MHF 4UI **Trigonometric Equations**

1. Solve for x, $0 \le x \le 2\pi$.

a) $\sin x = \frac{\sqrt{3}}{2}$ b) $\cos x = \frac{1}{2}$ c) $\tan x = -1$

d) $\sec x = -2$ e) $\sin x = \frac{-1}{2}$ f) $\cos^2 x = \frac{1}{4}$

2. Solve for x, $-\pi \le x \le 0$.

a) $\cos x = \frac{-1}{\sqrt{2}}$ b) $\tan^2 x = \tan x$ c) $\sin^2 x - \sin x = 2$

d) $\sin^2 x = \frac{3}{4}$ e) $4\cos^2 x - 3 = 0$ f) $(2\csc x - 1)^2 = 9$

3. Solve for x, $0 \le x \le 2\pi$.

a) $\cos(2x) + \cos x + 1 = 0$ b) $\cos(2x) = \sin x$ c) $3\tan x = \tan(2x)$

d) $\sin x = 6\sin(2x)$ e) $\sin(2x)\cos x + \sin^2 x = 1$ f) $\sin(2x) + \sin x = 0$

g) $\sin(2x) + \cos(2x) = 0$ h) $3\cos(2x) + 2 + \cos x = 0$ i) $\sin(2x) = \tan x$

j) $\cos^2 x - \sin^2 x = 2\sin x \cos x$ k) $3\sin(2x) + \cos x = 0$ l) $3\sin x + \cos(2x) = 2$

4. Solve for x in the given interval.

a) $\sin x - \sin x \tan x = 0$, $0 \le x \le \pi$ b) $\cos x \tan(3x) = 0$, $-\pi \le x \le 0$

c) $6\sin^2 x - 5\cos x - 2 = 0$, $\frac{-\pi}{2} \le x \le \frac{3\pi}{2}$ d) $\sqrt{2}\sin x + \tan x = 0$, $-\pi \le x \le \pi$

e) $\cos^2 x - 3\sin^2 x = 1$, $-2\pi \le x \le 2\pi$ f) $2\tan x = \sec x$, $-2\pi \le x \le 0$

5. Solve for x in the given interval.

a) $\cos(2x) = \cos^2 x$, $-\pi \le x \le \pi$ b) $\sin(2x) = \cos x$, $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$

c) $\cos^2 x - 2\sin x \cos x - \sin^2 x = 0$, $0 \le x \le \frac{\pi}{2}$ d) $2(\sin^4 x + \cos^4 x) = 1$, $-\pi \le x \le \pi$

6. Solve for x in the given interval.

a) $\cos x = \frac{1}{\sqrt{2}}$, $-2\pi \le x \le 2\pi$ b) $\sin(2x) = \frac{-\sqrt{3}}{2}$, $-\pi \le x \le \pi$

c) $\sin^2 x + \sin x = 0$, $\frac{-\pi}{2} \le x \le \frac{3\pi}{2}$ d) $2\sin^2 x + \sin x - 1 = 0$, $0 \le x \le 4\pi$

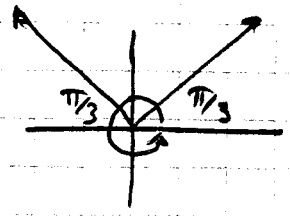
e) $2\sec^2 x + 3\sec x - 2 = 0$, $0 \le x \le 2\pi$ f) $\tan x + \sec(2x) = 1$, $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$

Trig Equations

1. $0 \le x \le 2\pi$

(a) $\sin x = \frac{\sqrt{3}}{2}$

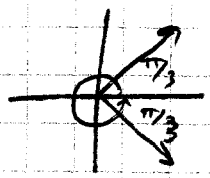
$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ →



$x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

(b) $\cos x = \frac{1}{2}$

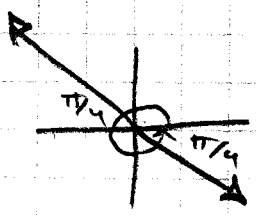
$x = \cos^{-1}\left(\frac{1}{2}\right)$ →



$x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ ↙

(c) $\tan x = -1$

$x = \tan^{-1}(-1)$ →

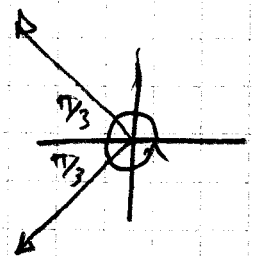


$x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ ↙

(d) $\sec x = -2$

$\cos x = -\frac{1}{2}$

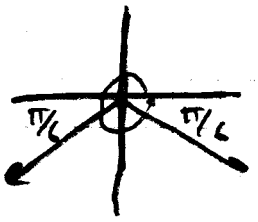
$x = \cos^{-1}\left(-\frac{1}{2}\right)$ ↗



$x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ ↙

1. (e) $\sin x = -\frac{1}{2}$

$x = \sin^{-1}\left(-\frac{1}{2}\right)$

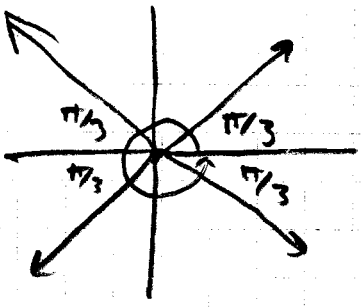


$x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

(f) $\cos^2 x = \frac{1}{4}$

$\cos x = \pm\frac{1}{2}$

$x = \cos^{-1}\left(\pm\frac{1}{2}\right)$

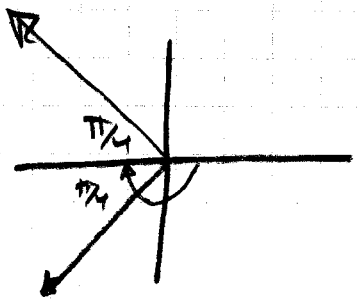


$x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ or $\frac{5\pi}{3}$

2. $-\pi \le x \le 0$

(a) $\cos x = \frac{-1}{\sqrt{2}}$

$x = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$



$x = -\frac{3\pi}{4}$

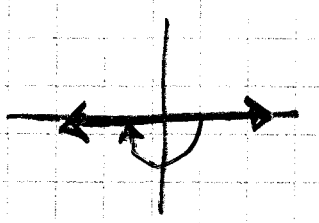
2 (b) $\tan^2 x = \tan x$

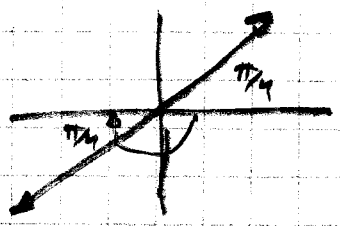
$\tan^2 x - \tan x = 0$

$\tan x (\tan x - 1) = 0$

$\tan x = 0$ or $\tan x = 1$

$x = \tan^{-1}(0)$ $x = \tan^{-1}(1)$





$x = 0$ or $-\pi$ or $-\frac{3\pi}{4}$

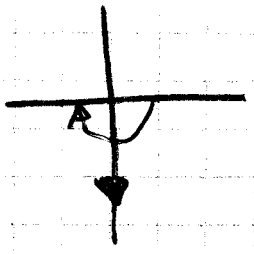
2 (c) $\sin^2 x - \sin x - 2 = 0$

$(\sin x - 2)(\sin x + 1) = 0$

$\sin x = 2$ or $\sin x = -1$

no solution! $x = \sin^{-1}(-1)$

$-1 \leq x \leq 1$

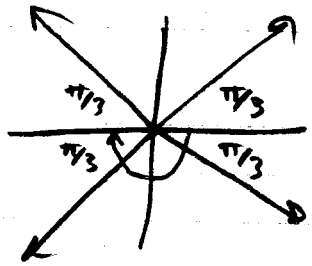


$x = -\frac{\pi}{2}$

2 (d) $\sin^2 x = \frac{3}{4}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

$x = \sin^{-1}\left(\pm \frac{\sqrt{3}}{2}\right)$ →



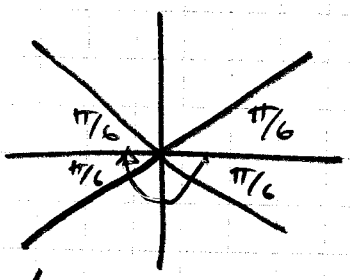
$x = -\frac{\pi}{3}$ or $-\frac{2\pi}{3}$ ←

2 (e) $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = \cos^{-1}\left(\pm \frac{\sqrt{3}}{2}\right)$ →



$x = -\frac{\pi}{6}$ or $-\frac{5\pi}{6}$

2 (f) $(2\csc x - 1)^2 = 9$

$2\csc x - 1 = \pm 3$

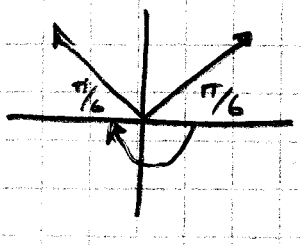
$2\csc x = \pm 3 + 1$

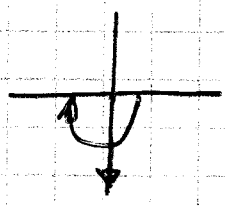
$2\csc x = 4$ or $2\csc x = -2$

$\csc x = 2$ $\csc x = -1$

$\sin x = \frac{1}{2}$ $\sin x = -1$

$x = \sin^{-1}\left(\frac{1}{2}\right)$ $x = \sin^{-1}(-1)$





No solutions here!

$x = -\pi/2$

3 (a) $\cos(2x) + \cos x + 1 = 0$

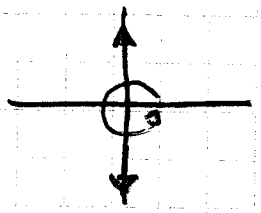
$2\cos^2 x - 1 + \cos x + 1 = 0$

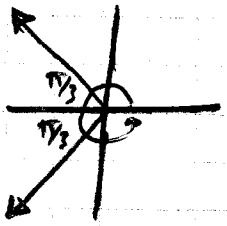
$2\cos^2 x + \cos x = 0$

$\cos x (2\cos x + 1) = 0$

$\cos x = 0$ or $\cos x = -\frac{1}{2}$

$x = \cos^{-1}(0)$ $\quad$ $x = \cos^{-1}(-\frac{1}{2})$





$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$

3 (b) $\cos(2x) = \sin(x)$

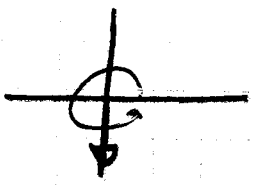
$1 - 2\sin^2 x = \sin x$

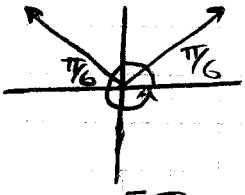
$2\sin^2 x + \sin x - 1 = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1$ or $\sin x = \frac{1}{2}$

$x = \sin^{-1}(-1)$ $\quad$ $x = \sin^{-1}(\frac{1}{2})$





$\therefore x = \frac{3\pi}{2}$ or $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

3 (c) $3\tan x = \tan(2x)$

$$(1-\tan^2 x)3\tan x = \frac{2\tan x}{1-\tan^2 x}(1-\tan^2 x)$$

$$3\tan x - 3\tan^3 x = 2\tan x$$

$$0 = 3\tan^3 x - \tan x$$

$$0 = \tan x(3\tan^2 x - 1)$$

$\tan x = 0$ or $3\tan^2 x - 1 = 0$

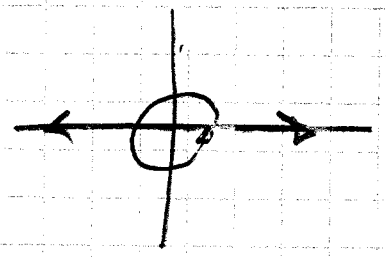
$x = \tan^{-1}(0)$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

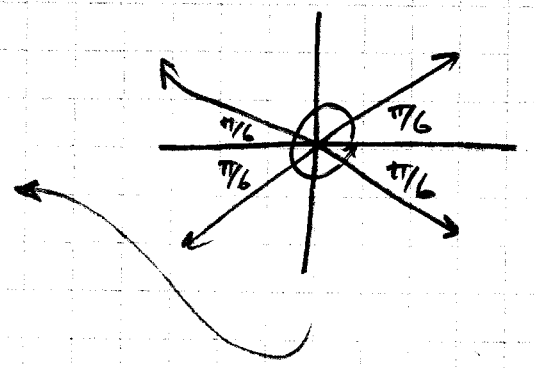
$x = \tan^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$



$x = 0$ or π or 2π or

$\frac{\pi}{6}$ or $\frac{5\pi}{6}$ or

$\frac{7\pi}{6}$ or $\frac{11\pi}{6}$



3 (d) $\sin x = 6\sin(2x)$

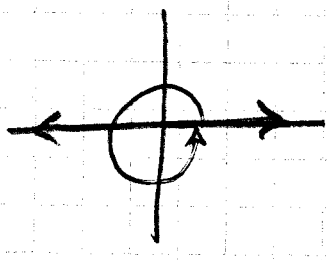
$\sin x = 6(2\sin x \cos x)$

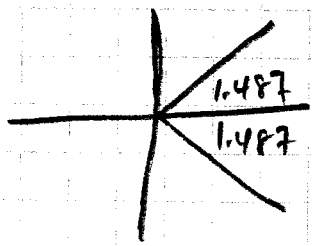
$0 = 12\sin x \cos x - \sin x$

$0 = \sin x(12\cos x - 1)$

$\sin x = 0$ or $\cos x = \frac{1}{12}$

$x = \sin^{-1}(0)$ $x = \cos^{-1}\left(\frac{1}{12}\right)$





$x = 0$ or π or 2π or 1.487 or 4.796

3 (e) $\sin(2x)\cos x + \sin^2 x = 1$

$2\sin x \cos x \cos x + 1 - \cos^2 x = 1$

$2\sin x \cos^2 x - \cos^2 x = 0$

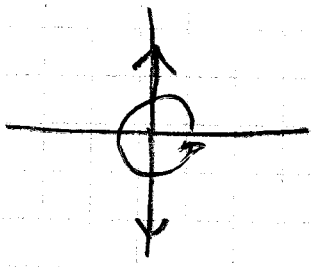
$\cos^2 x (2\sin x - 1) = 0$

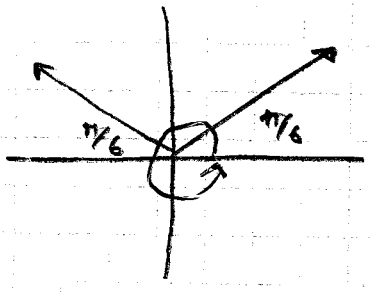
$\cos^2 x = 0$ or $\sin x = \frac{1}{2}$

$\cos x = 0$

$x = \cos^{-1}(0)$

$x = \sin^{-1}\left(\frac{1}{2}\right)$





$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

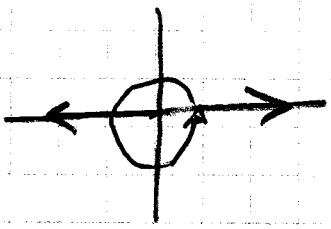
3 (f) $\sin(2x) + \sin x = 0$

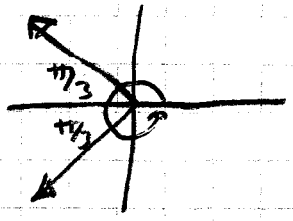
$2\sin x \cos x + \sin x = 0$

$\sin x (2\cos x + 1) = 0$

$\sin x = 0$ or $\cos x = -\frac{1}{2}$

$x = \sin^{-1}(0)$ $\quad$ $x = \cos^{-1}\left(-\frac{1}{2}\right)$





$x = 0$ or π or 2π or $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$

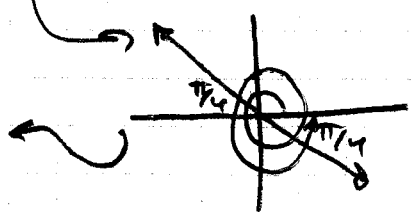
3 (g) $\sin(2x) + \cos(2x) = 0$

$\frac{\sin(2x)}{\cos(2x)} = \frac{-\cos(2x)}{\cos(2x)}$ } it is o.k. to divide by $\cos(2x)$ since we are using the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan(2x) = -1$

$2x = \tan^{-1}(-1)$

$0 \le x \le 2\pi$
$0 \le 2x \le 4\pi$ } $\times 2$



$2x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ or $\frac{11\pi}{4}$ or $\frac{15\pi}{4}$

$x = \frac{3\pi}{8}$ or $\frac{7\pi}{8}$ or $\frac{11\pi}{8}$ or $\frac{15\pi}{8}$

3 (h) $3\cos(2x) + 2 + \cos x = 0$

$3(2\cos^2 x - 1) + 2 + \cos x = 0$

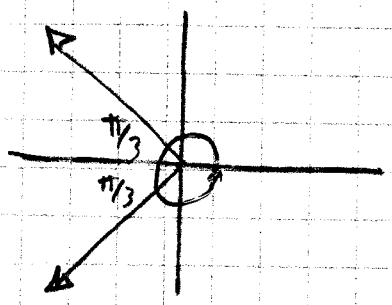
$6\cos^2 x - 3 + 2 + \cos x = 0$

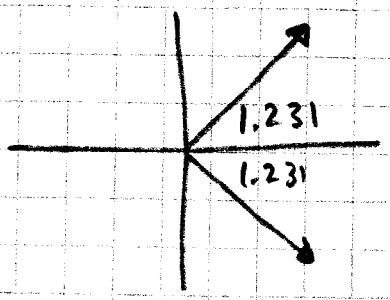
$6\cos^2 x + \cos x - 1 = 0$

$(2\cos x + 1)(3\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = \frac{1}{3}$

$x = \cos^{-1}\left(-\frac{1}{2}\right)$ $x = \cos^{-1}\left(\frac{1}{3}\right)$





$x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ or 1.231 or 5.052

3 (c) $\sin(2x) = \tan x$

$2\sin x \cos x = \frac{\sin x}{\cos x}$ $\Big\}\cdot \cos x$

$2\sin x \cos^2 x = \sin x$

$2\sin x \cos^2 x - \sin x = 0$

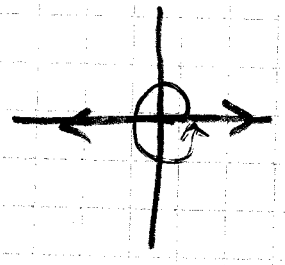
$\sin x(2\cos^2 x - 1) = 0$

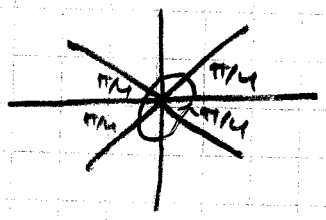
$\sin x = 0$ or $\cos^2 x = \frac{1}{2}$

$\cos x = \pm \frac{1}{\sqrt{2}}$

$x = \sin^{-1}(0)$

$x = \cos^{-1}\left(\pm \frac{1}{\sqrt{2}}\right)$





$x = 0$ or π or 2π or $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ or $\frac{5\pi}{4}$ or $\frac{7\pi}{4}$

3 (j) $\cos^2 x - \sin^2 x = 2\sin x \cos x$

$$\frac{\cos(2x)}{\cos(2x)} = \frac{\sin(2x)}{\cos(2x)}$$

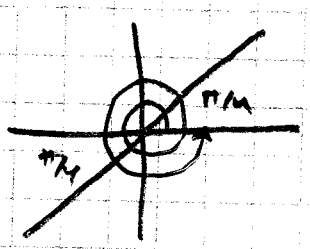
$$1 = \tan(2x)$$

$2x = \tan^{-1}(1)$

$0 \le x \le 2\pi$) $\times 2$

$0 \le 2x \le 4\pi$

$2x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$ or $\frac{9\pi}{4}$ or $\frac{13\pi}{4}$



$x = \frac{\pi}{8}$ or $\frac{5\pi}{8}$ or $\frac{9\pi}{8}$ or $\frac{13\pi}{8}$

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3 (k) $3\sin(2x) + \cos x = 0$

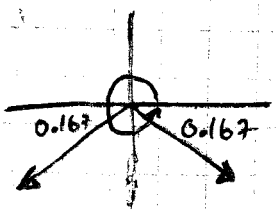
$3(2\sin x \cos x) + \cos x = 0$

$6\sin x \cos x + \cos x = 0$

$\cos x (6\sin x + 1) = 0$

$\cos x = 0$ or $\sin x = -\frac{1}{6}$

$x = \cos^{-1}(0)$ $\quad$ $x = \sin^{-1}\left(-\frac{1}{6}\right)$



$\therefore x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or 3.309 or 6.116

3 (a) $3\sin x + \cos(2x) = 2$

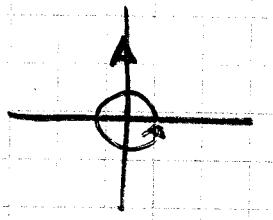
$3\sin x + 1 - 2\sin^2 x = 2$

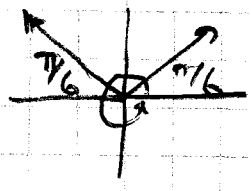
$2\sin^2 x - 3\sin x + 1 = 0$

$(\sin x - 1)(2\sin x - 1) = 0$

$\sin x = 1$ or $\sin x = \frac{1}{2}$

$x = \sin^{-1}(1)$ $x = \sin^{-1}\left(\frac{1}{2}\right)$





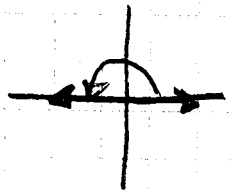
$x = \frac{\pi}{2}$ or $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

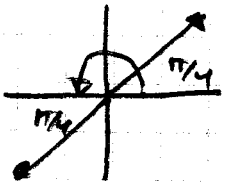
4 (a) $\sin x - \sin x \tan x = 0$ $\quad 0 \leq x \leq \pi$

$\sin x (1 - \tan x) = 0$

$\sin x = 0$ or $\tan x = 1$

$x = \sin^{-1}(0)$ $x = \tan^{-1}(1)$





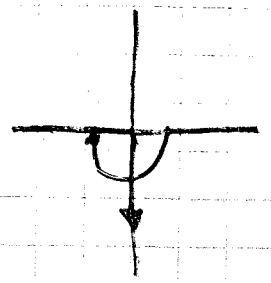
$x = 0$ or π or $\frac{\pi}{4}$

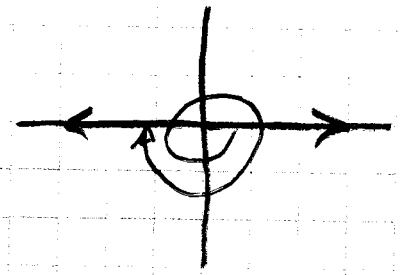
4 (b) $\cos x \tan(3x) = 0$ $\quad -\pi \le x \le 0$ $\quad \times 3$

$\cos x = 0$ or $\tan(3x) = 0$

$-3\pi \le 3x \le 0$

$x = \cos^{-1}(0)$ $\quad 3x = \tan^{-1}(0)$





$\therefore x = -\frac{\pi}{2}$ $\quad 3x = 0$ or $-\pi$ or -2π or -3π

OR

$x = 0$ or $-\frac{\pi}{3}$ or $-\frac{2\pi}{3}$ or $-\pi$

4 (c) $6\sin^2 x - 5\cos x - 2 = 0$ $\quad -\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$6(1-\cos^2 x) - 5\cos x - 2 = 0$

$6 - 6\cos^2 x - 5\cos x - 2 = 0$

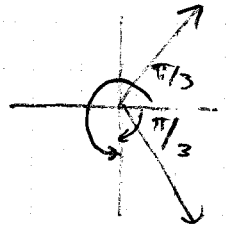
$0 = 6\cos^2 x + 5\cos x - 4$

$0 = (2\cos x - 1)(3\cos x + 4) = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -\frac{4}{3}$

no solutions $\because -1 \le \cos x \le 1$

$x = \cos^{-1}\left(\frac{1}{2}\right)$



$\therefore x = -\frac{\pi}{3}$ or $\frac{\pi}{3}$

4 (d) $\sqrt{2}\sin x + \tan x = 0 \qquad -\pi \le x \le \pi$

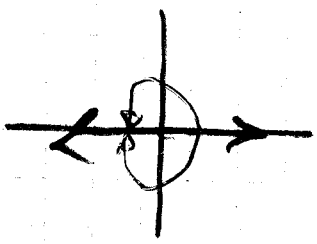
$\sqrt{2}\sin x + \frac{\sin x}{\cos x} = 0 \quad \Big\} \cdot \cos x$

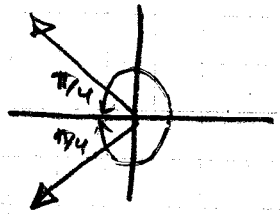
$\sqrt{2}\sin x\cos x + \sin x = 0$

$\sin x(\sqrt{2}\cos x + 1) = 0$

$\sin x = 0 \quad$ or $\quad \cos x = -\frac{1}{\sqrt{2}}$

$x = \sin^{-1}(0) \qquad x = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$





$x = 0$ or $-\pi$ or π or $-\frac{3\pi}{4}$ or $\frac{3\pi}{4}$

4 (e) $\cos^2 x - 3\sin^2 x = 1 \qquad -2\pi \le x \le 2\pi$

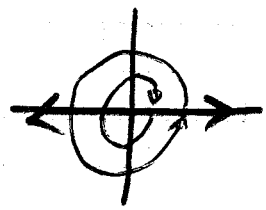
$1 - \sin^2 x - 3\sin^2 x = 1$

$-4\sin^2 x = 0$

$\sin^2 x = 0$

$\sin x = 0$

$x = \sin^{-1}(0)$



$\therefore\ x = 0$ or $-\pi$ or -2π or π or 2π

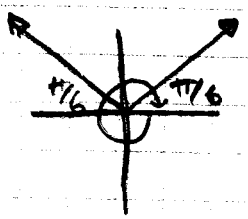
4 (f) $2\tan x = \sec x \qquad -2\pi \le x \le 0$

$$\frac{2\sin x}{\cos x} = \frac{1}{\cos x} \quad \} \cdot \cos x$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right)$$



$$x = -\frac{2\pi}{6} \text{ or } -\frac{11\pi}{6}$$

5 (a) $\cos(2x) = \cos^2 x \qquad -\pi \le x \le \pi$

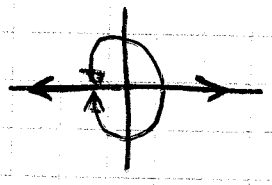
$$\cos^2 x - \sin^2 x = \cos^2 x$$

$$-\sin^2 x = 0$$

$$\sin^2 x = 0$$

$$\sin x = 0$$

$$x = \sin^{-1}(0)$$



$\therefore x = 0$ or $-\pi$ or π

5 (b) $\sin(2x) = \cos x$ $\quad -\frac{\pi}{2} \le x \le \pi/2$

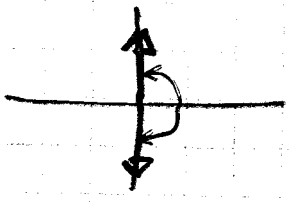
$2\sin x \cos x = \cos x$

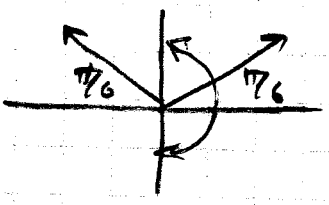
$2\sin x \cos x - \cos x = 0$

$\cos x (2\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

$x = \cos^{-1}(0)$ $\quad x = \sin^{-1}\left(\frac{1}{2}\right)$





$x = -\frac{\pi}{2}$ or $\frac{\pi}{2}$ or $\frac{\pi}{6}$

5 (c) $\cos^2 x - 2\sin x \cos x - \sin^2 x = 0$ $\quad 0 \le x \le \pi/2$

$\cos^2 x - \sin^2 x - 2\sin x \cos x = 0$

$\cos(2x) - \sin(2x) = 0$

$\frac{\sin(2x)}{\cos(2x)} = \frac{\cos(2x)}{\cos(2x)}$

$\tan(2x) = 1$

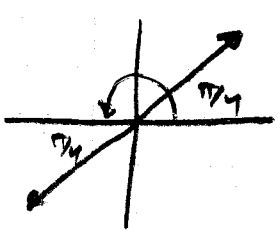
$2x = \tan^{-1}(1)$

$2x = \frac{\pi}{4}$

$x = \frac{\pi}{8}$

$\times 2$

$0 \le 2x \le \pi$



5 (all) $2(\sin^4 x + \cos^4 x) = 1$ $\quad -\pi \le x \le \pi$

$2(\sin^4 x + (1-\sin^2 x)^2) = 1$

$2(\sin^4 x + 1 - 2\sin^2 x + \sin^4 x) = 1$

$2(2\sin^4 x - 2\sin^2 x + 1) = 1$

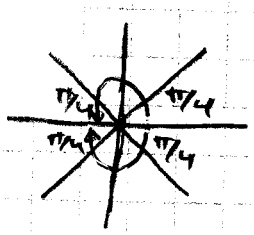
$4\sin^4 x - 4\sin^4 x + 1 = 0$

$(2\sin^2 x - 1)^2 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{1}{\sqrt{2}}$

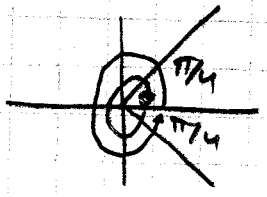
$x = \sin^{-1}\left(\pm \frac{1}{\sqrt{2}}\right)$



$x = -\frac{\pi}{4}$ or $-\frac{3\pi}{4}$ or $\frac{\pi}{4}$ or $\frac{3\pi}{4}$

6 (a) $\cos x = \frac{1}{\sqrt{2}}$ $\quad -2\pi \le x \le 2\pi$

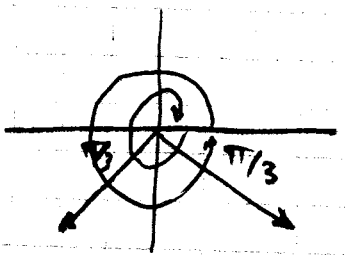
$x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$



$x = -\frac{\pi}{4}$ or $-\frac{7\pi}{4}$ or $\frac{\pi}{4}$ or $\frac{7\pi}{4}$

6 (b) $\sin(2x) = \frac{-\sqrt{3}}{2}$ $\quad -\pi \le x \le \pi$ $\quad \times 2$

$2x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\quad -2\pi \le 2x \le 2\pi$



$2x = -\frac{\pi}{3}$ or $-\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ or $\frac{5\pi}{3}$

$x = \frac{-\pi}{6}$ or $-\frac{2\pi}{6}$ or $\frac{4\pi}{6}$ or $\frac{5\pi}{6}$

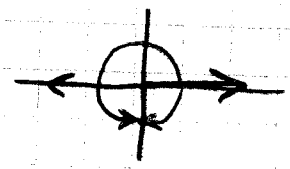
$x = -\frac{\pi}{6}$ or $\frac{-\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{5\pi}{6}$

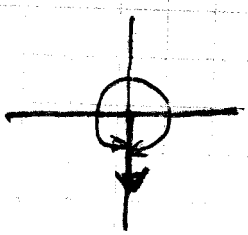
6 (c) $\sin^2 x + \sin x = 0$ $\quad -\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$\sin x(\sin x + 1) = 0$

$\sin x = 0$ or $\sin x = -1$

$x = \sin^{-1}(0)$ $\quad x = \sin^{-1}(-1)$





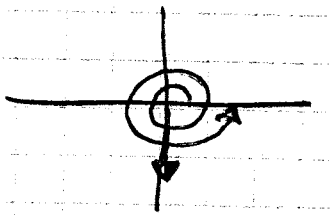
$x = 0$ or π or $-\frac{\pi}{2}$ or $\frac{3\pi}{2}$

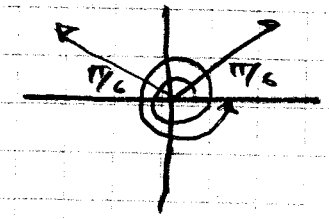
6 (d) $2\sin^2 x + \sin x - 1 = 0 \qquad 0 \le x \le 4\pi$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1$ or $\sin x = \frac{1}{2}$

$x = \sin^{-1}(-1)$ $\qquad$ $x = \sin^{-1}\left(\frac{1}{2}\right)$





$x = \frac{3\pi}{2}$ or $\frac{7\pi}{2}$ or $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\frac{13\pi}{6}$ or $\frac{17\pi}{6}$

6 (e) $2\sec^2 x + 3\sec x - 2 = 0 \qquad 0 \le x \le 2\pi$

$(\sec x + 2)(2\sec x - 1) = 0$

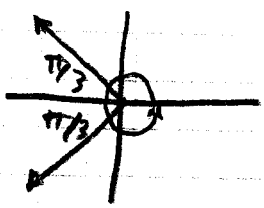
$\sec x = -2$ or $\sec x = \frac{1}{2}$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = 2$

no solution

$-1 \le \cos x \le 1$

$x = \cos^{-1}\left(-\frac{1}{2}\right)$



$\therefore x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

6 (f) $\tan x + \sec(2x) = 1$

$$\frac{\sin x}{\cos x} + \frac{1}{\cos(2x)} = 1$$

↑ 3 choices!!

try $\cos^2 x - \sin^2 x$ to see what happens

$$\frac{\sin x}{\cos x} + \frac{1}{\cos^2 x - \sin^2 x} = 1 \qquad \cdot \cos x(\cos^2 x - \sin^2 x)$$

$$\sin x(\cos^2 x - \sin^2 x) + \underline{\underline{\cos x}} = \cos x(\cos^2 x - \sin^2 x)$$

$$\sin x \cos^2 x - \sin^3 x + \cos x = \cos^3 x - \cos x \sin^2 x$$

- difficult to deal with
- easier to turn
  $\cos^2 x = 1 - \sin^2 x$ or $\sin^2 x = 1 - \cos^2 x$
  in the previous line

→ turning $\cos^2 x = 1 - \sin^2 x$ easier since "$\underline{\underline{\cos x}}$" on left side will disappear

∴ using $\cos(2x) = 1 - 2\sin^2 x$ at the beginning is "best"

Starting again using $\cos(2x) = 1 - 2\sin^2 x$ in the second line

6 (f) $\tan x + \sec(2x) = 1$

$$\frac{\sin x}{\cos x} + \frac{1}{\cos(2x)} = 1$$

$$\frac{\sin x}{\cos x} + \frac{1}{1-2\sin^2 x} = 1 \quad \Big\} \cdot \cos x(1-2\sin^2 x)$$

$$\sin x(1-2\sin^2 x) + \cos x = \cos x - 2\sin^2 x \cos x$$

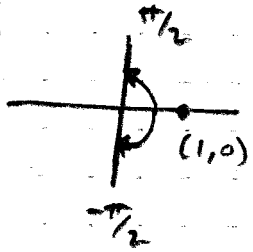
$$\sin x - 2\sin^3 x + \cancel{\cos x} - \cancel{\cos x} + 2\sin^2 x \cos x = 0$$

$$\sin x\,(1 - 2\sin^2 x + 2\sin x \cos x) = 0$$

$$\sin x\,(\cos 2x + \sin 2x) = 0$$

$\sin x = 0$ or $\cos 2x + \sin 2x = 0$

$x = \sin^{-1}(0)$



$x = 0$

$$\frac{\sin 2x}{\cos 2x} = \frac{-\cos 2x}{\cos 2x}$$

$\tan(2x) = -1$

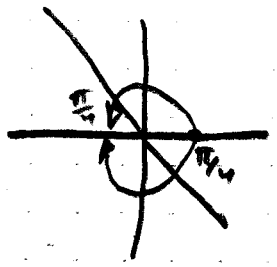
restriction

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$2x = \tan^{-1}(-1)$

$-\pi \le 2x \le \pi$

$2x = -\frac{\pi}{4}$ or $\frac{3\pi}{4}$



$x = -\frac{\pi}{8}$ or $\frac{3\pi}{8}$