Chapters 1 to 8 COURSE REVIEW

Chapter 1 Rates of Change

1. Consider the function $f(x) = 2x^2 - 3x + 4$.

a) Determine the average rate of change between the point where $x = 1$ and the point determined by each x-value.

i) $x = 4$ **ii)** $x = 1.5$ **iii)** $x = 1.1$

b) Describe the trend in the average slopes. Predict the slope of the tangent at $x = 1$.

2. State whether each situation represents average or instantaneous rate of change. Explain your reasoning.

a) Ali calculated his speed as 95 km/h when he drove from Toronto to Windsor.

b) The speedometer on Eric's car read 87 km/h.

c) The temperature dropped by 2°C/h on a cold winter night.

d) At a particular moment, oil was leaking from a container at 20 L/s.

3. a) Expand and simplify each expression, then evaluate for $a = 2$ and $h = 0.01$.

i) $\dfrac{3(a+h)^2 - 3a^2}{h}$ **ii)** $\dfrac{(a+h)^3 - a^3}{h}$

b) What does each answer represent?

4. A ball was tossed into the air. Its height, in metres, is given by $h(t) = -4.9t^2 + 6t + 1$, where t is time, in seconds.

a) Write an expression that represents the average rate of change over the interval $2 \le t \le 2 + h$.

b) Find the instantaneous rate of change of the height of the ball after 2 s.

c) Sketch the curve and the rate of change.

5. Determine the limit of each infinite sequence. If the limit does not exist, explain.

a) 2, 2.1, 2.11, 2.111, ... **b)** $5, 5\frac{1}{2}, 5\frac{1}{3}, 5\frac{1}{4}, \ldots$

c) 2, −2, 2, −2, 2, ... **d)** 450, −90, 18, ...

6. Evaluate each limit. If it does not exist, provide evidence.

a) $\lim\limits_{x \to 2}(3x^2 - 4x + 1)$ **b)** $\lim\limits_{x \to -8} \dfrac{5x + 40}{x + 8}$

c) $\lim\limits_{x \to 6^+} \sqrt{x - 6}$ **d)** $\lim\limits_{x \to 3} \dfrac{2x - 9}{x - 3}$

7. a) Sketch a fully labelled graph of

$$f(x) = \begin{cases} 2x + 1, & x < 0 \\ 3, & x = 0 \\ 2x^2 - 1, & x > 0. \end{cases}$$

b) Evaluate each limit. If it does not exist, explain why.

i) $\lim\limits_{x \to 3} f(x)$ **ii)** $\lim\limits_{x \to 0} f(x)$

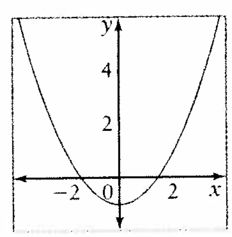
8. Use first principles to determine the derivative of each function. Then, determine an equation of the tangent at the point where $x = 2$.

a) $y = 4x^2 - 3$ **b)** $f(x) = x^3 - 2x^2$

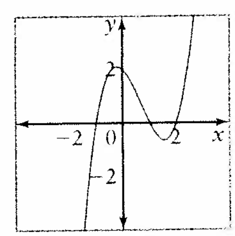
c) $g(x) = \dfrac{3}{x}$ **d)** $h(x) = 2\sqrt{x}$

9. Given the graph of $f(x)$, sketch a graph that represents its rate of change.

a)



b)



Chapter 2 Derivatives

10. Differentiate each function.

a) $y = -3x^2 + 4x - 5$

b) $f(x) = 6x^{-1} - 5x^{-2}$

c) $f(x) = 4\sqrt{x}$

d) $y = (3x^2 - 4x)(\sqrt{x} - 1)$

e) $y = \dfrac{x^2 + 4}{3x}$

11. Show that each statement is false.

a) $(f(x) \cdot g(x))' = f'(x) \cdot g'(x)$

b) $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)}{g'(x)}$

12. Determine the equation of the tangent to each function at the given value of x.

a) $f(x) = 2x^2 - 1$ at $x = -2$

b) $g(x) = \sqrt{x} + 5$ at $x = 4$

13. The position of a particle, s metres from a starting point, after t seconds, is given by the function $s(t) = 2t^3 - 7t^2 + 4t$.

a) Determine its velocity at time t.

b) Determine the velocity after 5 s.

14. Determine the points at which the slope of the tangent to $f(x) = x^3 - 2x^2 - 4x + 4$ is zero.

15. A fireworks shell is shot upward with an initial velocity of 28 m/s from a height of 2.5 m.

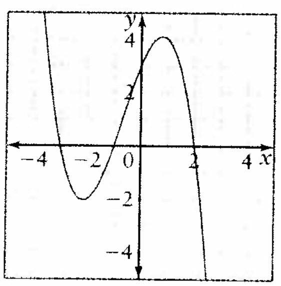
a) State an equation to represent the height of the shell at time t, in seconds.

b) Determine equations for the velocity and acceleration of the shell.

c) State the height, velocity, and acceleration after 2 s.

d) After how many seconds will the shell have the same speed, but be falling downward?

16. Copy this graph of the position function of an object. Draw graphs to represent the velocity and acceleration.



17. Determine the derivative of each function.

a) $y = 2(3x - x^{-1})^2$

b) $g(x) = \sqrt{2x + 5}$

c) $y = \frac{1}{\sqrt{3x - 1}}$

d) $\frac{-1}{\sqrt[3]{x^2 + 3x}}$

18. Determine an equation of the tangent to $f(x) = x^2(x^3 - 3x)^3$ at the point $(-1, 8)$.

19. The cost, in dollars, of water consumed by a factory is given by the function $C(w) = 15 + 0.1\sqrt{w}$, where w is the water consumption, in litres. Determine the cost and the rate of change of the cost when the consumption is 2000 L.

20. A fast-food restaurant sells 425 large orders of fries per week at a price of \$2.75 each. A market survey indicates that for each \$0.10 decrease in price, sales will increase by 20 orders of fries.

a) Determine the demand function.

b) Determine the revenue function.

c) Determine the marginal revenue function.

d) With what price will the marginal revenue be zero? Interpret the meaning of this value.

21. A car has constant deceleration of 10 km/h/s until it stops. If the car's initial velocity is 120 km/h, determine its stopping distance.

Chapter 3 Curve Sketching

22. Evaluate each limit.

a) $\lim_{x \to \infty} (2x^3 - 5x^2 + 9x - 8)$

b) $\lim_{x \to \infty} \frac{x + 1}{x - 1}$

c) $\lim_{x \to -\infty} \frac{x^2 - 3x + 1}{x^2 + 4x + 8}$

23. Sketch the graph of $f(x)$ based on the information in the table.

x	$(-\infty, -2)$	-2	$(-2, -1)$	-1	$(-1, 0)$	0	$(0, \infty)$
$f(x)$		-5		-1		3	
$f'(x)$	$-$	0	$+$	$+$	$+$	0	$-$
$f''(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$

24. For each function, determine the coordinates of the local extrema, the points of inflection, the intervals of increase and decrease, and the concavity.

a) $f(x) = x^3 + 2x^2 - 4x + 1$

b) $f(x) = \frac{3}{4}x^4 - x^3 - x^2 + 5x - 3$

c) $f(x) = \frac{3}{x^2 + 1}$

25. Analyse and sketch each function.

a) $f(x) = 3x^4 - 8x^3 + 6x^2$

b) $y = -x^3 + x^2 + 8x - 3$

c) $f(x) = \frac{x}{x^2 + 1}$

26. The power, in amps, transmitted by a belt drive from a motor is given by the function $P = 100v - \frac{3}{16}v^3$ where v is the linear velocity of the belt, in metres per second.

a) For what value of v is the power at a maximum value?

b) What is the maximum power?

27. A ship is sailing due north at 12 km/h while another ship is observed 15 km ahead, travelling due east at 9 km/h. What is the closest distance of approach of the two ships?

28. The Perfect Pizza Parlor estimates the average daily cost per pizza, in dollars, to be $C(x) = \frac{0.00025x^2 + 8x + 10}{x}$, where x is the number of pizzas made in a day.

a) Determine the marginal cost at a production level of 50 pizzas a day.

b) Determine the production level that would minimize the average daily cost per pizza.

c) What is the minimum average daily cost?

Chapter 4 Derivatives of Sinusoidal Functions

29. Differentiate each function.

a) $y = \cos^3 x$

b) $y = \sin(x^3)$

c) $f(x) = \cos(5x - 3)$

d) $f(x) = \sin^2 x \cdot \cos\left(\frac{x}{2}\right)$

e) $f(x) = \cos^2(4x^2)$

f) $g(x) = \frac{\cos x}{\cos x - \sin x}$

30. Determine the equation of the tangent to $y = 2 + \cos 2x$ at $x = \frac{5\pi}{6}$.

31. Use the derivatives of $\sin x$ and $\cos x$ to develop the derivatives of $\sec x$, $\csc x$, and $\tan x$.

32. Find the local maxima, local minima, and inflection points of the function $y = \sin^2 x - \frac{x}{2}$. Use technology to verify your findings and to sketch the graph.

33. The height above the ground of a rider on a large Ferris wheel can be modelled by $h(t) = 10\sin\left(\frac{2\pi}{30}t\right) + 12$, where h is the height above the ground, in metres, and t is time, in seconds. What is the maximum height reached by the rider, and when does this first occur?

34. A weight is oscillating up and down on a spring. Its displacement, from rest is given by the function $d(t) = \sin 6t - 4\cos 6t$, where d is in centimetres and t is time, in seconds.

a) What is the rate of change of the displacement after 1 s?

b) Determine the maximum and minimum displacements and when they first occur.

Chapter 5 Exponential and Logarithmic Functions

35. a) Compare the graphs of $y = e^x$ and $y = 2^x$.

b) Compare the graphs of the rates of change of $y = e^x$ and $y = 2^x$.

c) Compare the graphs of $y = \ln x$ and $y = e^x$, and their rates of change.

36. Evaluate, accurate to two decimal places.

a) $\ln 5$ b) $\ln e^2$ c) $(\ln e)^2$

37. Simplify.

a) $\ln(e^x)$ b) $e^{\ln x}$ c) $D^{\ln D}$

38. Determine the derivative of each function.

a) $y = -2e^x$ b) $g(x) = 5 \cdot 10^x$

c) $h(x) = \cos(e^x)$ d) $f(x) = xe^{-x}$

39. Determine the equation of the line perpendicular to $f(x) = \frac{1}{2}e^{x+1}$ at its y-intercept.

40. Radium decays at a rate that is proportional to its mass, and has a half-life of 1590 years. If 20 g of radium is present initially, how long will it take for 90% of this mass to decay?

41. Determine all critical points of $f(x) = x^2e^x$. Sketch a graph of the function.

42. The power supply, in watts, of a satellite is given by the function $P(t) = 200e^{-0.001t}$, where t is the time, in days, after launch. Determine the rate of change of power

a) after t days   b) after 200 days

43. The St. Louis Gateway Arch is in the shape of a catenary defined by the function $y = -20.96\left(\frac{e^{0.0329x} + e^{-0.0329x}}{2} - 10.06\right)$, with all measurements in metres.

a) Determine an equation for the slope of the arch at any point x m from its centre.

b) What is the slope of the arch at a point 2 m horizontally from the centre?

c) Determine the width and the maximum height of the arch.

44. Show how the function $y = e^{-x}\sin x$ could represent dampened oscillation.

a) Determine the maximum and minimum points for a sequence of wavelengths.

b) Show that the maximum points represent exponential decay.

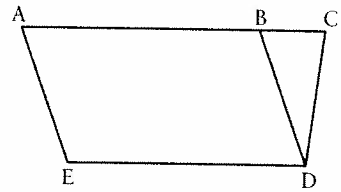
Chapter 6 Geometric Vectors

45. Use an appropriate scale to draw each vector.

a) displacement of 10 km at a bearing of 15°

b) velocity of 6 m/s upward

46. ACDE is a trapezoid, such that $\overrightarrow{AB} = \vec{u}$ and $\overrightarrow{AE} = \vec{v}$, BD ∥ AE, and AB = 5BC. Express each vector in terms of $\vec{u}$ and $\vec{v}$.



a) $\overrightarrow{DE}$ b) $\overrightarrow{DB}$ c) $\overrightarrow{BC}$

d) $\overrightarrow{AD}$ e) $\overrightarrow{CD}$ f) $\overrightarrow{CE}$

47. Simplify algebraically.

a) $3\vec{u} + 5\vec{u} - 7\vec{u} - 6\vec{u}$

b) $-5(\vec{c} + \vec{d}) - 8(\vec{c} - \vec{d})$

48. Determine the resultant of each vector sum.

a) 12 km north followed by 15 km east

b) a force of 60 N upward with a horizontal force of 40 N

49. A rocket is propelled vertically at 450 km/h. A horizontal wind is blowing at 15 km/h. What is the ground velocity of the rocket?

50. Two forces act on an object at 20° to each other. One force has a magnitude of 200 N, and the resultant has a magnitude of 340 N.

a) Draw a diagram illustrating this situation.

b) Determine the magnitude of the second force and direction it makes with the resultant.

51. An electronic scoreboard of mass 500 kg is suspended from a ceiling by four cables, each making an angle of 70° with the ceiling. The weight is evenly distributed. Determine the tensions in the cables, in newtons.

52. A ball is thrown with a force that has a horizontal component of 40 N. The resultant force has a magnitude of 58 N. Determine the vertical component of the force.

53. Resolve the velocity of 120 km/h at a bearing of 130° into its rectangular components.

54. A 100-N box is resting on a ramp inclined at 42° to the horizontal. Resolve the weight into the rectangular components keeping it at rest.

Chapter 7 Cartesian Vectors

55. Express each vector in terms of $\vec{i}$, $\vec{j}$, and $\vec{k}$.

a) $[5, 6, -4]$ **b)** $[0, -8, 7]$

56. Given points P(2, 4), Q(−6, 3), and R(4, −10), determine each of the following.

a) $|\overrightarrow{PQ}|$ **b)** $\overrightarrow{PR}$

c) $3\overrightarrow{PQ} - 2\overrightarrow{PR}$ **d)** $\overrightarrow{PQ} \cdot \overrightarrow{PR}$

57. Write each of the following as a Cartesian vector.

a) 30 m/s at a heading of 20°

b) 40 N at 80° to the horizontal

58. A ship's course is set at a heading of 214°, with a speed of 20 knots. A current is flowing from a bearing of 93°, at 11 knots. Use Cartesian vectors to determine the resultant velocity of the ship, to the nearest knot.

59. Use examples to explain these properties.

a) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

b) $k(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (k\vec{b})$

60. Calculate the dot product for each pair of vectors.

a) $|\vec{u}| = 12$, $|\vec{v}| = 21$, $\theta = 20°$

b) $|\vec{s}| = 115$, $|\vec{t}| = 150$, $\theta = 42°$

61. Given $\vec{u} = [3, -4]$, $\vec{v} = [6, 1]$, and $\vec{w} = [-9, 6]$, evaluate each of the following, if possible. If it is not possible, explain why.

a) $\vec{u} \cdot (\vec{v} + \vec{w})$ **b)** $\vec{u} \cdot (\vec{v} \cdot \vec{w})$

c) $\vec{u}(\vec{v} \cdot \vec{w})$ **d)** $(\vec{u} - \vec{v}) \cdot (\vec{u} + \vec{v})$

62. Determine the value of k so that $\vec{u} = [-3, 7]$ and $\vec{v} = [6, k]$ are perpendicular.

63. Determine whether or not the triangle with vertices A(3, 5), B(−1, 4), and C(6, 2) is a right triangle.

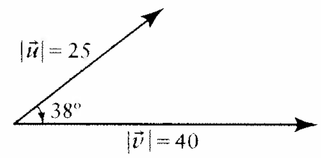
64. Determine the angle between the vectors $\vec{u} = [4, 10, -2]$ and $\vec{v} = [1, 7, -1]$, accurate to the nearest degree.

65. Determine the work done by each force $\vec{F}$, in newtons, for an object moving along the vector $\vec{d}$, in metres.

a) $\vec{F} = [1, 4]$, $\vec{d} = [6, 3]$

b) $\vec{F} = [320, 145]$, $\vec{d} = [32, 15]$

66. Determine the projection of $\vec{u}$ on $\vec{v}$.



67. Roni applies a force at 10° to the horizontal to move a heavy box 3 m horizontally. He does 100 J of mechanical work. What is the magnitude of the force he applies?

68. Graph each position vector.

a) $[4, 2, 5]$ **b)** $[-1, 0, -4]$ **c)** $[2, -2, -2]$

69. The initial point of vector $\overrightarrow{AB} = [6, 3, -2]$ is A(1, 4, 5). Determine the coordinates of the terminal point B.

70. Consider the vectors $\vec{a} = [7, 2, 4]$, $\vec{b} = [-6, 3, 0]$, and $\vec{c} = [4, 8, 6]$. Determine the angle between each pair of vectors.

71. Prove for any three vectors $\vec{u} = [u_1, u_2, u_3]$, $\vec{v} = [v_1, v_2, v_3]$, and $\vec{w} = [w_1, w_2, w_3]$, that $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$.

72. Determine two possible vectors that are orthogonal to each vector.

a) $\vec{u} = [-1, 5, 4]$ **b)** $\vec{v} = [5, 6, 2]$

73. A small airplane takes off at an airspeed of 180 km/h, at an angle of inclination of 14°, toward the east. A 15-km/h wind is blowing from the southwest. Determine the resultant ground velocity.

74. Determine $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$. Confirm that the cross product is orthogonal to each vector. $\vec{a} = [4, -3, 5]$, $\vec{b} = [2, 7, 2]$

75. Determine the area of the triangle with vertices P(3, 4, 8), Q(−2, 5, 7), and R(−5, −1, 6).

76. Use an example to verify that $k\vec{u} \times \vec{v} = (k\vec{u}) \times \vec{v} = \vec{u} \times (k\vec{v})$.

77. Determine two vectors that are orthogonal to both $\vec{u} = [-1, 6, 5]$ and $\vec{v} = [4, 9, 10]$.

78. Find the volume of the parallelepiped defined by the vectors $\vec{u} = [-2, 3, 6]$, $\vec{v} = [6, 7, -4]$, and $\vec{w} = [5, 0, 1]$.

79. When a wrench is rotated, the magnitude of the torque is 26 N·m. A 95-N force is applied 35 cm from the fulcrum. At what angle to the wrench is the force applied?

Chapter 8 Lines and Planes

80. Write vector and parametric equations of the line through.

A(−2, −1, −7) and B(5, 0, 10).

81. Graph each line.

a) $[x, y] = [2, 3] + t[-2, 5]$

b) $x = -3 + 5t$
$y = 4 + 3t$

82. Write the scalar equation of each line, given the normal vector $\vec{n}$ and point P_0.

a) $\vec{n} = [4, 8]$, $P_0(3, -1)$

b) $\vec{n} = [-6, -7]$, $P_0(-5, 10)$

83. Determine the vector equation of each line.

a) parallel to the y-axis and through P(6, −3)

b) perpendicular to $2x + 7y = 5$ and through A(1, 6)

84. Determine the coordinates of two points on the plane with equation $5x + 4y - 3z = 6$.

85. Write the parametric and scalar equations of the plane given its vector equation $[x, y, z] = [4, 3, -5] + s[2, 1, 4] + t[6, 6, -3]$.

86. Determine the intercepts of each plane.

a) $[x, y, z] = [1, 8, 6] + s[1, -12, -12] + t[2, 4, -3]$

b) $2x - 6y + 9z = 18$

87. Determine an equation for each plane. Verify your answers using 3-D graphing technology.

a) containing the points A(5, 2, 8), B(−9, 10, 3), and C(−2, −6, 5)

b) parallel to both the x-axis and y-axis and through the point P(−4, 5, 6)

88. Write the equation of the line perpendicular to the plane $7x - 8y + 5z = 1$ and passing through the point P(6, 1, −2).

89. Determine the angle between the planes with equations $8x + 10y + 3z = 4$ and $2x - 4y + 6z = 3$.

90. Determine whether the lines intersect. If they do, find the coordinates of the point of intersection.
$[x, y, z] = [2, -3, 2] + s[3, 4, -10]$
$[x, y, z] = [3, 4, -2] + t[-4, 5, 3]$

91. Determine the distance between these skew lines.
$[x, y, z] = [2, -7, 3] + s[3, -10, 1]$
$[x, y, z] = [3, 4, -2] + t[1, 6, 1]$

92. Determine the distance between A(2, 5, −7) and the plane $3x - 5y + 6z = 9$.

93. Does the line $r = \vec{r}[-5, 1, -2] + k[1, 6, 5]$ intersect the plane with equation $[x, y, z] = [2, 3, -1] + s[1, 3, 4] + t[-5, 4, 7]$? If so, how many solutions are there?

94. State equations of three planes that

a) are parallel

b) are coincident

c) intersect in a line

d) intersect in a point

95. Solve each system of equations.

a)
$2x - 4y + 5z = -4$
$3x + 2y - z = 1$
$4x + 3y - 3z = -1$

b)
$5x + 4y + 2z = 7$
$3x + y - 3z = 2$
$7x + 7y + 7z = 12$

22. Answers may vary **a)** Intersect at $(-4, 1, 3)$ **b)** Intersect in pairs **c)** Intersect at $(5.5, -2.5, 0)$ **d)** One plane intersects two parallel planes **e)** Intersect in a line **f)** Intersect in a line
23. a) The three planes do not have a common intersection **b)** Intersect at a point

Chapter 8 Practice Test, pages 504–505

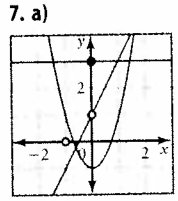
1. D **2.** B **3.** B **4.** D **5.** D **6.** A **7.** D **8.** B **9.** D **10.** A
11. Answers may vary **a)** $[x, y, z] = [1, 5, -4] + t[1, -14, 4]$
b) $(3, -23, 4)$ and $(0, 19, -8)$ **12.** $x = \dfrac{46}{3} + t, y = 2t$
13. $x = -6 + 6t; y = 4 - 5t; z = 3 - 2t$ **14.** $y + 1 = 0$
15. a) No **b)** $(3, 5)$ **c)** $(4, -2, 1)$ **d)** No
16. a) Lines do not intersect and are not parallel
b) 2.45 **17.** Yes **18.** Answers may vary
19. a) Intersect at a line **b)** Intersect at a point
c) Two parallel planes intersected by third plane
d) Intersect at a line **20. a)** Answers may vary; $[x, y, z] = [1, 13, 2] + s[3, 19, -3] + t[2, 14, 5]$; $x = 1 + 3s + 2t, y = 13 + 19s + 14t, z = 2 - 3s + 5t$
b) $137x - 21y + 4z + 128 = 0$ **c)** Answers may vary, $(4, 32, -1), (3, 27, 7)$ **21.** Answers will vary. For example: No such s and t exist where the x-, y-, and z-coordinates are simultaneously equal to zero.
b) 0.71 **22.** Answers will vary **23. a)** Yes **b)** $(10, -4, 4), (6, 4, 0), (12, 8, -6)$ **c)** 34.9 units **d)** 5.39 units^2

Chapters 6 to 8 Review, pages 506–507

1. a) 340° **b)** 144° **c)** 260° **2. a)** S50°E **b)** N70°W
c) S86°E **3.** Answers may vary **a)** $\overline{DB}$ **b)** Yes
c) $\angle FAE = \angle BDC$ **d)** $\overline{AE} - \overline{BE}$ **4.** 50 N in the northwest direction **5.** Answers may vary. **6.** Answers may vary.
7. Answers may vary **8. a)** 12.8 N N38.7° W
b) 80.9 m/s 8.5° up from the horizontal
c) 54.8 N 4.5° from the horizontal
9. Answers may vary. 75 cos 20° N of normal force; 75 sin 20°N of frictional force
10. a) Answers may vary; $[0.55, 0.83], [-0.55, -0.83]$
b) $\sqrt{13}$ **c)** $Q(-5,8)$ **11.** Answers may vary
12. 10 knots at a bearing of 232.4° **13.** 7446.0 N
14. a) -18 **b)** 7 **15.** Answers may vary; $\vec{u} = [5, 6]$
16. 45.3° **17.** 1575.7 J **18.** $\vec{v} = [6, -6, 7]$; 11
19. a) -316 **b)** $(-6, 46, -24)$
20. Answers may vary; $\vec{u} \times \vec{v} = -\vec{u} \times \vec{v}$
21. $(-11, 34, 18), (11, -34, -18)$
22. 52.3°, 127.7° **23.** 7.2 Nm
24. a) $[x, y] = [3, 5] + t[7, -2]; x = 3 + 7t, y = 5 - 2t$
b) Answers may vary; $[x, y, z] = [6, -1, 5] + t[8, 2, -1]$; $x = 6 + 8t, y = -1 + 2t, z = 5 - t$ **25.** $3x + 7y - 4 = 0$
26. $5x + y + 14 = 0; [x, y] = [-4, 6] + t[-1, 5]$
27. a) $z = -1$ **b)** $35x + y - 6z - 194 = 0$
c) $2x + 6y + 4z - 11 = 0$ **28.** Answers may vary
29. a) $10x - 26y + 19z - 37 = 0$
b) $x + 2y - 5z + 13 = 0$ **30.** 48.6°
31. Intersect at $(-13, 10, -1)$ **32.** 2 units
33. a) $(1, 2, 1)$ **b)** No intersection **34.** 1.96
35. a) Parallel and distinct **b)** Coincident
36. a) Intersect at a line **b)** Intersect at $(0, 2, -1)$
c) No solution; planes intersect in pairs

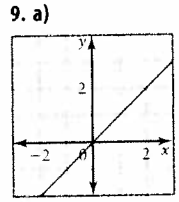
Chapters 1 to 8 Course Review, pages 509–514

1. a) i) 7 **ii)** 2 **iii)** 1.2 **b)** 1 **2. a)** Average
b) Instantaneous **c)** Average **d)** Instantaneous
3. a) i) $6a + h$; 12.01 **ii)** $3a^2 + 3ah + h^2$; 12.06001
b) i) An approximation of the slope of the tangent to $f(x) = 3x^2$ at $x = 2$ **ii)** An approximation of the slope of the tangent to $f(x) = x^3$ at $x = 2$
4. a) $(-13.6 - 4.9h)$ m/s **b)** -13.6 m/s
5. a) 2.111 111 111 **b)** 5 **c)** Limit does not exist
d) 0 **6. a)** 5 **b)** 5 **c)** 0 **d)** Limit does not exist
7. a)

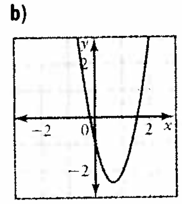


b) i) 17 **ii)** Limit does not exist **8. a)** $\dfrac{dy}{dx} = 8x$; 16
b) $\dfrac{dy}{dx} = 3x^2 - 4x$; 4 **c)** $\dfrac{dy}{dx} = -\dfrac{3}{x^2}; -\dfrac{3}{4}$ **d)** $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x}}; \dfrac{1}{\sqrt{2}}$

9. a)



b)

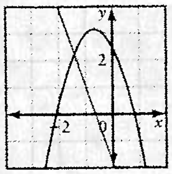


10. a) $\dfrac{dy}{dx} = -6x + 4$ **b)** $\dfrac{dy}{dx} = -6x^{-2} + 10x^{-3}$ **c)** $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{x}}$
d) $\dfrac{dy}{dx} = (6x - 4)(\sqrt{x} - 1) + (3x^2 - 4)\left(\dfrac{1}{2\sqrt{x}}\right)$
e) $\dfrac{dy}{dx} = \dfrac{(x - 2)(x + 2)}{3x^2}$
11. Answers may vary **12. a)** -8 **b)** $\dfrac{1}{4}$
13. a) $v(t) = 6t^2 = 14t + 4$ **b)** 84 m/s

14. $(-2, -4), \left(\frac{2}{3}, \frac{59}{27}\right)$

15. a) $h(t) = -4.9t^2 + 28t + 2.5$ **b)** $v(t) = -9.8t + 28$, $a(t) = -9.8$
c) 38.9 m; 8.4 m/s; -9.8 m/s^2 **d)** 3.7 s

16.

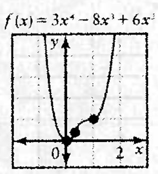


17. a) $\frac{dy}{dx} = 4(3x - x^{-1})(3 + x^{-2})$ **b)** $\frac{dy}{dx} = \frac{1}{\sqrt{2x+5}}$
c) $\frac{dy}{dx} = -\frac{3}{2(\sqrt{3x}+1)^3}$ **d)** $\frac{dy}{dx} = \frac{2x+3}{3(\sqrt[3]{x^2+3x})^4}$

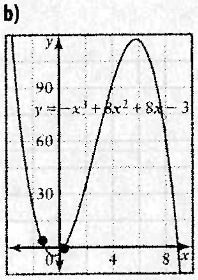
18. $32x + 2y = 1 = 0$ **19.** $19.47; $0.0011 per L
20. a) $D(x) = 425 + 20x$
b) $R(x) = (425 + 20x)(2.75 = 0.1x)$
c) $R'(x) = -4x + 12.5$ **d)** $2.44 **21.** 66.7 m
22. a) ∞ **b)** 1 **c)** 1 **23.** Answers will vary

24. a) Local max: $(-2, -4)$; local min: $\left(\frac{2}{3}, \frac{59}{27}\right)$; POI: $\left(-\frac{2}{3}, \frac{115}{27}\right)$; concave down: $x < -\frac{2}{3}$; concave up: $x > -\frac{2}{3}$ **b)** Local min: $(-1.07, -7.29)$; POI: $\left(-\frac{2}{3}, \frac{115}{27}\right)$; concave down: $x < -\frac{2}{3}$; concave up: $x > -\frac{2}{3}$
c) Local max: $(0, 3)$; POI: $\left(\frac{1}{\sqrt{3}}, \frac{9}{4}\right), \left(-\frac{1}{\sqrt{3}}, \frac{9}{4}\right)$; concave down: $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$; concave up: $x > \frac{1}{\sqrt{3}}, x < -\frac{1}{\sqrt{3}}$

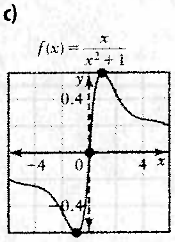
25. a)



b)



c)



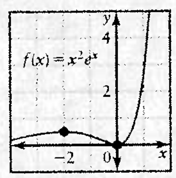
26. a) $v = \frac{40}{3}$ m/s **b)** 889 amps **27.** 9 km **28. a)** $410.63
b) 200 pizzas per day **c)** $8.10 **29. a)** $\frac{dy}{dx} = -3\cos^2 x \sin x$
b) $\frac{dy}{dx} = 3x^2 \cos(x^3)$ **c)** $\frac{dy}{dx} = -5\sin(5x - 3)$
d) $\frac{dy}{dx} = 2\sin x \cos x \cos\left(\frac{x}{2}\right) - \frac{1}{2}\sin^2 x \sin\left(\frac{x}{2}\right)$
e) $\frac{dy}{dx} = -16x\cos(4x^2)\sin(4x^2)$ **f)** $\frac{dy}{dx} = \frac{1}{(\cos x - \sin x)^2}$

30. $y = \sqrt{3}x + 3 - \frac{5\sqrt{3}}{6}\pi$

31. $y = \sec x$: $\frac{dy}{dx} = \sec x \tan x$; $y = \csc x$: $\frac{dy}{dx} = -\csc x \cot x$; $y = \tan x$: $\frac{dy}{dx} = \sec^2 x$

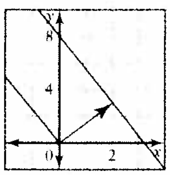
32. Local max: $x = 2\pi n + \frac{\pi}{6}$; local min: $x = 2\pi n + \frac{5\pi}{6}$; POI: $x = 2\pi n + \frac{\pi}{4}$, $x = 2\pi n + \frac{5\pi}{4}$ **33.** 22 m; 7.5 s
34. a) -0.945 cm/s **b)** Max: 4.123 cm, 0.48 s; min: -4.123 cm, 1.01 s **35.** $y = e^x$ increases faster as x increases; curves have same horizontal asymptote and go through (0, 1) **36. a)** 1.61 **b)** 2 **c)** 1 **37. a)** x **b)** x **c)** x^2
38. a) $\frac{dy}{dx} = -2e^x$ **b)** $g'(x) = 5 \ln 10 \cdot 10^x$ **c)** $h'(x) = -e^x \sin(e^x)$
d) $f'(x) = e^{-x}(1 = x)$ **39.** $y = \frac{e}{2}x + \frac{e}{2}$ **40.** 5282 years

41.

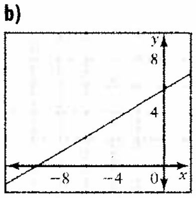


42. a) $P'(t) = -0.2e^{-0.001t}$ **b)** -0.164 Watts/day
43. a) $\frac{dy}{dx} = -0.344792(e^{0.0329x} - e^{-0.0329x})$ **b)** -0.0454
c) Width: 182.4 m; max height: 189.9 m
44. a) Max and min are π units apart **b)** Min's and max's get closer and closer to zero as x increases
45. Answers may vary **46. a)** $\vec{u}$ **b)** $-\vec{v}$ **c)** $\frac{1}{5}\vec{u}$ **d)** $\vec{u} + \vec{v}$
e) $-\frac{1}{5}\vec{u} + \vec{v}$ **f)** $-\frac{6}{5}\vec{u} + \vec{v}$ **47. a)** $-5\vec{u}$ **b)** $-13\vec{c} + 2\vec{d}$
48. a) 19.2 km [N 51.3° E] **b)** 72.1 N 56.3° up from the horizontal **49.** 450.2 km/h 1.9° from the vertical
50. b) 521 N at an angle of 11.6° from the resultant force
51. 1303.6 N each **52.** 42 N **53.** $120\cos 40°$ km/h to the right and $120\sin 40°$ km/h down **54.** $100\cos 42°$ N of normal force by the surface and $100\sin 42°$ N of friction.
55. a) $5\vec{i} + 6\vec{j} - 4\vec{k}$ **b)** $-8\vec{j} + 7\vec{k}$ **56. a)** $\sqrt{65}$
b) $\overline{PR} = [2, -14]$ **c)** $[-28, 25]$ **d)** -4
57. Answers may vary **58.** 27 km on a bearing of 234.2°
59. Answers will vary **60. a)** 236.8 **b)** 12819.2
61. a) -37 **b)** Not possible **c)** $[-144, 192]$ **d)** -12

62. $\frac{18}{7}$ **63.** No **64.** 14° **65. a)** 18 J **b)** 12415 J **66.** 19.7
67. 33.8 N
68. a, b, c) Answers may vary.
69. B (7, 7, 3) **70.** $\vec{a}$ and $\vec{b}$: 130.2°; $\vec{a}$ and $\vec{c}$: 40.5°; $\vec{b}$ and $\vec{c}$: 90° **72.** Answers may vary **a)** [5, 1, 0], [0, −4, 5] **b)** [0, −1, 3], [6, −5, 0] **73.** 185.6 km/h [N86.7°E]
74. $\vec{a}\cdot(\vec{a}\times\vec{b})=0$, $\vec{b}\cdot(\vec{a}\times\vec{b})=0$ **75.** 16.9 units^2
76. Answers will vary **77.** Answers will vary
78. 260 units^3 **79.** 51.4° **80.** Answers may vary; $[x, y, z] = [5, 0, 10] + t[7, 1, 17]$; $x = 5 + 7t$, $y = t$, $z = 10 + 17t$
81. a)



b)



82. a) $4x + 8y - 80 = 0$ **b)** $6x + 7y - 40 = 0$
83. a) $[x, y] = [6, -3] + t[0, 1]$ **b)** $[x, y] = [1, 6] + t[2, 7]$
84. Answers may vary **85.** $9x = 10y = 2z = 16 = 0$; $x = 4 + 2s + 6t$, $y = 3 + s + 6t$, $z = -5 + 4s - 3t$
86. a) x-intercept: −3; y-intercept: $\frac{4}{3}$; z-intercept: $-\frac{9}{7}$ **b)** x-intercept: 9; y-intercept: −3; z-intercept: 2 **87. a)** $64x + 7y - 168z + 1010 = 0$ **b)** $z = 6$ **88.** Answers may vary; $[x, y, z] = [6, 1, -2] + t[7, -8, 5]$ **89.** 86.5° **90.** No, they are skew **91.** 5.51
92. 8.37 units **93.** One **94.** Answers may vary
95. a) (−1, 3, 2) **b)** $[x, y, z] = \left[\frac{1}{7}, \frac{12}{7}, 0\right] + t[-3, -3, 1]$

PREREQUISITE SKILLS APPENDIX

Analyzing Polynomial Graphs

1. a) decreasing for $-\infty < x < -3.5$ and $3 < x < \infty$; increasing for $-3.5 < x < 3$ **b)** Positive for $-\infty < x < 6$ and $1 < x < 4$; negative for $-6 < x < 1$ and $4 < x < \infty$ **c)** zero slope at $x = -3.5$ and $x = 3$; negative slope for $-\infty < x < -3.5$ and $3 < x < \infty$; positive slope for $-3.5 < x < 3$

Arc Measure and Arc Length

1. a) 270° **b)** 225° **c)** 210° **d)** 240° **e)** 150° **f)** 360°
2. a) $\frac{\pi}{[illegible]}$ rad **b)** $\frac{\pi}{[illegible]}$ rad **c)** $\frac{\pi}{[illegible]}$ rad **d)** π rad **e)** 2π rad
f) 3π rad **3. a)** $\frac{1}{2}$ rad **b)** 5 rad

Applying Exponent Laws and Laws of Logarithms

1. a) bk^4 **b)** $16x^2y^6$ **c)** b^{3n-2} **2. a)** 2 **b)** $\frac{2}{3}$ **c)** 3

Changing Bases of Exponential and Logarithmic Expressions

1. a) $y = 2^{2x}$ **b)** $y = 2^{20x}$ **c)** $y = 2^{-6x}$ **2. a)** $\frac{\log 20}{\log 4} = 2.16$ **b)** $\frac{\log 112}{\log 3} = 4.29$ **c)** $\frac{\log \frac{1}{7}}{\log 8} = -0.94$ **d)** $\frac{\log \frac{1}{8}}{\log 2} = -3$

Construct and Apply an Exponential Model

1. a) $N = 4(2)^{\frac{t}{10}}$ **b)** 16 million **c)** 9.85 million
2. a) $N = 15\left(\frac{1}{2}\right)^{\frac{t}{6}}$ **b)** 3.75 g **c)** 1.33 g

Converting a Bearing to an Angle in Standard Position

1. a) 330° **b)** 40° **c)** 90° **d)** 180° **e)** 225° **f)** 120° **g)** 30° **h)** 215° **i)** 0° **j)** 135°

Create Composite Functions

1. a) $\frac{3x+8}{x+2}$ **b)** $\frac{1}{2x+5}$ **c)** $\sqrt{6x+5}$ **d)** $4x + 9$
2. a) $f(x) = x^4$ and $g(x) = 5x - 2$ **b)** $f(x) = \sqrt{x}$ and $g(x) = 3x - 4$ **c)** $f(x) = \frac{1}{x^3}$ and $g(x) = 2x^3 - 4$

Determining Slopes of Perpendicular Lines

1. a) $-\frac{1}{4}$ **b)** $\frac{3}{2}$ **c)** $\frac{4}{3}$

Differentiating Rules

1. $f'(x) = 12x^2 + 10x - 3$
2. $f'(x) = 6x(12x^2 + 5)(6x^4 + 5x^2)^2$
3. $f'(x) = 6(5t^3 + t - 3)$

Distance Between Two Points

1. a) $3\sqrt{2}$ **b)** $\sqrt{65}$

Domain of a Function

1. a) no restrictions **b)** $x \neq 4$ **c)** no restrictions

Dot and Cross Products

1. a) −5 **b)** (7, 11, −7) **c)** 131.9°

Equations and Inequalities

1. a) $x = 2, -8$ **b)** $x = \pm\frac{4}{5}$ **c)** $b = 7, -2$
d) $t = -0.31, 1.02$ **e)** $x = -2, 1, 4$
g) $x = -3, -0.83, 4.83$
2. a) $x > -4$ **b)** $x > 3$ **c)** $x < -5$ or $x > 0$
d) $-6 < x < -3$ **e)** $-2 < x < 1$ or $x > 4$