7.1 – Intro to Sequences and Recursion

A number sequence is a set of numbers in a particular order.
Ex. 3, 5, 7, 9, 11

t_1 t_2 Comma to separate

The n^{th} term is referred to as t_n

The sequence may stop ay some value, or continue indefinitely.
Ex. 2, 8, 32, 128, …

Ellipsis indicate that it continues indefinitely

Examples:

1) Find the first five terms of the sequence given by the formula for the n^{th} term.
a) $t_n = 3n - 2$ b) $t_n = 2^n - 5$

$t_1 =$ $t_1 =$
$t_2 =$ $t_2 =$
$t_3 =$ $t_3 =$
$t_4 =$ $t_4 =$
$t_5 =$ $t_5 =$

2) List the next three terms in the sequence.
a) 4, 8, 12, … b) -3, -2, -1, … c) 4, 7, 10, …

3) Can you find the formula for the n^{th} term for each sequence from #2?
a) b) c)

An explicit formula allows you to calculate any term in a sequence without knowing the previous term. So far, we have only seen explicit formulas.

Another type of formula is a recursion formula – which calculates terms from one or more previous terms. (*Happens a lot in Computer Science*)

Definition: Recursion→ See "Recursion". If you still don't get it, see "Recursion".

4) Find the next four terms.

$t_1 = 3$

$t_n = t_{n-1} + 2$

In fact, this is just another way of expressing $t_n =$

Some Famous Examples of Recursion:

a) Fibonacci Sequence

$t_1 = 1$

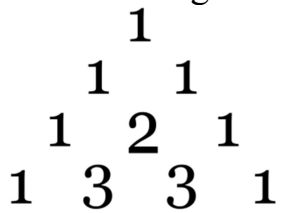
$t_2 = 1$

$t_n = t_{n-1} + t_{n-2}$

1, 1, 2, 3, 5, 8, 13, 21, 34, …

https://www.youtube.com/watch?v=wTlw7fNcO-0

b) Pascal's Triangle



Try to find the next four rows. Why is it recursion?

https://www.google.ca/search?q=pascal%27s+triangle&tbm=isch&tbo=u&source=univ&sa=X&ved=0ahUKEwjfw6GclP_TAhXi7YMKHc-RBy4QsAQIOw&biw=1920&bih=974#imgrc=9FelDcw_vAQgYM:

https://www.youtube.com/watch?v=XMriWTvPXHI

c) Sierpinski's Triangle

d) Koch's Snowflake

https://www.youtube.com/watch?v=zwb3ZXEdpos

7.2 – The Binomial Theorem

Definitions:

Sigma Notation: A compact way of writing a lot of addition.

$\sum$

Example:

$$\sum_{i=1}^{n} 2^i x^i = 2x + 4x^2 + 8x^3 + \cdots + 2^n x^n$$

$3 + 4 + 5 + \ \ldots$

$25 + 5 + 1 + \ldots$

Pascal's Triangle:

Binomial Theorem:

Expand:

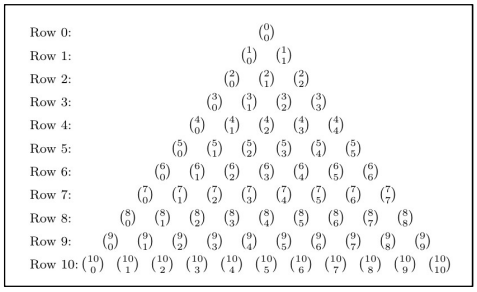
$(x + y)^0 =$ $(x + y)^3 =$

$(x + y)^1 =$ $(x + y)^4 =$

$(x + y)^2 =$ $(x + y)^5 =$

The Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$



Examples:

1. Expand and simplify:

a) $(x+y)^8$

b) $(x+3)^4$

c) $(1+x^2)^3$

d) $(a-2b)^5$

e) $\left(x-\frac{1}{4}\right)^4$

7.3 – Arithmetic Sequences

An arithmetic sequence is a sequence where each new term is obtained by adding (or subtracting) the same number to the previous term.

$d \rightarrow common\ difference$
$a \rightarrow the\ first\ term$

Examples:

1) State the common difference.

a) 3, 7, 11, 15, … $d =$ $a =$

b) 5, 2, -1, … $d =$ $a =$

c) $2x, 4x, 6x, ...$ $d =$ $a =$

2) Find the formula for t_n

a) 3, 7, 11, 15, …

b) 5, 2, -1, …

In general: for the sequence $a, a + d, a + 2d, a + 3d, ...$

$\boxed{t_n = a + d(n - 1)}$

3) Given the sequence 5, 9, 13, 17, … find:

a) t_n

b) t_{250}

c) the term # whose value is 401

4) In an arithmetic sequence, $t_4 = 21$ and $t_{40} = 237$. Find a, d, t_n.

7.4 – Geometric Sequences

A geometric sequence is a sequence of numbers such that each new term can be found by multiplying the previous term by some number. This number is called the common ratio, and is denoted by r. The first term is still a.

The general geometric sequence is:
$a, ar, ar^2, ar^3, \ldots$

$\therefore t_n =$

Notice:

Examples:

1) Given the formula for t_n, find the first four terms:

a) $t_n = 2(6)^{n-1}$

$t_1 =$
$t_2 =$
$t_3 =$
$t_4 =$

b) $t_n = 4(-3)^{n-1}$

$t_1 =$
$t_2 =$
$t_3 =$
$t_4 =$

2) Find the common ratio:

a) 5, 10, 20, 40, …

b) 2, -14, 98, -686, …

3) Given t_n, find t_7:

a) $t_n = -3(7)^{n-1}$

b) $t_n = -4(-5)^{n-1}$

4) Find the formula for the n^{th} term:

a) $a = 3, r = 2$

b) $5, -15, 45, -135, \ldots$

5) Find the number of terms in the sequence.

a) $3, 6, 12, \ldots, 1536$

b) $\frac{1}{25}, \frac{1}{5}, 1, \ldots, 625$

6) In a geometric sequence, $t_2 = 4 \; and \; t_4 = 64$. Find t_n.

7.5 – Arithmetic Series

A series is the sum of the terms of a sequence.

An arithmetic series is the sum of the terms of an arithmetic sequence.

S_n denotes the sum of the first n terms.

Find the sum of the first 100 natural number by writing the series in order, and then in reverse order.:

Examples:

1) Find the sum of the first 40 terms of each series:

a) $4 + 7 + 10+, \ldots$

b) $-2 - 4 - 6-, \ldots$

2) Find the sum of $4 + 10 + 16 + \cdots + 70$:

7.6 – Geometric Series

A geometric series is the sum of the terms of a geometric sequence.

In general, a geometric series looks like:
$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$

Let's multiply both sides by r

Examples:

1) Find S_7 for the series $2 + 6 + 18 + \cdots$

2) Find S_8 for the series $4 - 20 + 100 - \cdots$

3) Find S_{10} for $4 + 2 + 1 + \cdots$

4) Find the sum of $5 - 15 + 45 - \cdots + 3645$.

5) Find the value of "a" given $S_4 = 11480\ and\ r = 9$.