Graphing Rational Functions III

1\. (a) $y = 2x^3$

$\big\downarrow$ replace x w/ $-x$

$y = 2(-x)^3$

$y = -2x^3$

$\big\downarrow$ replace y w/ $-y$

$-y = -2x^3 \quad \} \times (-1)$

$y = 2x^3$

$\therefore$ symmetric in the origin

(b) $y = 1 - x^2$

$\big\downarrow$ replace x w/ $-x$

$y = 1 - (-x)^2$

$y = 1 - x^2$

$\therefore$ symmetric in the y-axis

(f) $y = \dfrac{x}{x^3+1}$

$\big\downarrow$ replace x w/ $-x$

$y = \dfrac{-x}{(-x)^3+1}$

$y = \dfrac{-x}{-x^3+1}$

replace y w/ $-y$

$-y = \dfrac{-x}{-x^3+1} \quad \} \times (-1)$

$y = \dfrac{x}{-x^3+1}$

$\therefore$ not symmetric in the y-axis or origin

- could be symmetric elsewhere!

2 (i) $f(x) = \dfrac{x^2}{x^2-9}$

(a) replace x w/ $-x$

$$y = \frac{(-x)^2}{(-x)^2-9}$$

$$y = \frac{x^2}{x^2-9}$$

$\therefore$ symmetric in the y-axis

(b) for x-int, y=0

$$0 = \frac{x^2}{x^2-9}$$

$$0 = x^2$$
$$0 = x$$

for y-int, x=0

$$f(0) = \frac{0^2}{0^2-9}$$

$$f(0) = 0$$

(c) for restrictions

$$x^2 - 9 \neq 0$$
$$(x-3)(x+3) \neq 0$$
$$x \neq 3 \ \& \ x \neq -3$$

(d) $\therefore$ vertical asymptotes are

$x = 3$ & $x = -3$

SINCE THE CURVE IS SYMMETRIC IN THE Y-AXIS I NEED TO TEST ONLY ONE OF THE ASYMPTOTES

for behaviour $x = 3$

$\lim_{x \to 3^+} f(x) = +\infty$

$f(3.001) = 1501$

$\lim_{x \to 3^-} f(x) = -\infty$

$f(2.999) = -1499$

(e) for horizontal asymptote

$$\lim_{x\to\infty} \frac{x^2}{x^2-9} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

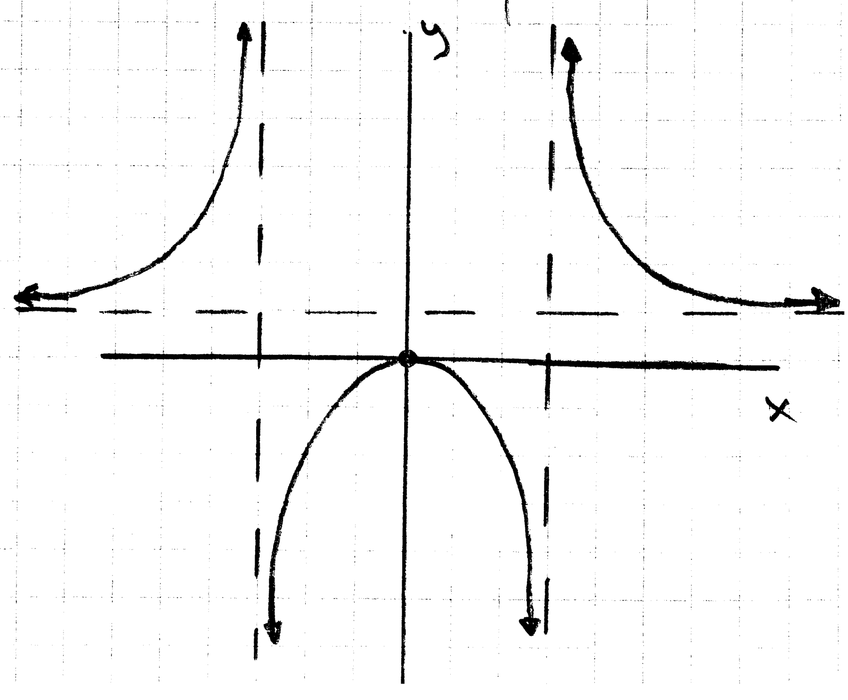
$$= \lim_{x\to\infty} \frac{1}{1-\frac{9}{x^2}}$$

$$= 1$$

$\therefore$ $y=1$ is the horizontal asymptote

SINCE CURVE IS SYMMETRIC IN THE Y-AXIS I ONLY NEED TO DO $f(100)$ or $f(-100)$

$f(100) = 1.0009$ (above)

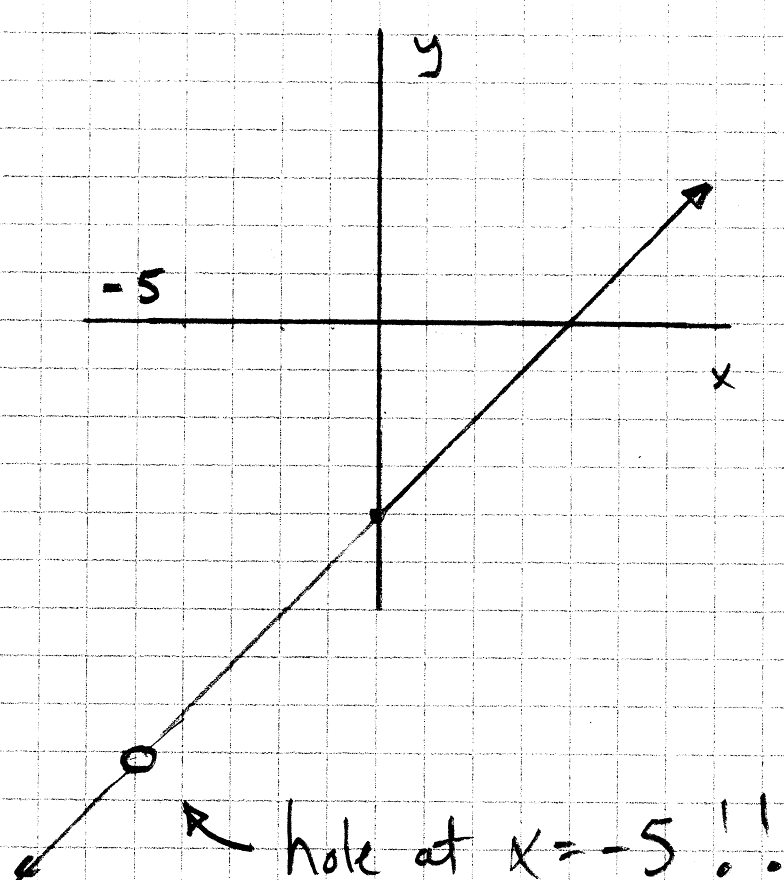


(ii) $f(x) = \dfrac{x^2+x-20}{x+5}$

$f(x) = \dfrac{(x-4)(x+5)}{x+5}$

$f(x) = x-4\ ,\ \ x \neq -5$

must state the restriction along with equation after reducing!



v) $f(x) = x^3 - x^2 - 14x + 24$ → polynomial function

→ just need x- & y-intercepts

(b) for x-int, y = 0

$0 = x^3 - x^2 - 14x + 24$

$0 = (x-2)(x^2 + x - 12)$

$0 = (x-2)(x+4)(x-3)$

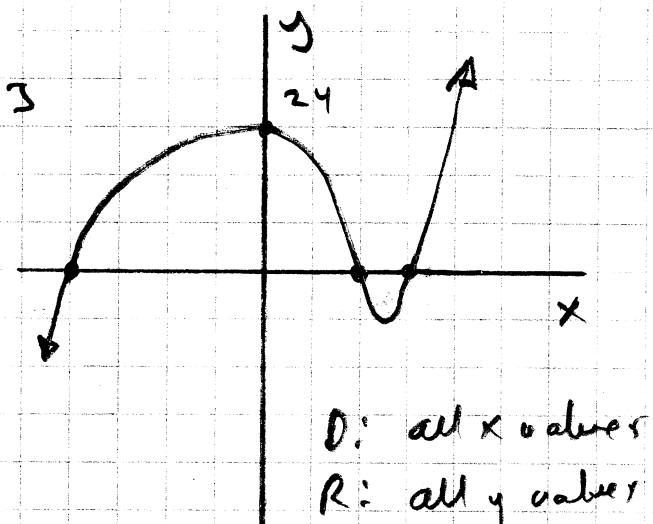
x	$x^3 - x^2 - 14x + 24$
2	$8 - 4 - 28 + 24 = 0$

∴ $(x-2)$ is a factor

$x = 2$ or $x = -4$ or $x = 3$

for y-int, x = 0

$f(0) = 24$



D: all x values

R: all y values

ix) $f(x) = -x(5-x)^2(x+2)$

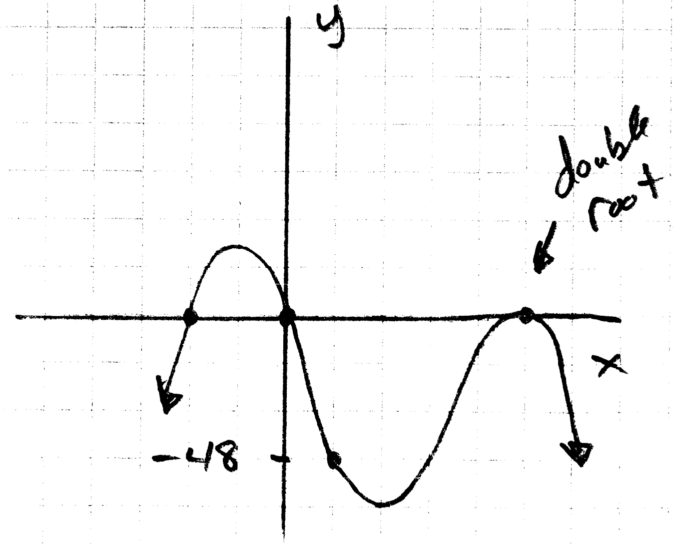
(b) for x-int, y = 0

$0 = -x(5-x)^2(x+2)$

$x = 0$ or $x = 5$ or $x = -2$

for y-int, x = 0

$f(0) = -0(5-0)^2(0+2)$
$= 0$



(f) "other info" $f(1) = -1(5-1)^2(1+2) = -48$

xviii) $f(x) = \dfrac{x^3+2x^2-15x}{x^2-5x-14}$

(b) for x-int, y = 0

$$0 = \frac{x^3+2x^2-15x}{x^2-5x-14}$$

$$0 = x(x^2+2x-15)$$

$$0 = x(x+5)(x-3)$$

$x=0$ or $x=-5$ or $x=3$

for y-int, x=0

$f(0)=0$

(c) for restrictions

$$x^2-5x-14 \neq 0$$

$$(x-7)(x+2) \neq 0$$

$x \neq 7$ AND $x \neq -2$

(a) ∴ vertical asymptotes are

$x=7$ AND $x=-2$

for behaviour

$\lim\limits_{x\to 7^+} f(x) = +\infty$

$f(7.001) = 37347$

$\lim\limits_{x\to 7^-} f(x) = -\infty$

$f(6.999) = -37320$

$\lim\limits_{x\to -2^+} f(x) = -\infty$

$f(-1.999) = -3332$

$\lim\limits_{x\to -2^-} f(x) = +\infty$

$f(-2.001) = 3334$

(d) $y = \dfrac{x^3+2x^2-15x}{x^2-5x-14}$

replace x w/ -x

$$y = \frac{-x^3+2x^2+15x}{x^2+5x-14}$$

replace y w/ -y

$$-y = \frac{-x^3+2x^2+15x}{x^2+5x-14} \times(-1)$$

$$y = \frac{x^3-2x^2-15x}{x^2+5x-14}$$

∴ not symmetric in the y-axis or origin

could be symmetric elsewhere!

(e) for slant asymptote

$$\lim_{x\to\infty} \frac{x^3+2x^2-15x}{x^2-5x-14} \rightarrow$$

$$\begin{array}{r} x+7 \\ x^2-5x-14 \overline{\smash{)}\, x^3+2x^2-15x+0} \\ \underline{x^3-5x^2-14x} \\ 7x^2-x+0 \\ \underline{7x^2-35x-98} \\ 34x+98 \end{array}$$

$$= \lim_{x\to\infty} x+7+\frac{34x+98}{x^2-5x-14}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} x+7+\frac{\frac{34}{x}+\frac{98}{x^2}}{1-\frac{5}{x}-\frac{14}{x^2}}$$

$$\therefore \frac{x^3+2x^2-15x}{x^2-5x-14} = \frac{(x^2-5x-14)(x+7)}{x^2-5x-14} + \frac{34x+98}{x^2-5x-14}$$

∴ slant asymptote is $y = x+7$

(e) continued...

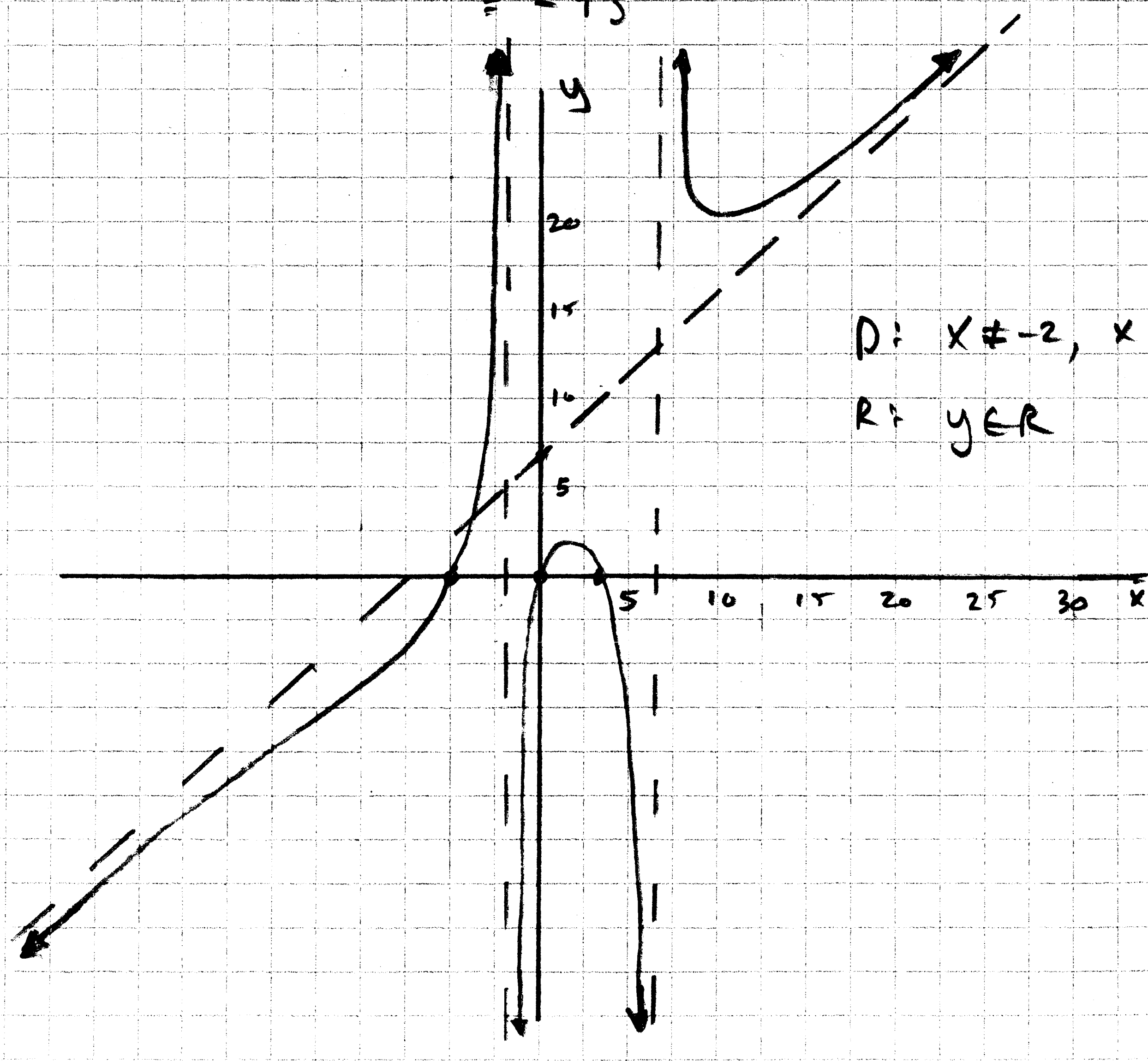
slant asymptote is $y = x + 7$

for behaviour $f(100) = 107.369$ (from above asymptote)

in asymptote $y = 100 + 7$
$= 107$

$f(-100) = -93.315$ (from below asymptote)

in asymptote $y = -100 + 7$
$= -93$



D: $x \neq -2,\ x \neq 7$
R: $y \in \mathbb{R}$