$Ax + By + C = 0$

$m = \tan\theta$

$m = -\frac{a}{b}$ $\quad$ $x\text{ int} = -\frac{c}{a}$ $\quad$ $y\text{ int} = -\frac{c}{B}$ $\quad$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$y = -2x^2 - 4x + 1$

$y = -2(x^2 + 2x + 1) + 1 + 2$ $\quad$ $\left(\frac{2x}{2}\right)^2 = 1x$

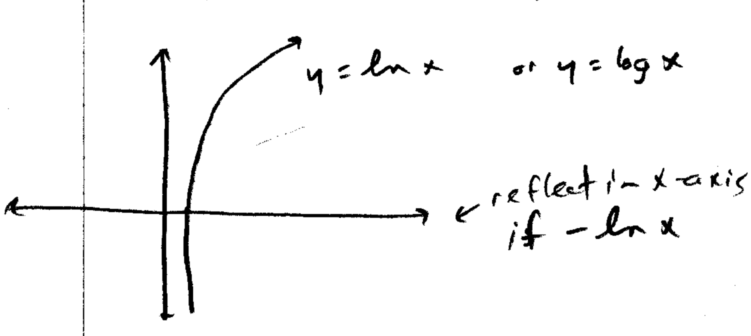
$y = -2(x+1) + 3$

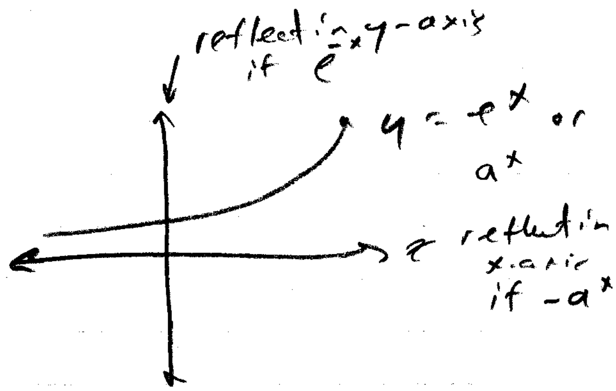
$V(-1, 3)$

$8x^3 - 27$

$= (2x)^3 - (3)^3$

$= (2x-3)(4x^2 + 9 + 6x)$





Rates of change first principles

$S(t) = \frac{80}{3}t^2 + \frac{140}{3}t$ S is distance of t, time

$S(10)$ is distance travelled after 10 minutes

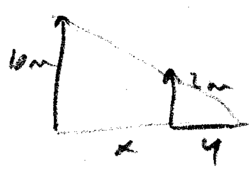
$S(10) - S(5)$ change in distance between 5 and 10 minutes in km

$\frac{S(10)-S(5)}{10-5}$ average rate of change of distance with respect to time in km/h between 5 and 10 minutes.

$\frac{S(10+h)-S(10)}{10+h-10}$ average rate of change of distance with respect to time in km/h between 10 and h minutes

$\lim_{h\to 0} \frac{S(10+h)-S(10)}{10+h-10}$ instantaneous velocity at 10 minutes in km/h

Find rate of change of y with respect to x



$\frac{2}{10} = \frac{y}{y+x}$

$\therefore 2(y+x) = 10y$

$\therefore y = \frac{1}{4}x$

$y(x) = \frac{1}{4}x$

For rate of change $\lim_{h\to 0} \frac{y(x+h)-y(x)}{h}$

$= \lim_{h\to 0} \frac{\cancel{\frac{1}{4}x} + \frac{1}{4}h - \cancel{\frac{1}{4}x}}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{4}h}{h}$

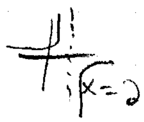
$= \frac{1}{4}$ m/m distance travelled

Continuity of a Function

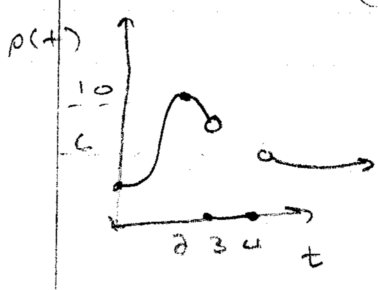
function $y=f(x)$ continuous at $x=a$ if

① if $f(a)$ defined
② if $\lim_{x\to a} f(x)$ exists
③ if $\lim_{x\to a} f(x) = f(a)$

eg graph dis at $x=2$ Explain condition fail



① $f(2)$ = undefined
② $\lim_{x\to 2} f(x)$ does not exist
③ $f(2) \neq \lim_{x\to 2} f(x)$



$\lim_{t\to 2} p(t) = 10$ val t is p(t) discont

$\lim_{t\to 4^+} p(t) = 6$ $t = 3, 4$ not 0

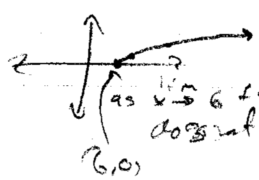
$\lim_{t\to 4^-} p(t) = 6$

Dir sub eval limits

$\lim_{x\to -1} (2x^2-5)$

$= 2(-1)^2-5$
-3

$\lim_{x\to 6} \sqrt{x-6}$ does not work $y=\sqrt{x-6}$



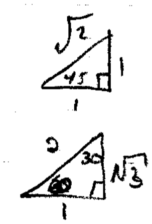
as $\lim_{x\to 6} f(x)$ does not exist

trig fund $\lim_{x\to \frac{\pi}{4}} \sin x = \sin\frac{\pi}{4}$

$\sin 45°$

$= \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$



One sided limits

$\lim_{x\to 3} f(x)$ if $f(x) = \begin{cases} x^2 & x \le 3 \\ -2x+15 & x > 3 \end{cases}$

Using one sided limits

$\therefore \lim_{x\to 3^+} f(x)$

$= \lim_{x\to 3^+} (-2x+15)$

$= -2(3)+15$

$= 9$

$\therefore \lim_{x\to 3^-} f(x)$

$= \lim_{x\to 3^-} x^2$

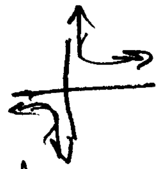
$= 9$

$\therefore \lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x) = 9$

$\therefore \lim_{x\to 3} f(x) = 9$

(4)

Not - $y = \frac{a}{x}$



a = any constant

eg $f(x) = Ax + B$ A and B constants

If $\lim\limits_{x \to 1} f(x) = 2$ and $\lim\limits_{x \to -1} f(x) = 4$ Find A + B

$1a + b = 2$ $-1a + b = 4$

$$\left[\begin{array}{cc|c} 1 & 1 & -2 \\ -1 & 1 & 4 \end{array}\right]$$

$$① + ② \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & 2 & 2 \end{array}\right]$$

$B = 1$ $A = -3$

Indeterminate forms $\frac{0}{0}$

Factor $\lim\limits_{x \to 10} \frac{x^2 - 100}{x - 10}$

Direct substitution yields indeterminate form $\frac{0}{0}$

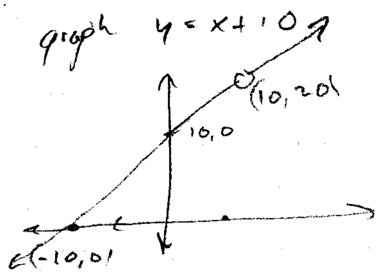
$\therefore \lim\limits_{x \to 10} \frac{x^2 - 100}{x - 10}$

$= \lim\limits_{x \to 10} \frac{\cancel{(x-10)}(x+10)}{\cancel{(x-10)}}$

$= \lim\limits_{x \to 10} (x+10)$

$= 20$

graph $y = x + 10$



Rationalizing $\lim\limits_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$ $\frac{\sqrt{x+1} - 1}{x} \times \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}$

$= \frac{x + 1 - 1}{x(\sqrt{x+1} + 1)}$

$= \frac{1}{\sqrt{x+1} + 1}$

$\lim\limits_{x \to 2} \frac{2x^3 - 5x^2 + 3x - 2}{2x - 4}$

direct sub yield indet form $\frac{0}{0}$

$\therefore \lim\limits_{x \to 2} \frac{2x^3 - 5x^2 + 3x - 2}{2x - 4}$

Let $P(x) = 2x^3 - 5x^2 + 3x - 2$

$P(2) = 0$

$\therefore (x - 2)$ is factor of $2x^3 - 5x^2 + 3x - 2$

$$\begin{array}{r} 2x^2 - x + 1 \\ x - 2 \overline{) 2x^3 - 5x^2 + 3x - 2} \\ \underline{2x^3 - 4x^2} \\ -x^2 + 3x \\ \underline{-x^2 + 2x} \\ x - 2 \\ \underline{x - 2} \\ 0 \end{array}$$

$\therefore \lim\limits_{x \to 2} \frac{(2x^3 - 5x^2 + 3x - 2)}{2x - 4}$

$= \lim\limits_{x \to 2} \frac{(x-2)(2x^2 - x + 1)}{2(x-2)}$

$\lim\limits_{x \to 1} \frac{x^3 - 1}{x - 1}$ cube, keep sign

$x^3 - 1$

$\lim \frac{(x - 1)(x^2 + 1 + x)}{}$ sq one first, sec — change sign mult first by sec

(5)

$8x^3+27$

$(2x+3)(4x^2+9-6x)$

one sided limits

$\lim_{x\to 2} \frac{(x+2)^3}{|x+2|}$ dir sub yield indeterminate form $\frac{0}{0}$

Let $f(x) = \frac{(x+2)^3}{|x+2|}$

If $x+2>0$ then $|x+2| = x+2$
$x>-2$
$\therefore f(x) = \frac{(x+2)^3}{x+2}$
$= (x+2)(x+2)$
$= x^2+4x+4$

If $x+2<0$ then $|x+2| = -x-2$
$x<-2$
$\therefore f(x) = \frac{(x+2)^3}{-x+2}$
$= \frac{(x+2)^3}{-(x+2)}$
$\equiv -(x+2)(x+2)$
$= -x^2-4x-4$

$\therefore \lim_{x\to 2} \frac{(x+2)^3}{|x+2|}$

$= \lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} x^2+4x+4 & x>2 \\ -x^2-4x-4 & x<2 \end{cases}$

Using one sided limits

Substitution

$\lim_{x\to 27} \frac{x^{\frac{1}{3}}-3}{x-27}$ Dir sub yield indet form $\frac{0}{0}$

$\therefore \lim_{x\to 27} \frac{x^{\frac{1}{3}}-3}{x-27}$ Let $x^{\frac{1}{3}} = u$ also as $x\to 27$, $u \to 3$

$\therefore u^3 = x$

$\therefore \lim_{u\to 3} \frac{u-3}{u^3-27}$ $\therefore u^3 = 27$

$u = 3$

$\lim_{x\to 1} \frac{x^{\frac{1}{6}}-1}{x^{\frac{1}{3}}-1}$

Let $u = x^{\frac{1}{6}}$ also as $x \to$ $u \to 1$

$\therefore u^6 = x$

$u = \sqrt[6]{1}$

in terms of u what is

$\therefore \lim_{x\to 1} \frac{u-1}{u^2-1}$ $x^{\frac{1}{3}} = u^2$

$u = \sqrt[2]{x^{\frac{1}{3}}}$ $u = x$

$\lim_{y\to 3} \frac{\frac{1}{y}-\frac{1}{3}}{y-3}$ dir sub yield indet form

$\therefore \lim_{y\to 3} \frac{\frac{1}{y}-\frac{1}{3}}{y-3}$

$= \lim_{y\to 3} \frac{\frac{3}{3y}-\frac{y}{3y}}{y-3}$

$= \lim_{y\to 3} \frac{3-y}{3y} \cdot \frac{1}{y-3}$

$= \lim_{y\to 3} \frac{3-y}{3y^2-9y}$

$= \lim_{y\to 3} \frac{3-y}{-3y(-y+3)}$

$= \frac{1}{-3y}$

What Val of x is $f(x)\ \frac{2x-2}{x^2+x-2}$ discon when $x=1$

$\frac{2(x-1)}{(x+2)(x-1)}$ or x^2+x-2 $(x+2)(x-1)$ $x=-2$, $x=1$

$= \frac{2}{x+2}$

when $x=-2$ also

Val A+B this statement correct?

$\lim_{x\to 0} \frac{\sqrt{Ax+B}-3}{x} = 1$

rationalize to

$\lim_{x\to 0} \frac{Ax+B-9}{x(\sqrt{Ax+B}+3)}$ $B=9$

$\lim_{x\to 0} \frac{A}{\sqrt{Ax+B}+3}$ $\frac{A}{\sqrt{B}+3}$ $\frac{A}{6}$ $A=6$

For what value of A does limit exist

$\lim_{x\to 1} \frac{x^2+Ax-3}{x-1}$ → make indeterminate

$(1)^2+A(1)-3=0$

$1+a-3=0$

$LS = 0$ $a=2$

$\lim_{x\to 1} \frac{1}{(x-1)}\left(\frac{1}{x+3} - \frac{2}{3x+5}\right)$

dir sub yields indet form

$\therefore \lim_{x\to 1} \frac{1}{(x-1)(x+3)} - \frac{2}{(3x+5)(x-1)}$

$\therefore \lim_{x\to 1} \frac{3x+5-2(x+3)}{(x-1)(x+3)(3x+5)}$

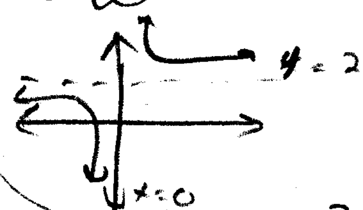
6

Undefined for $\frac{k}{0}$, $k \in \mathbb{R}$

$\lim_{x \to 0} \frac{2x+1}{x}$ ∴ direct substitution yields undefined for $\frac{1}{0}$

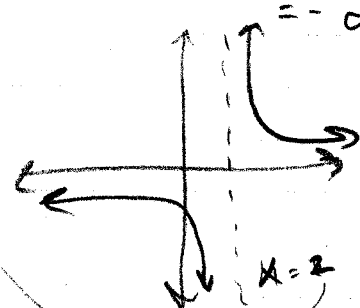
∴ $\lim_{x \to 0^+} \frac{2x+1}{x} = \infty$ $\qquad \lim_{x \to 0^-} \frac{2x+1}{x} = -\infty$

graph $y = 2 + \frac{1}{x}$



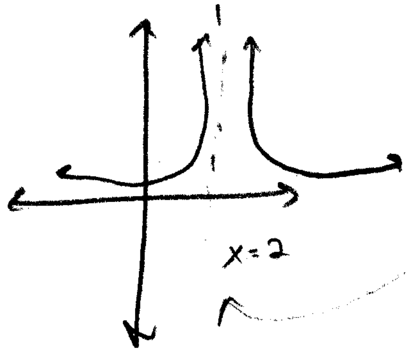
$\lim_{x \to 2} \frac{3}{x-2}$ direct sub yields undefined form $\frac{3}{0}$

$\lim_{x \to 2^+} \frac{3}{x-2} = \infty$ $\qquad \lim_{x \to 2^-} \frac{3}{x-2} = -\infty$ $\qquad y = \frac{3}{x} - \frac{3}{2}$



when bottom squared

$\lim_{x \to 2} \frac{4}{(x-2)^2}$



derivative is slope to tangent at a point on curve

derivatives from first principles $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

alternatives $f'(x)$

$\frac{d(fn)}{dx}$

$D_x y$

Power rule for derivatives

if $y = ax^N$ $\frac{dy}{dx} = Nax^{(N-1)}$

Product rule

$y = f(x) \cdot g(x)$ $\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$

Find equation of tangent to curve $y = 2x^2+3$ that passes through $(2,-7)$

Let the contact pt be (a,b)

$\therefore$ slope to curve at any point is

$\frac{dy}{dx} = 4x$ $\therefore$ slope at (a,b) $\frac{dy}{dx} = 4a$

slope is also $= \frac{\Delta y}{\Delta x} = \frac{b+7}{a-2}$

$\therefore 4a = \frac{b+7}{a-2}$ ①

but (a,b) is on parabola $\therefore$ $b = 2a^2+3$ ②

sub ① into ②

$\therefore 4a = \frac{2a^2+3+7}{a-2}$ $\therefore 4a^2-8a = 2a^2+3+7$

$\therefore 2a^2-8a-10=0$

$\therefore a^2-4a-5=0$

$\therefore (a-5)(a+1)=0$

$a=5$ $a=-1$

use $a=5$ $b=53$

pt$(5,53)$ $\frac{dy}{dx} = 4(5) = 20$

$\therefore 20(x-5) = y-53$

eg $y = 7x^2+7x+7$

$\frac{dy}{dx} = 14x+7$

$y = (x^2+3)(5x+2)$

$\frac{dy}{dx} = (x^2+3)(5) + (5x+2)(2x)$

$= 5x^2+5+10x^2+4x$

$= 15x^2+4x+5$

$y = (x^2+3)(x^2+2)(x^2+3)$

$\frac{dy}{dx} = (x^2+3)(x^2+2)(2x) + (x^2+3)(2x)(x^2+3) + (2x)(x^2+2)(x^2+3)$

$y = au^N$ in la'binitz $\frac{dy}{dx} = Nau^{N-1} \cdot \frac{du}{dx}$

Quotient Rule

$$y = \frac{f(x)}{g(x)} \qquad \frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Power of a function rule

$$y = a[f(x)]^N \quad \frac{dy}{dx} = Na[f(x)]^{N-1}[f'(x)]$$

eg $y = 3(\pi^2 - x^2)^3$

$$\frac{dy}{dx} = (9)(\pi^2 - x^2)^2(-2x)$$
$$= (-18x)(\pi^2 - x^2)^2$$

$f(x) = (x^2+3)^3(x^3+3)^2$

$f'(x) = 3(x^2+3)^2(2x)(x^3+3)^2 + (x^2+3)^3(2)(x^3+3)(3x^2)$

$= (6x)(x^2+3)^2(x^3+3)^2 + (6x)(x^2+3)^3(x^3+3)$

$= (6x)(x^2+3)^2(x^3+3)[(x^3+3) + x(x^2+3)]$

$= 6x(x^2+3)^2(x^3+3)(2x^3+3x+3)$

finding equation to normal of curves at pts.

normal slope is neg reciprocal which is 1

tangent slope = −1

$y = (x^2+x-3)^3 + 3$ show tangent at $(1,3)$ is also tgt to curve at another pt

well $\frac{dy}{dx} = 3(x^2+x-3)^2(2x+1)$

$\therefore$ slope at $(1,3)$ is 9 equation is $9(x-1) = y-3$

$$\therefore 9x - y - 6 = 0 \quad ①$$
$$-\ \ (x^2+x-3)^3 - y + 3 = 0 \quad ②$$
$$\overline{9x - (x^2+x-3)^3 - 9 = 0}$$

solve for x to find pt

eg $y = \frac{5(x+2)^2(3x+2)^4}{(3x+1)^2}$ $\quad \frac{dy}{dx} = \frac{(3x+1)^2(10)(x+2)(1)(3x+2)^4 + 5(x+2)^2(4)(3x+2)^3(3) - 5(x+2)^2(3x+2)^4(2)(3x+1)(3)}{(3x+1)^4}$

$y = x^2 + x^{-2}$

$y = 2x - 2x^{-3}$

$*\ \ln y = \ln \frac{5(x+2)^2(3x+2)^4}{(3x+1)^2}$

$\ln y = \ln 5(x+2)^2 + \ln(3x+2)^4 - \ln(3x+1)^2$

Find Pts when $y = \frac{3x}{x-4}$ when slopes of tans are $-\frac{3}{5}$

$$\frac{dy}{dx} = \frac{(x-4)(3) - (3x)(1)}{(x-4)^2}$$

$$= \frac{3x - 12 - 3x}{(x-4)^2} = \frac{-12}{(x-4)^2}$$

$$\therefore -\frac{3}{5} = \frac{-12}{(x-4)^2}$$

 [redacted]

Chain rule   a) $y = 5(3x^2-2x+1)^4 \therefore u = 3x^2-2x+1$
inner function

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ $\therefore y = 5u^4$ ← outer function

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (20u^3)(6x-2)$$

$$= 20(3x^2-2x+1)^3(6x-2)$$

Pg 99 #6

$$\frac{d\left(\frac{x}{y}\right)}{dx} = \frac{(y)(1)-(x)\frac{dy}{dx}}{y^2}$$

$$\frac{d(xy^2+x^2y)}{dx} = (1)(y^2)+(2)(y)\left(\frac{dy}{dx}\right)(x) + (2x)(y)+(x^2)\left(\frac{dy}{dx}\right)$$

Development of rules

Proof of power rule if $y = ax^N$ then $\frac{dy}{dx} = Nax^{(N-1)}$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{a(x+h)^N - ax^N}{h} = \frac{a((x+h)^N - x^N)}{h}$$

factor $\rightarrow$ $(x+h)^N - x^N$

$$= \lim_{h\to 0} \frac{a[(x+h)-x][(x+h)^{N-1} + (x+h)^{N-2}x + (x+h)^{N-3}x^2 + \dots + (x+h)^2 x^{N-3} + (x+h)x^{N-2} + x^{N-1}]}{h}$$

$$= a[x^{N-1} + x^{N-2}x + x^{N-2}x^2 + \dots + x^2 x^{N-3} + x \cdot x^{N-2} + x^{N-1})$$

$$= a[x^{N-1} + x^{N-1} + x^{N-1} + \dots + x^{N-1} + x^{N-1} + x^{N-1})$$

$Nx \rightarrow$ will be # the number of x^{N-1} terms

$$= a \cdot Nx^{(N-1)}$$

$$= Nax^{N-1}$$

Proof of Power of a function rule if $y = a[f(x)]^N$ then $\frac{dy}{dx} = Na[f(x)]^{N-1}[f'(x)]$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \therefore \lim_{h\to 0} \frac{a[f(x+h)]^N - a[f(x)]^N}{h}$$

$$= \lim_{h\to 0} \frac{a[f(x+h)-f(x)][f(x+h)^{N-1} + f(x+h)^{N-2}f(x) + \dots + f(x+h)f(x)^{N-2} + f(x)^{N-1}]}{h}$$

$$= \lim_{h\to 0} \frac{a[f(x+h)-f(x)]}{h} \times \frac{[f(x+h)^{N-1} + f(x+h)^{N-2}f(x) + \dots + f(x+h)f(x)^{N-2} + f(x)^{N-1}}{1}$$

$$= a[f'(x)][f(x)^{N-1} + f(x)^{N-2}f(x) + \dots + f(x)f(x)^{N-2} + f(x)^{N-1}]$$

$$= a[f'(x)][N][f(x)]^{N-1}$$

$$= Na[f(x)]^{N-1}[f'(x)]$$

Proof of Product rule if $y = f(x)\cdot g(x)$ then $\frac{dy}{dx} = f'(x)\cdot g(x) + f(x)g'(x)$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \therefore \frac{dy}{dx} = \frac{f(x+h)\cdot g(x+h) - f(x)g(x)}{h}$$

added in $-f(x)g(x+h)$ $+f(x)g(x+h)$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{g(x+h)[f(x+h)-f(x)] + f(x)[g(x+h)-g(x)]}{h}$$

$$= \lim_{h\to 0} g(x+h)\frac{[f(x+h)-f(x)]}{h} + f(x)\frac{[g(x+h)-g(x)]}{h}$$

these are $f'(x)$ & $g'(x)$

$$\therefore \frac{dy}{dx} = g(x)f'(x) + f(x)g'(x)$$

then $y = f(x)\cdot g(x)\cdot h(x)$

$\therefore y = f(x)[g(x)\cdot h(x)]$

\# $\therefore \frac{dy}{dx} = f(x)[g'(x)h(x) + h'(x)g(x)] + g(x)h(x)f'(x)$

$$= f(x)g'(x)h(x) + f(x)h'(x)g(x) + g(x)h(x)f'(x)$$

Proof of Quotient rule

$$F(x) = \frac{P(x)}{Q(x)}$$

$\therefore P(x) = F(x)\,Q(x)$

taking derivative

$\therefore P'(x) = F'(x)\,Q(x) + F(x)\,Q'(x)$

$\therefore Q(x)\,F'(x) = P'(x) - F(x)\,Q'(x)$

sub in $F(x) = \frac{P(x)}{Q(x)}$

$\therefore Q(x)\,F'(x) = P'(x) - \frac{P(x)}{Q(x)}\,Q'(x)$

on right everything over $Q(x)$

$\therefore Q(x)\,F'(x) = \dfrac{P'(x)Q(x) - P(x)Q'(x)}{Q'(x)}$

divide by $Q(x)$ to get $F'(x)$

$$\therefore F'(x) = \frac{P'(x)Q(x) - P(x)Q'(x)}{[Q'(x)]^2}$$

Proof of power rule for negative exponents

$y = x^N$ when N is a negative integer

Let $m = -N$

$\therefore y = x^N$

$\therefore y = x^{-m}$

$\therefore y = \frac{1}{x^m}$

By Quotient rule

$$\frac{dy}{dx} = \frac{x^m(0) - 1(m)x^{m-1}}{x^{2m}}$$

$$= -\frac{mx^{m-1}}{x^{2m}}$$

$$= -mx^{m-1-2m}$$

$$= -mx^{-m-1}$$

sub in $m = -N$

$\therefore Nx^{N-1}$

Proof of power of a function rule for negative exponents

$y = [f(x)]^N$ when N is a negative integer

let $m = -N$

$\therefore y = [f(x)]^{-m}$

$\therefore y = \frac{1}{[f(x)]^m}$

By Quotient rule

$$\frac{dy}{dx} = \frac{[f(x)]^m(0) - 1(m)[f(x)]^{m-1}[f'(x)]}{[f(x)]^{2m}}$$

$$= -\frac{m[f(x)]^{m-1}[f'(x)]}{[f(x)]^{2m}}$$

$$= -m[f(x)]^{m-1-2m}[f'(x)] = -m[f(x)]^{-m-1}[f'(x)]$$

sub in $m = -N$

$\therefore N[f(x)]^{N-1}[f'(x)]$

Proof of Power rule with rational exponents

$y = ax^N$ N is a rational exponent

Let $N = \frac{p}{q}$

$\therefore y = ax^{\frac{p}{q}}$

$\therefore y^q = (ax^{\frac{p}{q}})^q$

$\therefore y^q = a^q x^p$

taking derivative

$\therefore qy^{q-1}\frac{dy}{dx} = p \cdot a^q x^{p-1}$

$\therefore \dfrac{dy}{dx} = \dfrac{p \cdot a^q x^{p-1}}{qy^{q-1}}$

sub in $y = ax^{\frac{p}{q}}$

$$\therefore = \frac{p \cdot a^q x^{p-1}}{q(ax^{\frac{p}{q}})^{q-1}}$$

$$= \frac{p \cdot a^q x^{p-1}}{qa^{q-1}x^{(p-\frac{p}{q})}}$$

$$= \frac{p}{q}ax^{\frac{p}{q}-1}$$

sub in $N = \frac{p}{q}$

$= Nax^{N-1}$

Power of a Function rule $y = a[f(x)]$

do later

$C = \pi d$

Area of circle $= \pi r^2$

Vol Sphere $= \frac{4}{3}\pi r^3$

S.A sphere $= 4\pi r^2$

Vol Cylinder $= \pi r^2 h$

S.A cylinder $= 2\pi r^2 + 2\pi r h$

Volume cone $= \frac{1}{3}\pi r^2 h$

$$x^2 + y^2 = 25$$
$$2x + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{2x}{2y}$$

Implicit differentiation

$x^2 - xy + y^2 = 9$ Find tangents horizontal ie $m = 0$

For slope m at any point differentiate implicitly with respect to x

$\therefore 2x - x\frac{dy}{dx} + y(-1) + 2y\frac{dy}{dx} = 0$

$\therefore (2y - x)\frac{dy}{dx} = y - 2x$

$\therefore \frac{dy}{dx} = \frac{y-2x}{2y-x}$

$\therefore 0 = \frac{y-2x}{2y-x}$

$\therefore 0 = y - 2x$ Pts be (a, b)

$\therefore 0 = b - 2a$

$\therefore b = 2a$

$\therefore a^2 - ab + b^2 = 9$

$\therefore a^2 - a(2a) + (2a)^2 = 9$

$\therefore a^2 - 2a^2 + 4a^2 = 9$

$\therefore 3a^2 = 9$

$\therefore a^2 = 3$

$\therefore a = \pm\sqrt{3}$

b is ....

[redacted]

(11)

$8\frac{y}{x^2} - 8\frac{x}{y^2} = 7$

$= 8yx^{-2} - 8xy^{-2} = 7$

$\therefore (8y)(-2x^{-3}) + (x^{-2})(8\frac{dy}{dx}) - [(8x)(-2y^{-3}\frac{dy}{dx}) + (y^{-2})(8)] = 0$

$\therefore -16x^{-3}y + 8x^{-2}\frac{dy}{dx} + 16xy^{-3}\frac{dy}{dx} - 8y^{-2} = 0$

$\therefore (8x^{-2} + 16xy^{-3})\frac{dy}{dx} = 16x^{-3}y + 8y^{-2}$

$\therefore \frac{dy}{dx} = \frac{16x^{-3}y + 8y^{-2}}{8x^{-2} + 16xy^{-3}}$

time $t=0$, train halted deceleration $1.2\ m/s^2$, its position $s(t) = s_0 + v_0 t - \frac{1}{4}t^2$

(a) Show s_0 train's position when brakes applied

$\therefore t=0 \therefore s(0) = s_0 + 0 + 0$

(b) Show v_0 is its velocity

$v = \frac{ds}{dt} = v_0 - \frac{2}{4}t$ when $t=0$

$v = v_0$

(c) Find stopping distance if v_0 is $20\ m/s$ $\therefore$ Velocity is $0\ m/s$

$\therefore 0 = 20 - \frac{2}{4}t$ $\rightarrow$ $s(40)$ . . .

$\therefore t = 40s$

Obj moves so velocity v is related to position s $v = \sqrt{b^2 + 2gs}$ show acceleration is constant

$v^2 = b^2 + 2gs \therefore 2v\frac{dv}{dt} = 2g\frac{ds}{dt}$

$\therefore 2va = 2gv$

$\therefore a = \frac{2gv}{2v}$

$\therefore a = g$ $\therefore$ constant

#2

Sphere radius = 9 ft, springs leak loosing air at 171π ft^3/min. Find rate of decrease of radius after 4 min.

Let V represent volume, r, radius

$V = \frac{4}{3}\pi r^3$

$r = 9$ ft $\quad \frac{dV}{dt} = -171\pi$ ft^3/min $\quad \frac{dr}{dt} = ?$

To find the change in volume with respect to time in ft^3/min differentiate implicitly with respect to time

$\therefore \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

but $\frac{dV}{dt} = -171\pi$ ft^3/min

$\therefore -171\pi = 4\pi r^2 \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = -\frac{171\pi}{4\pi r^2} = -\frac{171}{4r^2}$

need r at 4 min find change in volume

when $r = 9$ ft

$V = \frac{4}{3}\pi 9^3$

$= 972\pi$ ft^3

looses 171π ft^3/min

looses $171\pi \times 4 = 684\pi$

$\therefore \Delta V = 972\pi - 684\pi = 288\pi$

$\therefore 288\pi = \frac{4}{3}\pi r^3$

$\therefore r^3 = \frac{288(3)\pi}{4\pi}$

$\therefore r^3 = 216$

$\therefore r = 6$ ft

$\therefore \frac{dr}{dt} = -\frac{171}{4(6)^2}$

$= -\frac{171}{144}$ ft/min

Conical pile, $\frac{dV}{dt} = 12$ ft^3/s $\quad h = \frac{1}{2}d \quad \frac{dh}{dt}$ when $h = 4$ ft

$\therefore h = \frac{2}{3}r$

$\therefore r = \frac{3}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (\frac{3}{2}h)^2 h$

$= \frac{9}{12}\pi h^3$

$\frac{dV}{dt} = \frac{27}{12}\pi h^2 \frac{dh}{dt}$

$\therefore 12 = \frac{27}{12}\pi h^2 \frac{dh}{dt}$

$\therefore 144 = 27\pi h^2 \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{144}{27\pi h^2}$

when $h = 4$ ft

$\therefore \frac{dh}{dt} = \frac{144}{27\pi(4)^2}$

$= \frac{144}{432\pi}$

$= \frac{1}{3\pi}$ ft/~~min~~.

(13)

Indicate $\frac{dA}{dt} = \frac{dr}{dt}$ of circle what's $r = ?$

$a = \pi r^2$

$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

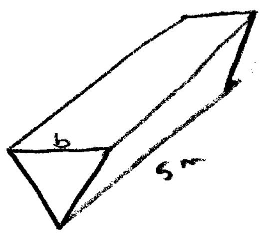
$\therefore \frac{dr}{dt} = \frac{\frac{dA}{dt}}{2\pi r}$

$\frac{dr}{dt} = \frac{dA}{dt}$

$\therefore \frac{\frac{dA}{dt}}{2\pi r} = \frac{dA}{dt}$

$\therefore 2\pi r \frac{dA}{dt} = \frac{dA}{dt}$

$\therefore r = \frac{1}{2\pi}$



$\updownarrow .25\text{ m}$

$\frac{dV}{dt} = 0.25\ m^3/min$

$\frac{dh}{dt} = ?$ when $h = 0.1$ m deep?

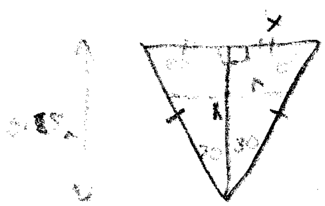
$V = \frac{1}{2}bhl \quad l = 5m$

$\therefore V = \frac{5}{2}bh$

$\therefore V = 5rh$

but $r = \frac{h}{\sqrt{3}}$

$\therefore V = \frac{5}{\sqrt{3}}h^2$



$\frac{h}{r} = \frac{0.25\ m}{x}$

$\tan 30^\circ = \frac{x}{0.25}$

$\therefore x = \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{4}\right)$

$= \frac{1}{4\sqrt{3}}$

$\therefore \frac{h}{r} = \frac{0.25}{1/4\sqrt{3}}$

$\therefore \frac{h}{4\sqrt{3}} = 0.25r$

$\therefore r = \frac{h}{\sqrt{3}}$

$\therefore \frac{dV}{dt} = \frac{10}{\sqrt{3}}h\frac{dh}{dt}$

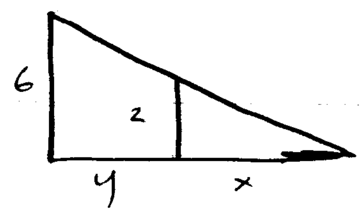
$\therefore 0.25 = \frac{10}{\sqrt{3}}h\frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{0.25\sqrt{3}}{10h}$

when $h = 0.1$ m

$\frac{dh}{dt} = \frac{0.25\sqrt{3}}{1}$

$= \frac{\sqrt{3}}{4}$ m/s



→ when x m from base

$\frac{dx}{dt} = ?$ $\frac{du}{dt} = 1.5$ m/s

$\frac{x}{2} = \frac{x+u}{6}$ $\therefore 4x = 2u$

$\therefore 4\frac{dx}{dt} = 2\frac{du}{dt}$

$\therefore \frac{dx}{dt} = \frac{2(1.5)}{4}$

$= 0.75$ m/s → at any dist from base

tip of shadow moving at any distance from base

$0.75 \times \frac{6}{2} = 2.25$ m/s

Cone $\frac{dr}{dt} = 4$ cm/min $\frac{dV}{dt} = ?$ when $r = h$

$\frac{dh}{dt} = -7$ cm/min

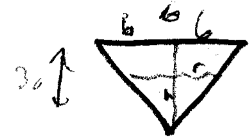
$V = \frac{1}{3}\pi r^2 h$ $\therefore \frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{2}{3}\pi r \frac{dr}{dt} h$

$= \frac{1}{3}\pi r^2(-7) + \frac{2}{3}\pi r(4)h$

$= -\frac{7}{3}\pi r^2 + \frac{8}{3}\pi r h$ $r = h$

$= \frac{8}{3}\pi r^2 - \frac{7}{3}\pi r^2$

$= \frac{1}{3}\pi r^2$ → cm^3/min increasing



$h = 30$ $d = 12$

$\therefore \frac{h}{r} = \frac{30}{6}$ $\therefore 30r = 6h$

$V = \frac{1}{3}\pi r^2 h$

r

$\therefore r = \frac{1}{5}h$

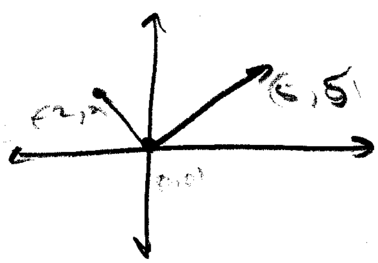
15

Abs max. and min.

For quadratic function the absolute max or min values must occur at either end pt in the domain, or at a "critical" point $f'(x) = 0$

For cubic relation the absolute max or min pt either can occur at either an end pt in the domain or a critical pt ($f'(x) = 0$)

However → Consider $y = |x|$ $\quad -2 \le x \le 5$



① abs. max at end pt
② abs. min at "critical" pt but $f'(x)$ does not exist at $x = 0$

ABS max or min with open ended domain
ie. What the number minimizes the sum of the number and its reciprocal

$$S = x + \frac{1}{x} \quad \{x \mid x \in R, x > 0\}$$

critical pts $S'(x) = 0$

$\therefore S'(x) = 1 - x^{-2}$

$\therefore -1 = -1x^{-2}$

$\therefore \frac{1}{x^2} = 1$

$\therefore x^2 = 1$

$\therefore x = 1 \quad \{x \in R, x > 0\}$

→ Consider $S''(x) = 2x^{-3}$

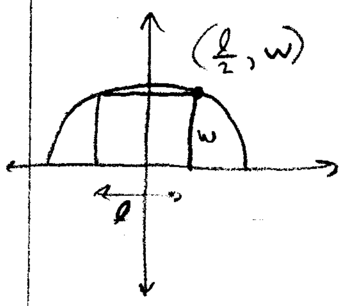
Since $S''(x) > 0$ for all $x \in D$
$S(x)$ is always concave up since $1 \in D$ abs min occurs at $x = 1$
when $x = 1$ $\quad S = 2$

if $S''(x) < 0$ opens concave down $\therefore$ max pt

if $S''(x) > 0$ opens concave up $\therefore$ min pt

Optimization

Rectangle inscribed in a semi-circle of radius 2cm. Find the largest area of such a rectangle



circle: $x^2+y^2=4$

$\{l \mid l,\ 0 \le l \le 4\}$

Maximize $a = l \times w$

$(\frac{l}{2}, w)$ on circle

$\therefore (\frac{l}{2})^2 + w^2 = 4$

$\therefore w^2 = 4 - \frac{l^2}{4}$

$\therefore w = \sqrt{4 - \frac{l^2}{4}}$

$\therefore a = l \cdot (4 - \frac{l^2}{4})^{\frac{1}{2}}$

For critical values of l

$a'(l) = 0$

$a'(l) = l(\frac{1}{2})(4 - \frac{l^2}{4})^{-\frac{1}{2}}(-\frac{l}{2}) + (4 - \frac{l^2}{4})^{\frac{1}{2}}$

$= (4 - \frac{l^2}{4})^{-\frac{1}{2}}\left[(l)(\frac{1}{2})(-\frac{l}{2}) + (4 - \frac{l^2}{4})\right]$

$= (4 - \frac{l^2}{4})^{-\frac{1}{2}}\left(\frac{l^2}{-4} + 4 - \frac{l^2}{4}\right)$

$= (4 - \frac{l^2}{4})^{-\frac{1}{2}}\left(-\frac{2l^2}{4} + 4\right)$

$0 = (4 - \frac{l^2}{4})^{-\frac{1}{2}}\left(-\frac{2l^2}{4} + 4\right)$

$\therefore \frac{2l^2}{4} = 4$

$\therefore l^2 = 8$

$\therefore l = \sqrt{8}$

$= 2\sqrt{2}$

Critical pts

$a(0) = 0$

$a(4) = 0$

$a(2\sqrt{2}) = 4$

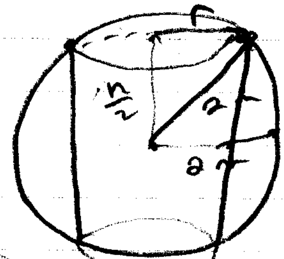
$\therefore$ ABS max area occurs when $l = 2\sqrt{2}$ and is $4\ cm^2$

---

right circular cylinder inscribe in circle radius 2 m what dimensions to maximize volume

$V = \pi r^2 h \leftarrow$ Maximize

$\{h \mid h \in R,\ 0 \le h \le 4\}$



$\therefore (\frac{h}{2})^2 + r^2 = 2^2$

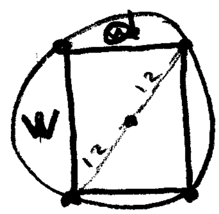
$\therefore r^2 = 4 - \frac{h^2}{4}$

$\therefore r = \sqrt{4 - \frac{h^2}{4}}$

Strength rectangular beam is proportional
to product of width and square of depth
Find dimensions of strongest beam cut from circular log 24 cm diameter

∴ $S \propto wd^2$

∴ $S = Kwd^2$ k - constant



$w^2+d^2=24^2$

∴ $d^2=24^2-w^2$

∴ $S=kw(24^2-w^2)$ $\{w \mid w\in\mathbb{R}, 0\le w\le 24\}$

∴ $S=576kw-kw^3$

$S'(0)=576k-3kw^2$

$0=576k-3kw^2$

$0=576-3w^2$

∴ $w^2=192$

∴ $w=\sqrt{192}=8\sqrt{3}$

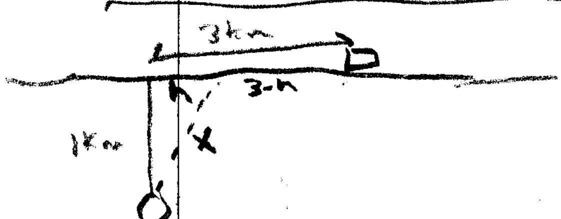
Find Pt on curve $x^2-y^2=16$ $4\le x\le 5$
closest to $(0,2)$

minimum

$l=\sqrt{(\Delta x)^2+(\Delta y)^2}$

∴ $l=\sqrt{x^2+(y-2)^2}$ $x^2=16+y^2$

∴ $l=\sqrt{16+y^2+y^2+4-4y}$



3 km/h canoe
5 km/h jog

minimize time $t=\frac{\Delta d}{\Delta v}$

$T=\frac{x}{3\,km/h}+\frac{3-h}{5}$ $1^2+h^2=x^2$

$x=\sqrt{h^2+1^2}$

$=\frac{\sqrt{h^2+1}}{3}+\frac{3-h}{5}$ Domain $\{h \mid h\in\mathbb{R}, 0\le h\le 3\}$

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* $\lim_{x\to\infty} \frac{x^2+2x+3}{x+2} = \lim_{x\to\infty} \frac{x+2+\cdots}{1+\frac{2}{x}} = \infty$

$\lim_{x\to\infty} \frac{x^2+2x+3}{x^2+2} = \lim_{x\to\infty} \frac{1+\frac{2}{x}+\frac{3}{x^2}}{1+\frac{2}{x^2}} = 1$

Function increases when $f'(x) > 0$
Function decreases when $f'(x) < 0$
Function concave up $f''(x) > 0$
Function concave down $f''(x) < 0$

$y = f(x)$

For critical values of $f(x)$ $f'(x) = 0$
Use 2nd derivative test to find nature of critical pts
For additional critical pts consider when $f'(x)$ is undefined
For possible cusp verify with $f'(x)$ test
For possible pts of inflection use $f''(x) = 0$
To ~~de~~ termine whether inflection use $f''(x)$ test on either side for alternation

For vertical asymptotes consider $\lim_{x\to a} f(x)$ (values which make undefined)

Dir sub yields undefined form $\frac{b}{0}$ use one sided limits direction of approach

For horizontal asymptotes consider $\lim_{x\to\infty} f(x) = \text{constant}$
* as abs val of both num and den increase without bound yielding indet form $\frac{\infty}{\infty}$ $f(x)$
$y = \text{constant}$ is asymptote
For direction of approach of it to $y = c$ consider

$f(x) - c$ for large positive vals of x

if > 0 the approach above
< 0 the approach below

Similarly $f'(x) - c$ large $-$'ve vals of x

* Slant asymptotes $y = \frac{2x^3-5x+1}{x^2+2x-3}$
dir approach

$$x^2+2x-3 \overline{\smash{)}\,2x^3-0x^2-5x+1}$$ quotient $2x-4$

$2x^3+4x^2-6x$

$-4x^2+x+1$

$-4x^2-8x+12$

$9x-11$

$f(x) - \text{asym} \Rightarrow$ large the small

$\therefore \frac{2x^3-5x+1}{x^2+2x-3} = y = 2x-4+\frac{9x-11}{x^2+2x-3}$

$\therefore y = 2x-4$ is slant asymptote

$y = |f(x)| \rightarrow$ reflect everything under x-axis

$y = 2|x+1| - 3x + |x-2|$

case 1: $x > 2$ $\quad |x+1| = x+1$
$\quad |x-2| = x-2$

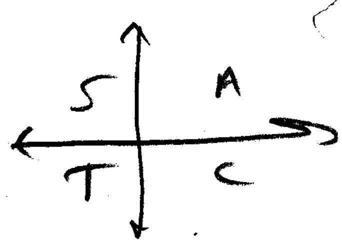
case 2: $x < -1$ $\quad |x+1| = -x-1$
$\quad |x-2| = -x+2$

case 3: $-1 < x < 2$ $\quad |x+1| = x+1$
$\quad |x-2| = -x+2$

Trig

$\sin^2\theta + \cos^2\theta = 1$

$\cos\theta = \sin(90-\theta)$



$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$
$\quad = 2\cos^2 A - 1$
$\quad = 1 - 2\sin^2 A$

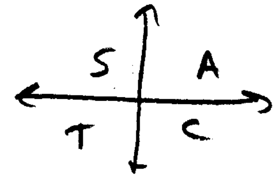
$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x+y}{2}$



Identities

$\cos^2\theta + \sin^2\theta = 1$ → $1 + \cot^2\theta = \csc^2\theta$

$\cos^2\theta + \sin^2\theta = 1$ → $1 + \tan^2\theta = \sec^2\theta$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

* $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

* $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$\cos\theta = \sin(90-\theta$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$ → $= 2\cos^2 A - 1$
 $= 1 - 2\sin^2 A$

* $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$\sin(195^\circ)$

$= \sin(150^\circ + 45^\circ)$

$= \sin 150^\circ\cos 45^\circ + \cos 150\sin 45^\circ$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

(2)

eg

$$\frac{\tan 7^\circ + \tan 8^\circ}{1 - \tan 7^\circ \tan 8^\circ} = \tan 15^\circ$$

$$\tan(45 - 30^\circ)$$

$$= \frac{\tan 45 - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

...

eg

$$\frac{\sin(x - 30^\circ) + \cos(60^\circ - x)}{\sin x}$$

$$= \frac{\sin(x-30) + \sin[90 - (60 - x)]}{\sin x}$$

$$= \frac{\sin(x-30) + \sin(30+x)}{\sin x}$$

$$= \frac{2\sin\frac{2x}{2}\cos\frac{-60}{2}}{\sin x}$$

* consider domains ...

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

ON exam

First principles $\quad y = 5\cos x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5\cos(x+h) - 5\cos x}{h}$$

$$= \lim_{h \to 0} \frac{5[\cos[x+h] - \cos x]}{h}$$

$$= \lim_{h \to 0} \frac{5\left[-2\sin\frac{2x+h}{2}\sin\frac{h}{2}\right]}{h}$$

$$= \lim_{h \to 0} \frac{5\left[-2\sin\frac{2x+h}{2}\cancel{\sin\frac{h}{2}}\right] \cdot \frac{1}{2}}{\cancel{\frac{h}{2}}}$$

$$= 5[-2\sin x]\frac{1}{2}$$

$$= -5\sin x$$

Derivitive rules for trig

$y = \sin f(x) \qquad \frac{dy}{dx} = f'(x)\cos f(x)$

$y = \cos f(x) \qquad \frac{dy}{dx} = -f'(x)\sin f(x)$

$y = \tan f(x) \qquad \frac{dy}{dx} = f'(x)\sec^2 f(x)$

$y = \csc f(x) \qquad \frac{dy}{dx} = -f'(x)\csc f(x)\cot f(x)$

$y = \sec f(x) \qquad \frac{dy}{dx} = f(x)\sec f(x)\tan f(x)$

$y = \cot f(x) \qquad \frac{dy}{dx} = -f'(x)\csc^2 f(x)$

eg $x = \sin y$

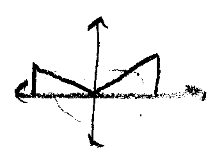
$\therefore 1 = \frac{dy}{dx}\cos y$

$\therefore \frac{dy}{dx} = \sec y$

<u>max / min</u>

$f(x) = 2\cos x + x \qquad -\pi \le x \le \pi$

$f'(x) = -2\sin x + 1$



$\therefore 0 = -2\sin x + 1$

$\therefore \sin x = \frac{1}{2}$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

$f(-\pi) = 2\cos(-\pi) + (-\pi)$
$= -2 - \pi$

$f(\pi) = 2\cos(\pi) + \pi$
$= -2 + \pi$

$f\left(\frac{5\pi}{6}\right) = -\sqrt{3} + \frac{5\pi}{6}$

$f\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right) + \frac{\pi}{6} = \sqrt{3} + \frac{\pi}{6}$

$f\left(\frac{\pi}{6}\right)$ max
$f(-\pi)$ min

$y = a\sin x + b\cos x$

$y = k\sin(x+\alpha)$ → graphing

$k^2 = a^2 + b^2 \qquad \tan\alpha = \frac{b}{a}$

limits

$\lim_{x \to \frac{3\pi}{4}} \sin x = \frac{1}{\sqrt{2}}$

$\lim_{x \to 0} \frac{\cos x}{\sin x}$ direct sub yields undefined form $\frac{1}{0}$

using one sided limits

$\lim_{x \to 0} \frac{\sin 3x}{x} \quad \frac{0}{0}$

$= \lim_{x \to 0} \frac{3\sin 3x}{3x}$

$= 3$

$y = 2\sin(3x + \frac{\pi}{2})$
Amp = 2
period = $\frac{360}{3} = 120°$
back $\frac{\pi}{2}$ radians

$y = a\sin x + b\cos x$
$\therefore y = k\sin(x+\alpha)$
$k^2 = a^2 + b^2$
$\tan\alpha = \frac{b}{a}$

* $\lim_{t \to 0} \frac{1-\cos t}{t^2}$

$= \lim_{t \to 0} \frac{1-(1-2\sin^2\frac{t}{2})}{t^2}$

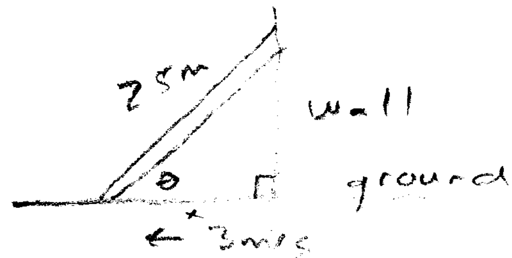
$= \lim_{t \to 0} \frac{2\sin^2\frac{t}{2}}{t^2}$

$= \lim_{t \to 0} 2 \cdot \frac{\sin\frac{t}{2} \cdot \frac{1}{2}}{\frac{t}{2}} \cdot \frac{\sin\frac{t}{2} \cdot \frac{1}{2}}{\frac{t}{2}}$

$= \frac{1}{2}$

Related rates

beam 25 m



$$\frac{d\theta}{dt} = ? \text{ when } x = 15 \;?$$

$$\frac{dx}{dt} = 3\text{ m/s}$$

$$\cos\theta = \frac{x}{25}$$

$$\therefore 25\cos\theta = x$$

$$\therefore -25\sin\theta \frac{d\theta}{dt} = \frac{dx}{dt}$$

$$\therefore \frac{d\theta}{dt} = -\frac{3}{25\sin\theta}$$

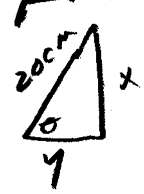
at $x = 15$
$\sin\theta = \frac{20}{25} = \frac{4}{5}$

$$\therefore \frac{d\theta}{dt} = -\frac{3}{25(\frac{4}{5})}$$

$$= -\frac{3}{20} \text{ radians / s}$$

Max, Min

Find maximum perimeter of right triangle



$$P = x + y + 20$$

$\cos\theta = \frac{y}{20}$ $\quad y = 20\cos\theta$

$\sin\theta = \frac{x}{20}$ $\quad x = 20\sin\theta$

$\therefore P = 20\sin\theta + 20\cos\theta + 20$ $\quad$ Feasible Domain $\{\theta \mid \theta \in R, 0 < \theta \leq \frac{\pi}{2}\}$

$P(\theta) = 20(\sin\theta + \cos\theta + 1)$

For critical values of $P(\theta)$, $P'(\theta) = 0$

$P'(\theta) = 20(\cos\theta - \sin\theta)$ $\quad 0 = \cos\theta - \sin\theta$

$\tan\theta = 1 \quad \theta = \frac{\pi}{4}$

logs

$$y = \log_a f(x) \qquad \frac{dy}{dx} = \log_a e \cdot \frac{f'(x)}{f(x)}$$

$$y = \ln f(x) \qquad \frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

$$e = \lim_{x \to 0} (1 + x)^{\frac{1}{x}}$$

$$y = a^{f(x)} \qquad \frac{dy}{dx} = a^{f(x)} \cdot f'(x) \cdot \ln a$$

$$y = e^{f(x)} \qquad \frac{dy}{dx} = e^{f(x)} \cdot f'(x)$$

$$y = f(x)^{f(x)} \Rightarrow \ln y = \ln f(x)^{f(x)}$$

$$\therefore \ln y = f(x) \ln f(x) \qquad \text{implicitly, ...}$$

logs first principles

$$y = \log_7 (3x - 2)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\log_7 (3x + 3h - 2) - \log_7 (3x - 2)}{h}$$

$$= \lim_{h \to 0} \frac{\log_7 \left(1 + \frac{3h}{3x-2}\right)}{h} = \lim_{h \to 0} \frac{1}{h} \cdot \frac{3h}{3x-2} \cdot \frac{3x-2}{3h} \cdot \log_7 \left(1 + \frac{3h}{3x-1}\right)$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \cdot \frac{3\cancel{h}}{3x-2} \log_7 \underbrace{\left(1 + \frac{1}{\frac{3x-2}{3h}}\right)^{\frac{3x-2}{3h}}}_{\to e}$$

$$= \frac{3}{3x-2} \cdot \log_7 e$$

Antiderivitie, Indefinite integral

$\int (ax^N)\,dx \qquad y = \frac{ax^{N+1}}{n+1} + c, \ c \in R$

After filling a reservoir for 1 minute, it has 10L in it. After this it fills at a rate of $\frac{10}{t^2}$ L/minute. What is the Volume five minutes after?

$\frac{dV}{dt} = \frac{10}{t^2} \qquad \therefore V(t) = \frac{10t^{-1}}{-1} + c, \ c \in R$

$= -\frac{10}{t} + c, \ c \in R$

But $V(1) = 10$

$\therefore 10 = -\frac{10}{1} + c$

$\therefore c = 20$

$\therefore V(t) = -\frac{10}{t} + 20 \qquad \therefore V(5) = \ldots$

$\int [f(x)]^N f'(x)\,dx \qquad y = \frac{[F(x)]^{N+1}}{N+1} + c, \ c \in R$

$\int \frac{f'(x)}{f(x)}\,dx \qquad y = \ln|f(x)| + c, \ c \in R \qquad x \neq 0, \ x \in R$

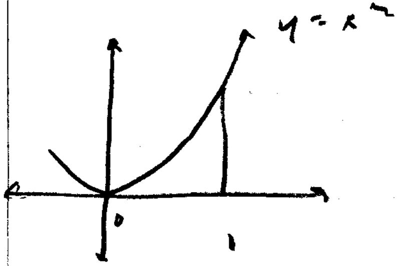
no ABS is $x > 0$

$\int f'(x) e^{f(x)}\,dx \qquad y = e^{f(x)} + c, \ c \in R$

for trig remember rules

✱ $\int (ky)\,dx \qquad y = ce^{kx},\ c \in \mathbb{R}$

Find the area bounded by $y = x^2$, x-axis, $x = 0$ and $x = 1$



$$A = \int_0^1 x^2\,dx$$

$$= \left.\frac{x^3}{3}\right|_0^1$$

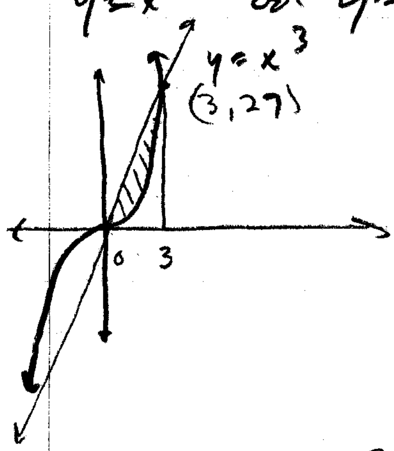
$$= \frac{1^3}{3} - \frac{0^3}{3}$$

$$= \frac{1}{3}\text{ units}^2$$

✱ For areas below x-axis

$$A = \left|\int_0^b f(x)\,dx\right|$$

$y = x^3$ and $y = 9x$



For intersection $y = x^3$ and $y = 9x$

$\therefore x^3 = 9x$

$\therefore x^3 - 9x = 0$

$\therefore x(x^2 - 9) = 0$

$\therefore x = 0$ or $x = \pm 3$

For $y = x^3$ $f(3) = 27$

$$A = \int_0^3 (9x)\,dx - \int_0^3 (x^3)\,dx$$

$$= \left.\frac{9x^2}{2}\right|_0^3 - \left.\frac{x^4}{4}\right|_0^3$$

$$= \left[\frac{9(3)^2}{2} - \frac{9(0)^2}{2}\right] - \left[\frac{3^4}{4} - \frac{0^4}{4}\right]$$

$$= \frac{81}{2} - \frac{81}{4}$$

$$= \frac{81}{4}\text{ units}^2$$