amount.
- Solve for PV to determine the present value.
- Solve for PMT to determine the regular payment.
- Use the ΣInt command to determine the interest earned.

Chapter Review

7.1

1. Determine the amount of each ordinary simple annuity.

a) Deposits of $2550 for 7 years at 9.7% per year compounded annually

b) Deposits of $1380 for 4 years at 10% per year compounded semi-annually

c) Deposits of $750 for 5 years at 12.6% per year compounded monthly

2. Calculate the interest earned on each annuity in question 1.

3. Yvonne and Teresa each make regular deposits into an annuity.

- Yvonne deposits $150 at the end of each month at 8% per year compounded monthly
- Teresa deposits $450 at the end of each quarter at 8% per year compounded quarterly

a) Who do you think will have the greater amount at the end of 4 years? Explain your reasoning.

b) Verify your prediction by calculating each amount.

4. Carlos and Renata each invest money at the end of each year in an RRSP.

- Carlos invests $4500 for 30 years at 7.5% per year compounded annually.
- Renata invests $9000 for 15 years at 7.5% per year compounded annually.

a) Determine the amount in each RRSP.

b) Does the amount remain the same when the regular deposit is doubled and the time period is halved? Explain.

7.2

5. Determine the principal that must be deposited today to provide for each ordinary simple annuity.

a) Payments of $3500 for 7 years at 6.5% per year compounded annually

b) Payments of $3575 for 12 years at 9% per year compounded semi-annually

6. Calculate the interest earned on each annuity in question 5.

7. Shawn buys a new computer. He will make monthly payments of $72 for the next 2 years, starting 1 month from now. He is charged 16% per year compounded monthly.

a) How much did Shawn borrow to purchase the computer?

b) How much interest will Shawn pay?

8. Consider these 3 annuities.

Annuity A: $50 per quarter for 4 years at 6% per year compounded quarterly

Annuity B: $100 per quarter for 4 years at 6% per year compounded quarterly

Annuity C: $50 per quarter for 8 years at 6% per year compounded quarterly

a) Determine the present value of each annuity.

b) Which of the following has the greater effect on the present value of an annuity?

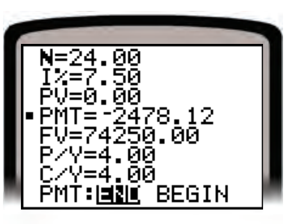
i) Doubling the payments

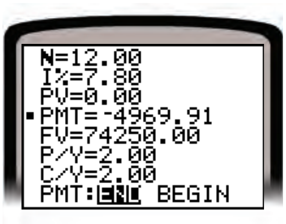
ii) Doubling the time period

Justify your answer.

7.3 **9.** Adrian wants to have $15 000 in 3 years to start a mechanic shop. He plans to save the money by making regular deposits into an annuity that earns 11.7% per year compounded semi-annually. What semi-annual deposits does Adrian have to make?

10. Tarak uses the TVM Solver to compare two annuities.





a) Describe each annuity.

b) Which annuity do you think Tarak should choose? Justify your answer.

11. Anisha obtains a small business loan for $6500 to start her roofing business. She can repay the loan in 24 months or 36 months. She is charged interest at 6.2% per year compounded monthly.

a) How much more will Anisha pay each month if she repays the loan in 24 months instead of 36 months?

b) How much interest will Anisha save if she repays the loan in 24 months instead of 36 months?

c) Why does Anisha pay less interest with a 24-month loan even though her monthly payments are greater than with the 36-month loan?

7.4 **12.** Suppose you make regular quarterly deposits to amount to $5000 over 12 years at 9.4% per year compounded quarterly.

a) Use a spreadsheet to determine the quarterly deposit required.

b) How much more could you save if the interest rate is 10% per year compounded quarterly?

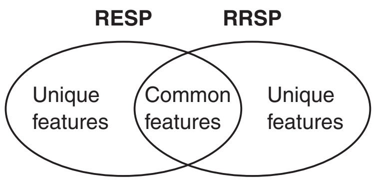
c) Suppose you could increase your regular payments by $100. How much more quickly could you reach your goal?

13. Suppose you borrow $8000 at 7.5% per year compounded monthly for 6 years. You repay the loan by making monthly payments of $138.32.

a) Use a spreadsheet to create a loan repayment schedule.

b) What is the total interest paid on the loan? How does this compare to the principal originally borrowed?

7.5 **14.** Use a Venn diagram to illustrate the similarities and differences between RESPs and RRSPs.



7.6 **15.** Wenfeng's friend has just purchased a house. She tells Wenfeng that she has arranged "a closed mortgage with bi-weekly payments at 6% per year compounded semi-annually with an amortization period of 30 years and a 5-year term." Wenfeng does not understand the terminology his friend is using. Explain the details of the mortgage in everyday language.

7.7

16. For each Canadian mortgage, determine the monthly payment and total interest paid.

	Principal borrowed	Interest rate	Length of mortgage
a)	$97 000	4.5%	25 years
b)	$145 000	3.25%	20 years
c)	$207 000	10%	15 years
d)	$299 000	5.5%	30 years

17. The amortization table shows the first 6 monthly payments of a mortgage.

Payment numbers	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$150 000.00
1	$959.71	$740.79	$218.92	$149 781.08
2	$959.71	$739.71	$220.00	$149 561.08
3	$959.71	$738.63	$221.08	$149 340.00
4	$959.71	$737.53	$222.18	$149 117.82
5	$959.71	$736.44	$223.27	$148 894.55
6	$959.71	$735.33	$224.38	$148 670.17

a) What is the principal borrowed?

b) What is the monthly payment?

c) How much of the 4th payment is interest?

d) What is the outstanding balance after the 3rd payment?

e) What is the total interest paid in the first 6 payments?

18. Part of an amortization table is shown below. Determine the missing values in the table. Justify your answers.

Payment numbers	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$180 000.00
1	$1,077.84	$786.36	$291.48	$179 708.52
2	$1,077.84	$785.09	$292.75	$179 415.77
3	$1,077.84	$783.81	$294.03	$179 121.74
4		$782.52	$295.32	$178 826.42
5	$1,077.84	$781.23		$178 529.81
6	$1,077.84		$297.90	

7.8

19. A mortgage of $190 000 is amortized over 15 years at 5.25% per year compounded semi-annually.

a) Use the TVM Solver or an online calculator to determine the monthly payment.

b) Use a spreadsheet to create an amortization table for the first 12 payments.

c) Determine the total amount and interest paid over the life of the mortgage.

7.9

20. How many payments will you make in 1 year if you repay your mortgage with:

a) Semi-monthly payments

b) Bi-weekly payments

c) Weekly payments

d) Accelerated bi-weekly payments

21. The Mahers were pre-approved for a mortgage of $250 000 amortized over 30 years at 7% per year compounded semi-annually. They will make monthly payments. If the Mahers apply for a mortgage over the Internet, they can choose to make one of these changes.

Option I: A 25-year amortization period

Option II: A 6.5% interest rate

Option III: Semi-monthly payments

Option IV: Accelerated bi-weekly payments

a) Determine the total interest under the original terms of the mortgage.

b) Predict which option would save the Mahers the most interest over the life of their mortgage. Explain your reasoning.

c) Verify your prediction in part b by calculating the interest saved under each option. Was your prediction correct? Explain.

Practice Test

Multiple Choice: Choose the correct answer for questions 1 and 2. Justify each choice.

1. Karyan deposits $675 at the end of each year into an account that earns 5.4% per year compounded annually. Which is the amount after 3 years?

A. $2025.00 **B.** $2136.32 **C.** $1791.01 **D.** $784.35

2. Which is the monthly payment on a mortgage of $230 000 amortized over 30 years at 5.75% per year compounded semi-annually?

A. $1320.93 **B.** $1332.34 **C.** $1342.22 **D.** $1277.78

Show your work for questions 3 to 6.

3. **Knowledge and Understanding** Determine the present value of quarterly payments of $250 for 2.5 years at 2.4% per year compounded quarterly.

4. **Application** The Zaidis arrange a mortgage amortized over 30 years at 9% per year compounded semi-annually. Here is part of an amortization table for the mortgage.

Payment number	Monthly payment	Interest paid	Principal paid	Outstanding balance
0				$80 000.00
1	$634.27	$589.05		$79 954.78
2				$79 909.23

a) How much did the Zaidis borrow?

b) How much of the 1st payment is principal? How much is interest?

c) Complete the row for payment 2. Explain why the interest payments decrease each month while the principal payments increase each month.

d) How much interest will the Zaidis pay over the life of the mortgage?

e) Suggest two strategies that the Zaidis can use to reduce the interest costs on their mortgage. Why will these strategies reduce the interest costs?

5. **Thinking** Mila will need $10 000 when she goes to college 5 years from now. She has 2 options for saving the money.

Option A: A regular deposit at the end of each month into an account that earns 7% per year compounded monthly

Option B: A regular deposit at the end of each year into an account that earns 7.25% per year compounded annually

Which option should Mila choose? Make a recommendation, then justify it.

6. **Communication** Explain the advantages of saving as early as possible for large expenses. Include examples to support your explanation.

Chapter Problem Planning Ahead

Materials

- TI-83 or TI-84 graphing calculator

Jamie and Sam have been saving money each month for the past 5 years. They now have $10 000 saved.

1. How much did Jamie and Sam set aside each month?
Assume an average interest rate of 4.8% per year compounded monthly.

2. Jamie and Sam plan to use the $10 000 as a down payment for a house. One house they are interested in buying has a selling price of $179 900. What monthly mortgage payment will Jamie and Sam make if they arrange a mortgage at 5.25% per year compounded annually for 20 years?

3. Jamie and Sam have different opinions about whether they should buy a house now.
- Sam suggests that they save the difference between the monthly payment in part b and their current monthly rental payment of $750 in an annuity that earns 3.5% per year compounded monthly for 5 years. He says that at the end of that time, they will have larger down payment and can afford a nicer house.
- Jamie suggests that they buy the house now since house values are expected to increase by an average of 3.5% per year for the next 5 years.
  What advice would you give Jamie and Sam?
  How would you convince them to follow your advice?
  Include calculations in your answer.