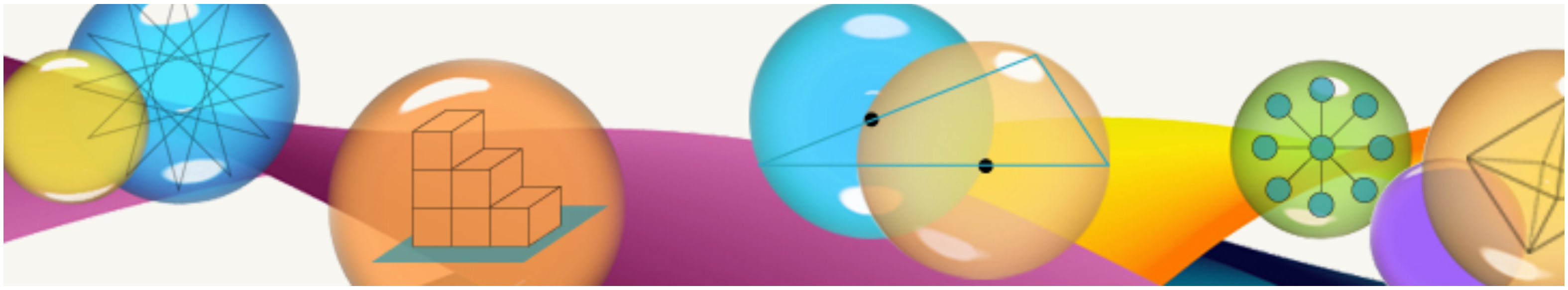


Problem of the Week
Grade 3/4 (A)

Problem and Solutions
2016 - 2017

Strands

Data Management & Probability
Geometry & Spatial Sense
Measurement
Number Sense & Numeration
Patterning & Algebra

(Click the strand name above to jump to that section)

The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 3 or higher.

Data Management & Probability

TAKE ME TO THE COVER

Problem of the Week
Problem A
Sweet Possibilities

At Ahmed's birthday party, his guests can build their own dessert. They start by choosing either a waffle cone, sugar cone, or a bowl. Then they choose from the following ice cream flavours: vanilla, chocolate, strawberry and rocky road. Next they add either chocolate sauce or sprinkles. Finally, they can add a cherry on top if they wish.

How many different dessert combinations are possible? Justify your answer.

Strand Data Management and Probability

Problem of the Week
Problem A and Solution
Sweet Possibilities

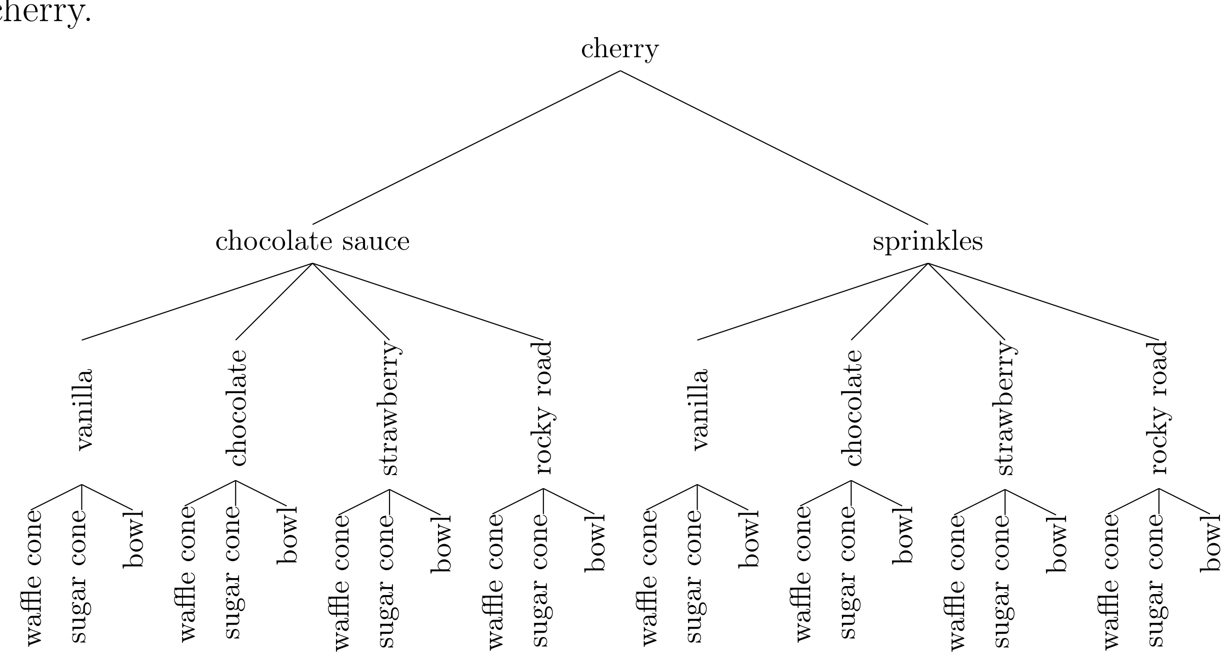
Problem

At Ahmed's birthday party, his guests can build their own dessert. They start by choosing either a waffle cone, sugar cone, or a bowl. Then they choose from the following ice cream flavours: vanilla, chocolate, strawberry and rocky road. Next they add either chocolate sauce or sprinkles. Finally, they can add a cherry on top if they wish.

How many different dessert combinations are possible? Justify your answer.

Solution

This tree shows all of the possible combinations of ice cream treats that include a cherry.



There are a total of 24 combinations. You could make a similar tree for the 24 combinations of ice cream without a cherry. This means that there are a total of 48 dessert combinations possible.

Note that this is not the only tree that shows all possible combinations. You could also build a tree that starts with the type of cone and works down to adding a cherry or not. In fact, you could have the choices appear in any order you like within the levels of the tree. However, all of these trees would show that there are a total of 48 combinations.

Teacher's Notes

We can calculate the number of combinations in a more formulaic way. Suppose we had just two choices to make: the base of the dessert and the type of ice cream. For each of the three bases there are four types of ice cream that can fill them. In other words, we multiply the number of choices we have in each case to determine the number of combinations. This means that there would be $3 \times 4 = 12$ different combinations of bases and ice cream. For this problem there are four sets of choices to make. This means the number of combinations can be calculated as:

$$\underset{\text{choices for the base}}{3} \times \underset{\text{choices of ice cream}}{4} \times \underset{\text{choices of topping}}{2} \times \underset{\text{cherry options}}{2} = 48 \text{ combinations}$$

The tree gives more information than just the number of combinations. The tree also shows the actual arrangements of the dessert. For example, one combination has a *cherry* on top of *chocolate sauce* on *vanilla ice cream* in a *bowl*. You can see that combination by following a **path** that starts at the top of the tree, known as the **root**, moves down one level on each step, and ends at the bowl under vanilla. There is a unique path from the top of the tree to each of the **nodes** at the bottom level of the tree.

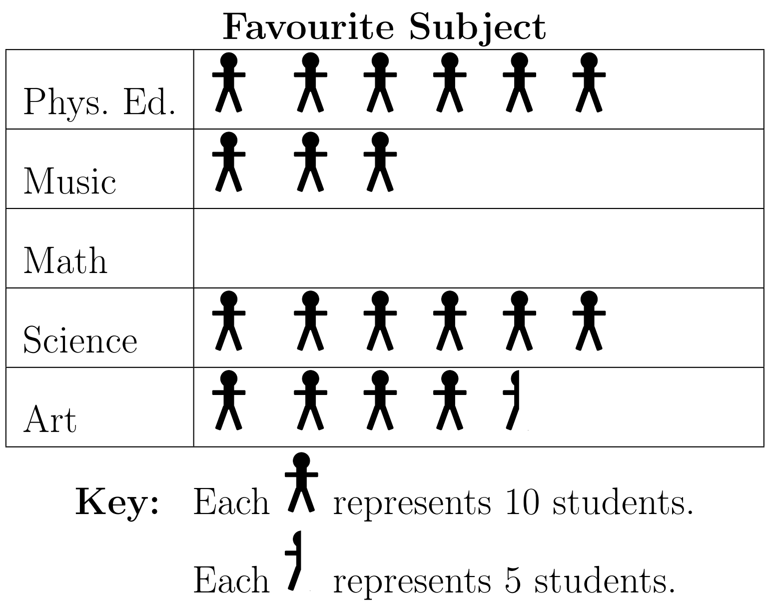
The properties and uses of trees are studied in an area of mathematics known as graph theory. Trees are also very useful structures used in many computer programs to solve a variety of problems.

Problem of the Week

Problem A

Mathematical Mystery

A school of 270 students completed a survey about their favourite subject in school. The results for all subjects except Math are shown in the pictograph below.



A) How many students selected Math as their favourite subject? Explain your reasoning.

B) Complete the pictograph.

C) What is the mode and median for this set of data?

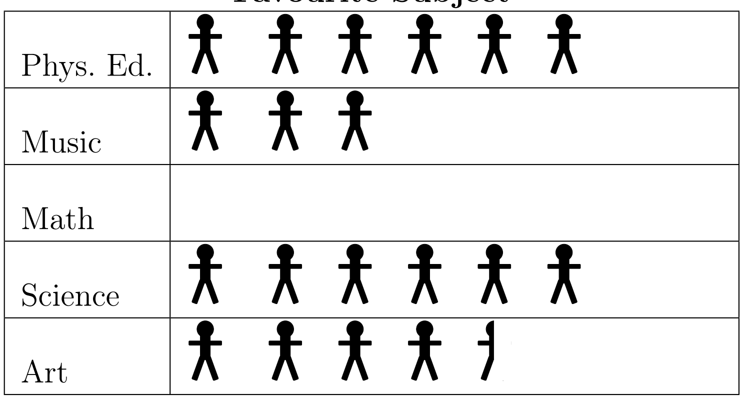
Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Mathematical Mystery

Problem

A school of 270 students completed a survey about their favourite subject in school. The results for all subjects except Math are shown in the pictograph below.

Favourite Subject



Key: Each (figure) represents 10 students.

Each (half figure) represents 5 students.

A) How many students selected Math as their favourite subject? Explain your reasoning.

B) Complete the pictograph.

C) What is the mode and median for this set of data?

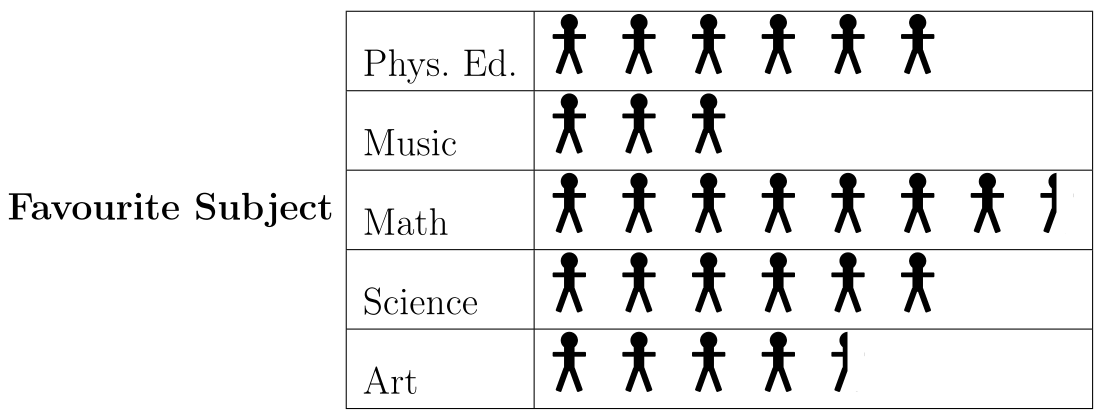
Solution

A) If each represents 10 students, and a half represents 5 then;

- $6 \times 10 = 60$ students chose Phys. Ed. as their favourite subject
- $3 \times 10 = 30$ students chose Music as their favourite subject
- $5 \times 10 = 50$ students chose Science as their favourite subject
- $(4 \times 10) + 5 = 45$ students chose Art as their favourite subject

The total of the known data is: $60 + 30 + 60 + 45 = 195$. To determine how many students voted for Math, you must remove the votes from the 195 students that have been counted, from the total 270 who voted. So, $270 - 195 = 75$ students voted for Math as their favourite subject.

B) Completed pictograph:



Key: Each represents 10 students.
Each represents 5 students.

C) In order to find the mode and the median for the data set, you should arrange the data from least to greatest. In this case, the data values are:
30, 45, 60, 60, 75

The mode is the value that occurs most often in the data set, so the mode is 60.

The median is the middle value in the data set, so the median is 60.

Teacher's Notes

The word average is often used in general conversation, but it is an ambiguous term. Many people equate *average* to *mean*, but statisticians use several different measurements of central tendency to describe averages of data. Central tendency is a formal way of saying "the typical values in a set of numbers", and *mean*, *median* and *mode* are three standard measurement tools. The *mean* tends to be easier to calculate, since the data does not have to be sorted. However, the *median* is often a better descriptor of an average value. The *mean* weighs all values in the set equally, so small numbers of extreme values can shift the calculated average. The *median* will not be as affected by a small number of outlining values.

Statisticians measure the data in many other ways. For example, *standard deviation* is a measurement tool that describes how clustered the data is around its *mean*. If a set of data has a low standard deviation, that indicates that the individual data values tend to be close to the *mean*. If the *standard deviation* is high, then the data values are much more spread out. If you are using data to predict the future, then a low standard deviation will generally lead to better predictions in applications such as weather, finance, or polling. However, there are never any guarantees.

Problem of the Week
Problem A
Pet Care

Rebecca decides to start her own pet-care business. Four neighbours are going on holidays and hire Rebecca to watch their animals while they are away. Rebecca is paid for each visit that she makes to the houses. She makes 30 visits overall.

- Rebecca visits the Smiths' house three times a day to feed and walk their dog for the three days they are away.
- The Kings' snake, needs one less visit per day than the Smiths' dog. But the Kings are away one more day than the Smiths.
- The Webers' turtle is fed the same number of times a day as the snake. The Webers are away for one more day than the Kings.
- Rebecca visits the Starks' fish once a day.

How many days were the Starks away? A chart might be helpful.

Family Name	Visits Each Day	Days Away	Total Number of Visits

Strand Data Management and Probability

Problem of the Week
Problem A and Solution
Pet Care

Problem

Rebecca decides to start her own pet-care business. Four neighbours are going on holidays and hire Rebecca to watch their animals while they are away. Rebecca is paid for each visit that she makes to the houses. She makes 30 visits overall.

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- The Webers' turtle is fed the same number of times a day as the snake. The Webers are away for one more day than the Kings.
- Rebecca visits the Starks' fish once a day.

How many days were the Starks away? A chart might be helpful.

Solution

First, fill in the chart with as much information as we can.

Family Name	Visits Each Day	Days Away	Total Number of Visits
Smith (dog)	3	3	$3 \times 3 = 9$
King (snake)	2	4	$2 \times 4 = 8$
Weber (turtle)	2	5	$2 \times 5 = 10$
Stark (fish)	1	?	?

If we add up the total number of visits we know about from the chart we get:

$$9 + 8 + 10 = 27$$

Since we know that Rebecca made 30 visits overall, she must have made

$$30 - 27 = 3$$

visits to the Starks. Since she only visited the Starks once a day, then they must have been away for 3 days.

Teacher's Notes

This problem can be solved using a spreadsheet. A spreadsheet contains cells that are identified by a row number and a column letter. In each cell we can put either a constant value or a formula. Constant values can be numbers or words. From the problem description, we know the numbers for the cells **B2**, **C2**, **B5**, and **D6**. For example, we could start filling in the spreadsheet as follows:

	A	B	C	D
1	**Family Name**	**Visits Each Day**	**Days Away**	**Total Visits**
2	Smith (dog)	3	3	
3	King (snake)			
4	Weber (turtle)			
5	Stark (fish)	1		
6			Total:	30

A formula starts with an equals sign (=) and includes references to other cells. Spreadsheets often use a star (*) for multiplication and a forward slash (/) for division. The rest of the cells can be filled in with formulae.

For example, since the Kings' snake needs one less visit per day than the Smiths' dog, the formula for cell **B3** is calculated by using the value from cell **B2** and subtracting 1. This formula is written as: = **B2 - 1**. Here is the complete spreadsheet:

	A	B	C	D
1	**Family Name**	**Visits Each Day**	**Days Away**	**Total Visits**
2	Smith (dog)	**3**	**3**	= B2 * C2
3	King (snake)	= B2 - 1	= C2 + 1	= B3 * C3
4	Weber (turtle)	= B3	= C3 + 1	= B4 * C4
5	Stark (fish)	**1**	= D5 / B5	= D6 - (D2 + D3 + D4)
6			Total:	**30**

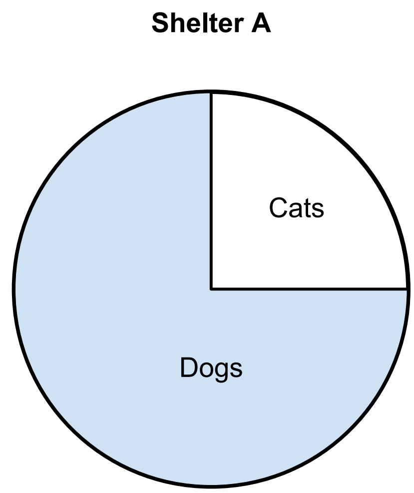
The most complicated formula in this case is the one that calculates the total number of visits made to the Starks' house. This formula takes the total number of visits to all families, and subtracts the sum of the total visits to the Smiths, Kings, and Webers.

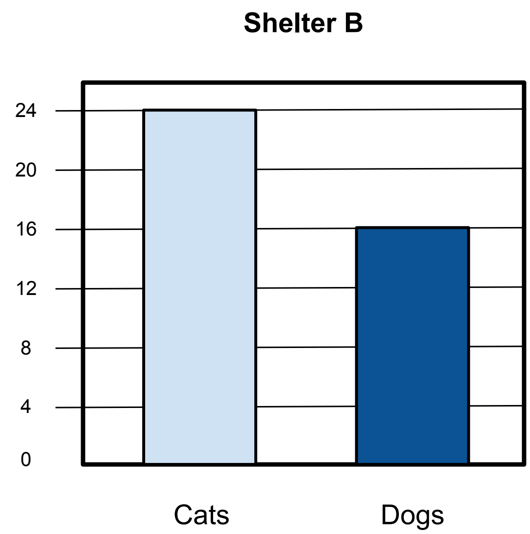
If you have access to a spreadsheet, you could copy the values and formulae given, and the results would be automatically calculated and displayed as follows:

	A	B	C	D
1	**Family Name**	**Visits Each Day**	**Days Away**	**Total Visits**
2	Smith (dog)	3	3	9
3	King (snake)	2	4	8
4	Weber (turtle)	2	5	10
5	Stark (fish)	1	3	3
6			Total:	30

Problem of the Week
Problem A
More Dogs

The graphs below represent the number of cats and dogs in two local animal shelters. Shelter A and Shelter B have the same number of animals.





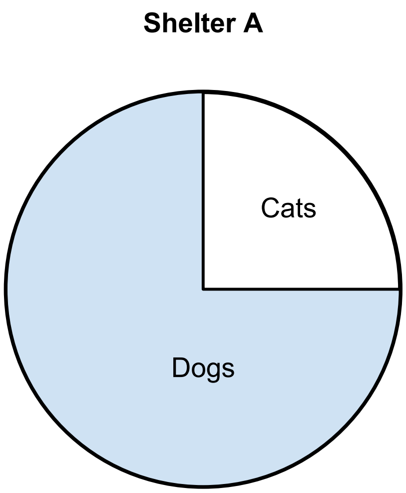
There are more dogs in Shelter A than in Shelter B. How many more? Justify your answer.

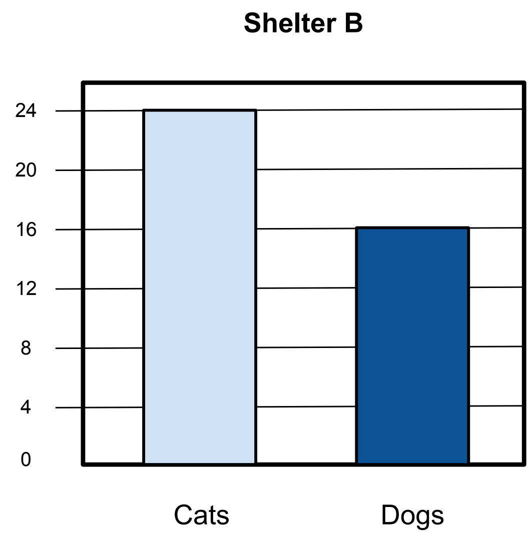
Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
More Dogs

Problem

The graphs below represent the number of cats and dogs in two local animal shelters. Shelter A and Shelter B have the same number of animals.





There are more dogs in Shelter A than in Shelter B. How many more? Justify your answer.

Solution

Looking at Shelter B, we see that there are 24 cats and 16 dogs. This means there is a total of $24 + 16 = 40$ animals in each of the two shelters.

It appears that in Shelter A, $\frac{3}{4}$ of the shelter is filled with dogs, and $\frac{1}{4}$ are cats. If the total number of animals at this shelter is 40, then we need to calculate $\frac{1}{4}$ of 40.

Since $10 + 10 + 10 + 10 = 40$, then $\frac{1}{4}$ of $40 = 10$. So there are 10 cats in Shelter A. This means that there are $40 - 10 = 30$ dogs in Shelter A.

Since there are 30 dogs at Shelter A and 16 dogs at Shelter B, then there are $30 - 16 = 14$ more dogs at Shelter A than Shelter B.

Teacher's Notes

Data can be visualized in many ways. A tool like a spreadsheet can automatically convert numeric data into a chart. The same data can be used to generate different styles of charts. Consider the bar chart showing the data for Shelter B. The spreadsheet automatically calculated the maximum value on the y-axis and chose the distance between the horizontal lines in the chart. Most spreadsheeets would give the user the option of changing the maximum value and changing the distance between each of the horizontal lines. Once those decisions are made, the spreadsheet will automatically recalculate the size of the chart and its elements.

If students want to create their own charts, they would need to do all of those calculations themselves. The work to determine the values of regular intervals from the minimum to the maximum as the locations of the horizontal lines is not trivial. Suppose you use graph paper to draw the chart. You need to determine a scale, such as 1 square represents 2 dogs. This is setting up a *ratio*, which is a fixed relationship between the number of squares on the paper and the number of dogs that distance represents.

The calculations involved in producing a pie chart can also be tricky. It is easy to divide a pie in half or in quarters, but other fractions can be more difficult. The size of a pie slice that represents some data can be determined by equivalent fractions. A whole circle contains 360 degrees. A slice of a circle is called a *sector*. The size of the sector can be described by the angle from one edge of the slice to the other. So a sector that is one quarter of the circle has an angle of 90 degrees, since 90 is one quarter of 360. To determine the size of a sector for any data value, you need to find a fraction with a denominator of 360 that is equivalent to the fraction of the data value divided by the total number in your set. For example, suppose we had a total of 40 animals at the shelter, and 4 of them are birds. If we wanted to show a pie slice representing the birds in this example, we need to find the value of x when $\frac{x}{360} = \frac{4}{40}$. In this case $x = 36$. So the sector will have an angle of 36 degrees.

Problem of the Week

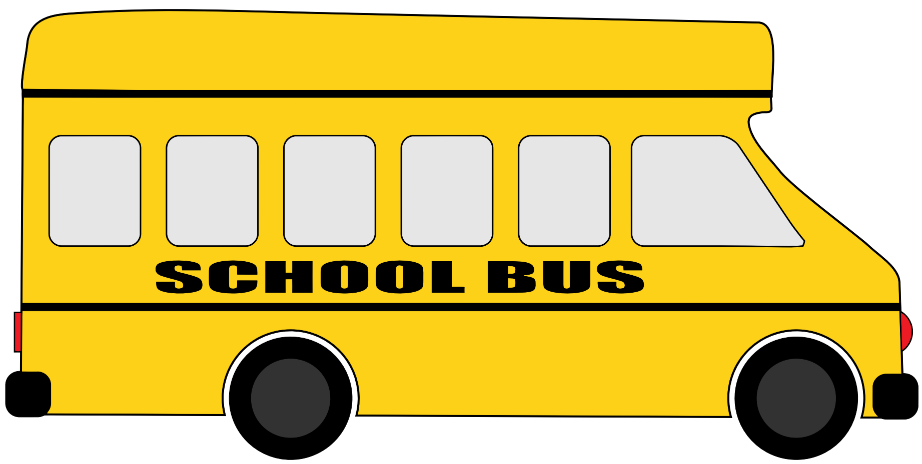
Problem A

Getting to School

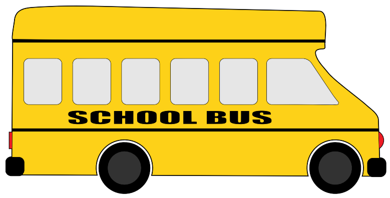
Twenty grade 3 students and 25 grade 4 students attend the School of Math. Twelve grade 3 students take the bus, and the rest walk to school. Out of all the students in grades 3 and 4, there are 23 who walk to school. At an assembly for grade 3 and 4 students, a name is drawn for a prize.

A) What is the probability that the prize winner is in grade 3 and walks to school?

B) What is the probability that the prize winner is in grade 4 and takes the bus to school?



Strands Data Management and Probability, Number Sense and Numeration



Problem of the Week

Problem A and Solution

Getting to School

Problem

Twenty grade 3 students and 25 grade 4 students attend the School of Math. Twelve grade 3 students take the bus, and the rest walk to school. Out of all the students in grades 3 and 4, there are 23 who walk to school. At an assembly for grade 3 and 4 students, a name is drawn for a prize.

A) What is the probability that the prize winner is in grade 3 and walks to school?

B) What is the probability that the prize winner is in grade 4 and takes the bus to school?

Solution

The solution for both parts is based on determining the total number of students at the assembly. There are a total of $20 + 25 = 45$ students at the assembly.

A) Since there are 20 students in grade 3, and 12 of them take the bus, then $20 - 12 = 8$ grade 3 students walk to school.

The probability that the prize winner is in grade 3 and walks to school is 8 out of 45 or $\frac{8}{45}$.

B) Since 23 students walk to school, and 8 of them are grade 3 students, then $23 - 8 = 15$ grade 4 students walk to school.

Since there are 25 students in grade 4 and 15 of them walk to school, then $25 - 15 = 10$ grade 4 students take the bus to school.

The probability that the prize winner is in grade 4 and takes the bus to school is 10 out of 45 or $\frac{10}{45}$ or $\frac{2}{9}$.

Teacher's Notes

The probability of some outcome can be described in many different ways. The solution uses the words "out of" essentially as a replacement for the line between the numerator and the denominator of a fraction. The solution also uses fractions to describe the probability.

We expect the numerator and denominator of a fraction describing probability to have specific characteristics. The numerator must be a non-negative number (i.e. greater than or equal to 0), and the denominator must be a positive number that is greater than or equal to the numerator. Since there are infinitely many equivalent fractions, there are many fractions we could use to describe the same probability. In this problem, the probability of the prize winner being in grade 4 was described as $\frac{10}{45}$ or $\frac{2}{9}$. But we could also describe the probability as $\frac{100}{450}$ or $\frac{12}{54}$.

If we compare all of these fractions by doing division with a calculator, the result is the same:

$$\frac{10}{45} = \frac{2}{9} = \frac{100}{450} = \frac{12}{54} = 0.\overline{2}$$

As shown here, all probabilities can also be described as a number between 0 and 1. So in this problem, we could describe the probability as approximately 0.22222.

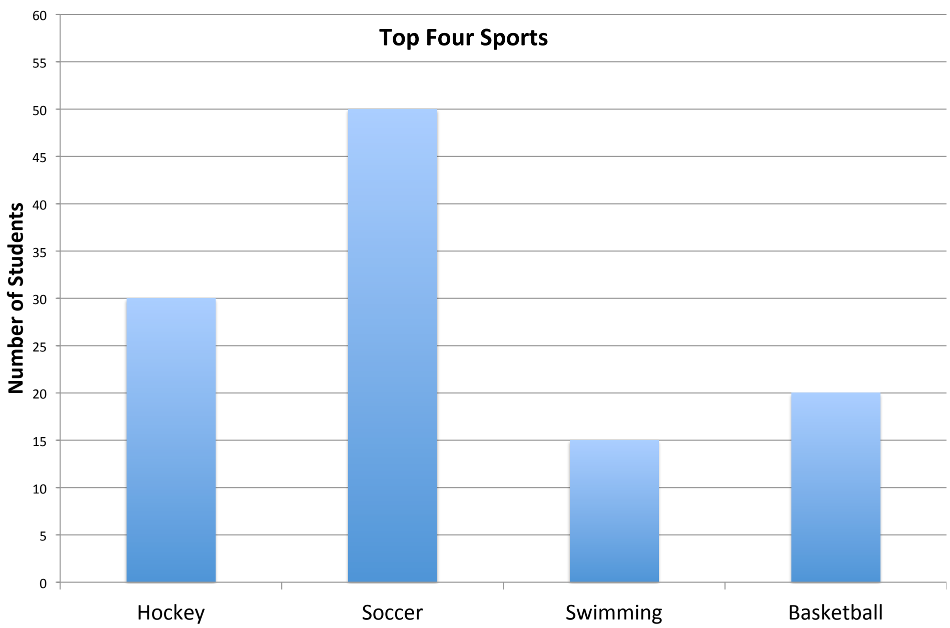
(Note that since dividing the numerator and denominator of any of the fractions produces a repeating decimal, we have to describe it as *approximately* the same.)

Problem of the Week

Problem A

Sporty Choices

Mr. King's class did a survey with the students of their school to determine favourite sports. They determined that the top four sports were soccer, basketball, swimming and hockey. They surveyed the students in the school again to have them choose from the four favourite sports. Here are the results:



A) How many students at the school were surveyed concerning the four sports?

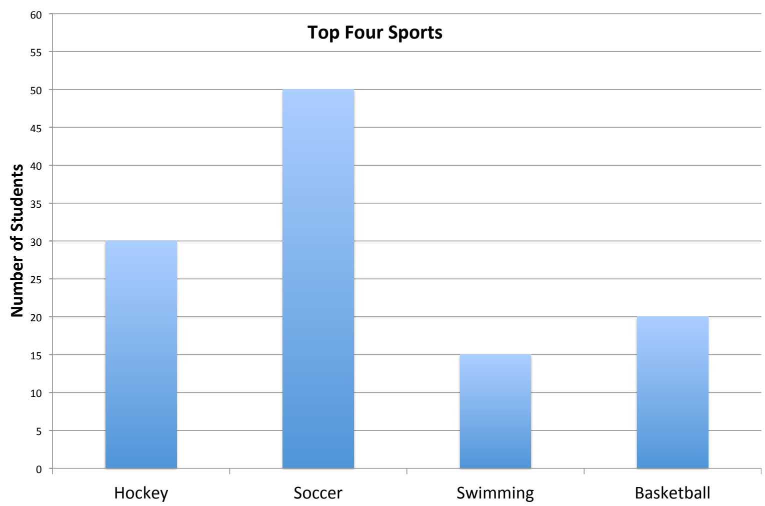
B) Of the top four most popular sports at the school, what is the difference between the number of students selecting the most popular sport and the number of students selecting the least favourite sport?

Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Sporty Choices

Problem

Mr. King's class did a survey with the students of their school to determine favourite sports. They determined that the top four sports were soccer, basketball, swimming and hockey. They surveyed the students in the school again to have them choose from the four favourite sports. Here are the results:



A) How many students at the school were surveyed concerning the four sports?

B) Of the top four most popular sports at the school, what is the difference between the number of students selecting the most popular sport and the number of students selecting the least favourite sport?

Solution

A) Based on the chart we can determine that 30 students chose hockey, 50 students chose soccer, 15 students chose swimming, and 20 students chose basketball.

Therefore, the total number of students in the school that were surveyed is: $30 + 50 + 15 + 20 = 115$.

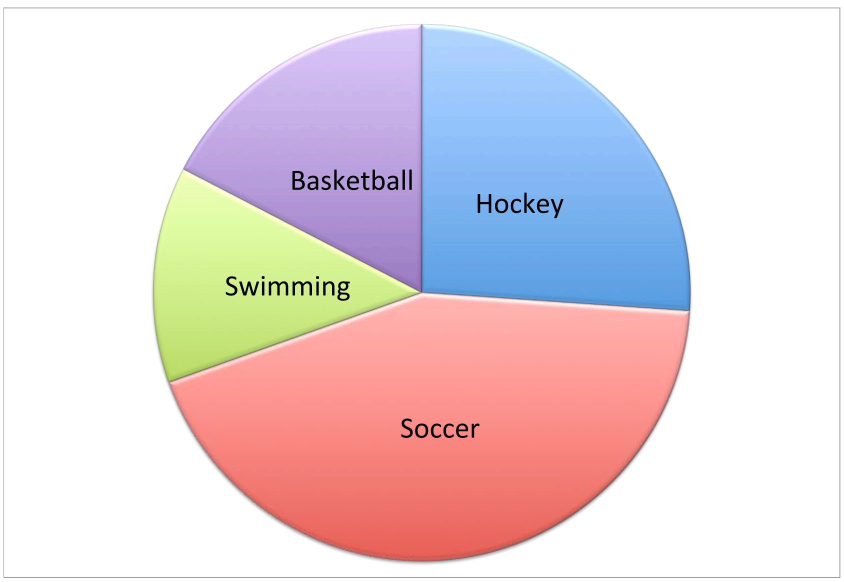
B) The most popular sport is soccer with 50 students. The least popular sport is swimming with 15 students.

The difference between most and least popular sports is: $50 - 15 = 35$ students.

Teacher's Notes

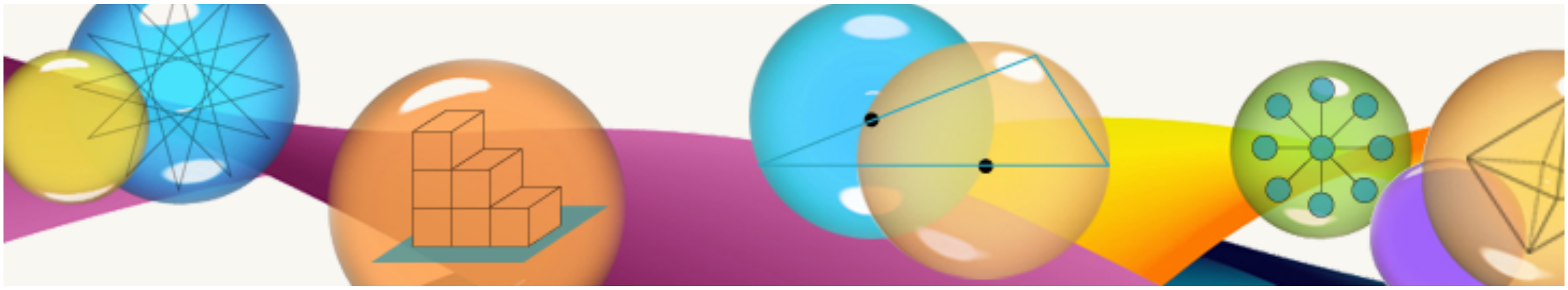
We use charts to visualize numeric data. The same data can be used to generate different types of charts. A column chart is a good way to identify relative sizes. It is immediately clear from the column chart that soccer is the most popular sport and that swimming is the least popular sport in this survey. Since the chart includes clearly marked grid lines, and the top of all the columns align with the grid lines, then it is also easy to determine the numbers that are being represented by the picture. Even if the columns do not align with the gridlines, we can reasonably estimate the values that the columns represent.

Other types of charts can emphasize different aspects of the data. A line chart is a good way to identify trends over time. A pie chart is a good visualization of proportion or the percentage each data point represents in the whole. Here is the same data from the original problem shown in the form of a pie chart:



Using this format, it is easy to see that, of the students surveyed in this school, close to half of them say that soccer is their favourite sport and approximately a quarter of them picked hockey.

Spreadsheet programs make it easy to generate different charts depending on how you want to present the data.



Geometry & Spatial Sense

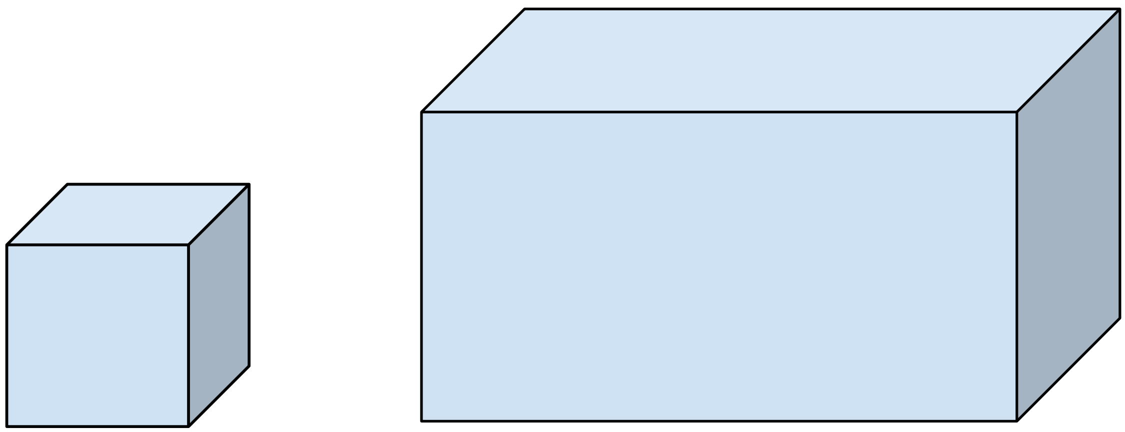
TAKE ME TO THE COVER

Problem of the Week

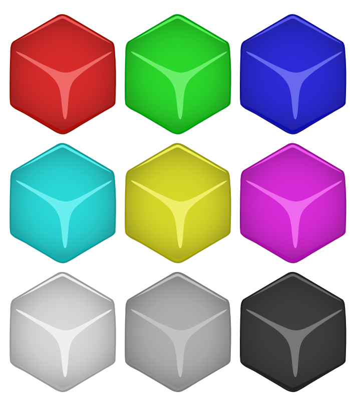
Problem A

Block Builder

Tabia has 18 blocks. Show the number of different solid, rectangular prisms she can build, using all 18 cubes.



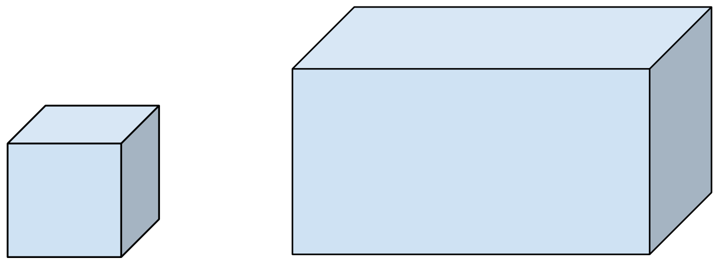
Note: A rectangular prism is a 3-D figure with six rectangular faces.



Strands Measurement, Geometry and Spatial Sense

Problem of the Week
Problem A and Solution
Block Builder

Problem
Tabia has 18 blocks. Show the number of different solid, rectangular prisms can she build, using all 18 cubes.



Note: A rectangular prism is a 3-D figure with six rectangular faces.

Solution
Since the rectangular prism is formed by solid objects, each dimension of the prism must be a whole number of blocks. According to the question, the volume of the prism is 18. The volume is calculated by multiplying the three dimensions together. So you need to find the total number of distinct combinations of three whole numbers that have a product of 18.

To find these combinations, look at the numbers that divide exactly into 18. They are: 1, 2, 3, 6, 9, and 18. Pick any two of these numbers and determine if there is a third value from this list so that the product of the three numbers is 18. There are four combinations that will work:

- $1 \times 1 \times 18 = 18$
- $1 \times 2 \times 9 = 18$
- $1 \times 3 \times 6 = 18$
- $2 \times 3 \times 3 = 18$

Note that a rectangular prism with dimensions $1 \times 3 \times 6$ is the same as a rectangular prism with dimensions $3 \times 1 \times 6$. The objects are simply rotations of each other.

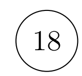
Teacher's Notes

In this problem, you are given 18 blocks and there are four different rectangular prisms you can assemble with those blocks. Interestingly, a larger number of blocks does not necessarily lead to a larger number of prisms. For example, given 35 blocks, there are only two possibilities: $1 \times 1 \times 35 = 35$ and $1 \times 5 \times 7 = 35$.

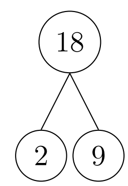
The number of different prisms that can be formed depends on the prime factorization of the number of blocks. A prime factorization is a product formed entirely of prime numbers. A prime number has exactly two factors: 1 and itself, and 2 is the smallest prime number. For example, the prime factorization of 48 is: $2 \times 2 \times 2 \times 2 \times 3$.

One way to find the prime factorization of a number is to use a factor tree. Here is how you can form a factor tree for the number 18.

Start with the number 18.

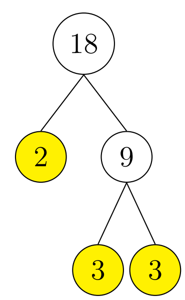


Since 18 is not a prime number, find a factor pair, and add two *branches* that lead to *nodes* (the circles) containing each number of the factor pair.



Look for any other *nodes* in the tree that are not prime numbers. Add two branches from those nodes that lead to *nodes* containing each number of a factor pair. Continue adding new *branches* and *nodes* to the tree as long as there are non-prime number nodes. In this case, the only non-prime number left is 9 with a factor pair 3×3. At this point, the factor tree for 18 is complete.

The prime factorization is composed of the nodes in the tree that contain prime numbers. So, the prime factorization of 18 is: $2 \times 3 \times 3$.

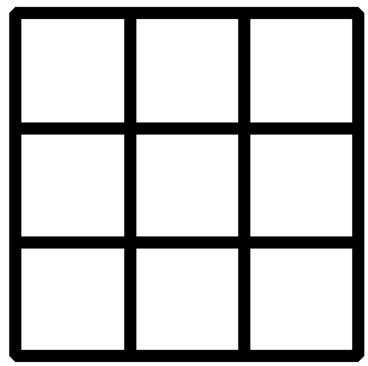


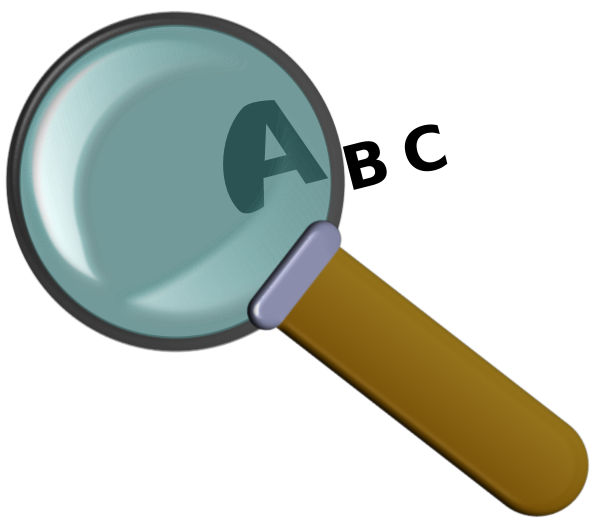
More factors in the prime factorization of a number means more possible divisors for that number, which means there are more ways you can write the number as a product of three numbers. *Highly composite numbers* are a special set of numbers with many divisors. They were first described in detail by Srinivasa Ramanujan, a mathematician from India, born in 1887, who unfortunately died at the age of 32. Despite very limited opportunities for higher education early in his life, Ramanujan eventually went on to be a well-respected mathematician. Eventually he went on to continue his research at the University of Cambridge. His life has been dramatized in the film *The Man Who Knew Infinity (2015)*.

Problem of the Week
Problem A
Letter Clues

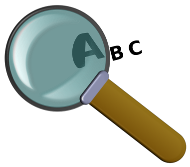
Using the following clues, place the letters A, B, C, D, E, F, G, H, and I in the correct boxes. Two boxes are touching if they share a side.

- A is touching C and D
- B is directly under H
- C is on the left
- D is on the bottom, on the right
- E is touching H
- F is not touching D
- G is in the centre
- H is on the left
- I is touching D





Strand Geometry and Spatial Sense



Problem of the Week
Problem A and Solution
Letter Clues

Problem

Using the following clues, place the letters A, B, C, D, E, F, G, H, and I in the correct boxes. Two boxes are touching if they share a side.

- A is touching C and D
- B is directly under H
- C is on the left
- D is on the bottom, on the right
- E is touching H
- F is not touching D
- G is in the centre
- H is on the left
- I is touching D

Solution

Two of the clues, *G is in the centre* and *D is on the bottom, on the right*, describe the exact placement letters in the grid. This is a good place to start.

	G	
		D

From the clues, *C is on the left* and *A is touching C and D*, then C must be on the bottom row, in the left corner, and A must be in the bottow row in the centre.

	G	
C	A	D

From the clues, *H is on the left* and *B is directly under H*, we can place H and B in the grid.

H		
B	G	
C	A	D

At this point, based on the clue, *E is touching H*, the only place that E fits is in the centre of the top row. Since *F is not touching D*, then F must be on the right side of the top row. Now, there is only one place for I, which is touching D.

H	E	F
B	G	I
C	A	D

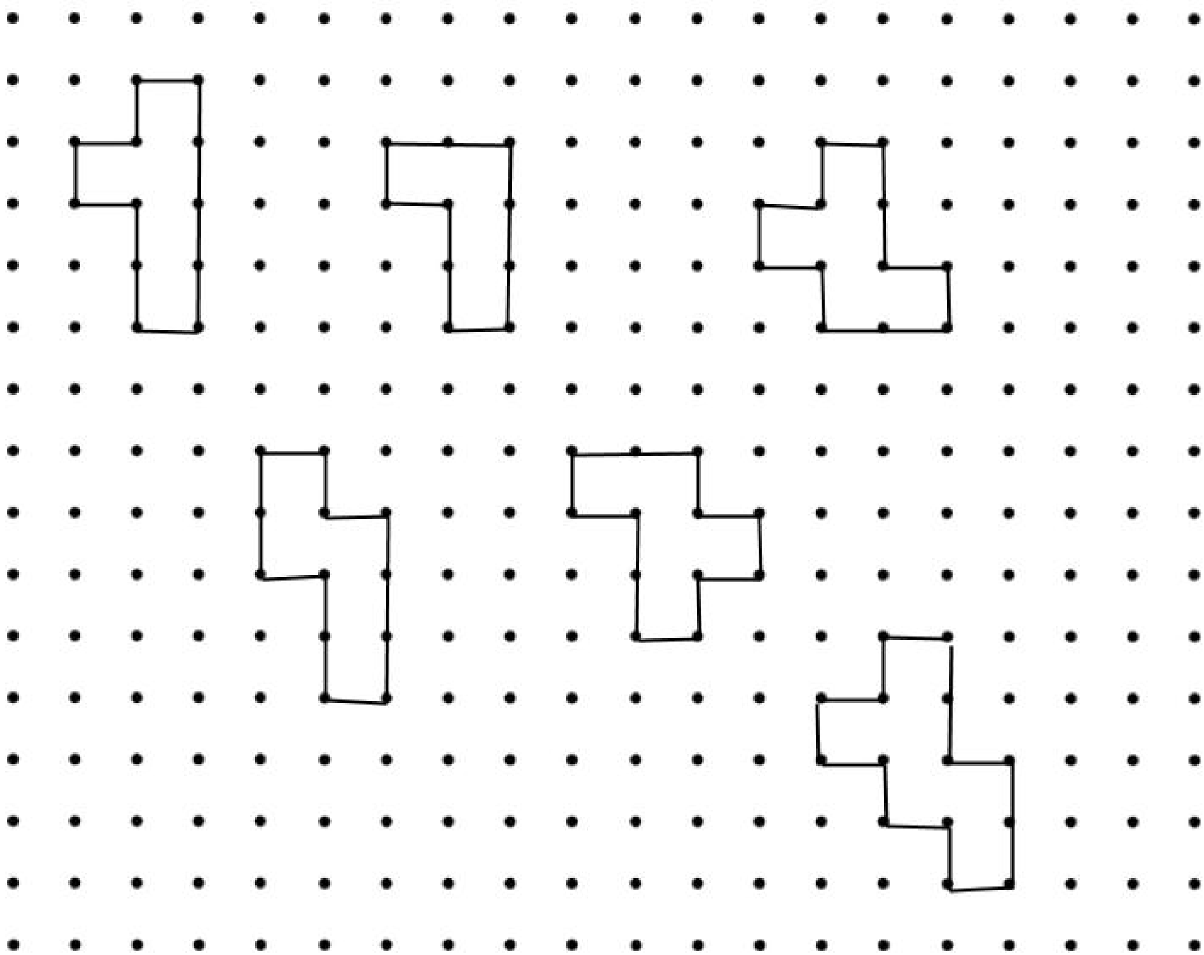
Teacher's Notes

This problem has exactly one solution. The solution can be determined through logical steps; it is not necessary to make any guesses. Some students may choose a "guess and check" approach. This is an example of a **brute force** algorithm. Unlike this one, some problems can only be solved using brute force. However, we try to avoid using brute force to solve problems whenever possible since it is very inefficient. At the very least, we want to limit the number of guesses that must be checked.

Sudoku puzzles are a good way to practice using logic to narrow the number of possible solutions to a problem. Some of the easier puzzles can be solved without having to make any guesses. For more difficult problems, it may be necessary to make a guess and see if it works out. If it does not work out, you need to undo all of the decisions you made based on that guess, make a different guess, and try again from that point. This type of approach is called **backtracking**, which is a standard algorithm used to solve many different types of problems. Backtracking is most effective when you have a small number of choices to make at any point during the process of solving the problem.

Problem of the Week
Problem A
What is Missing?

Add the fewest number of squares to each figure shown below, so that each has at least one line of symmetry?



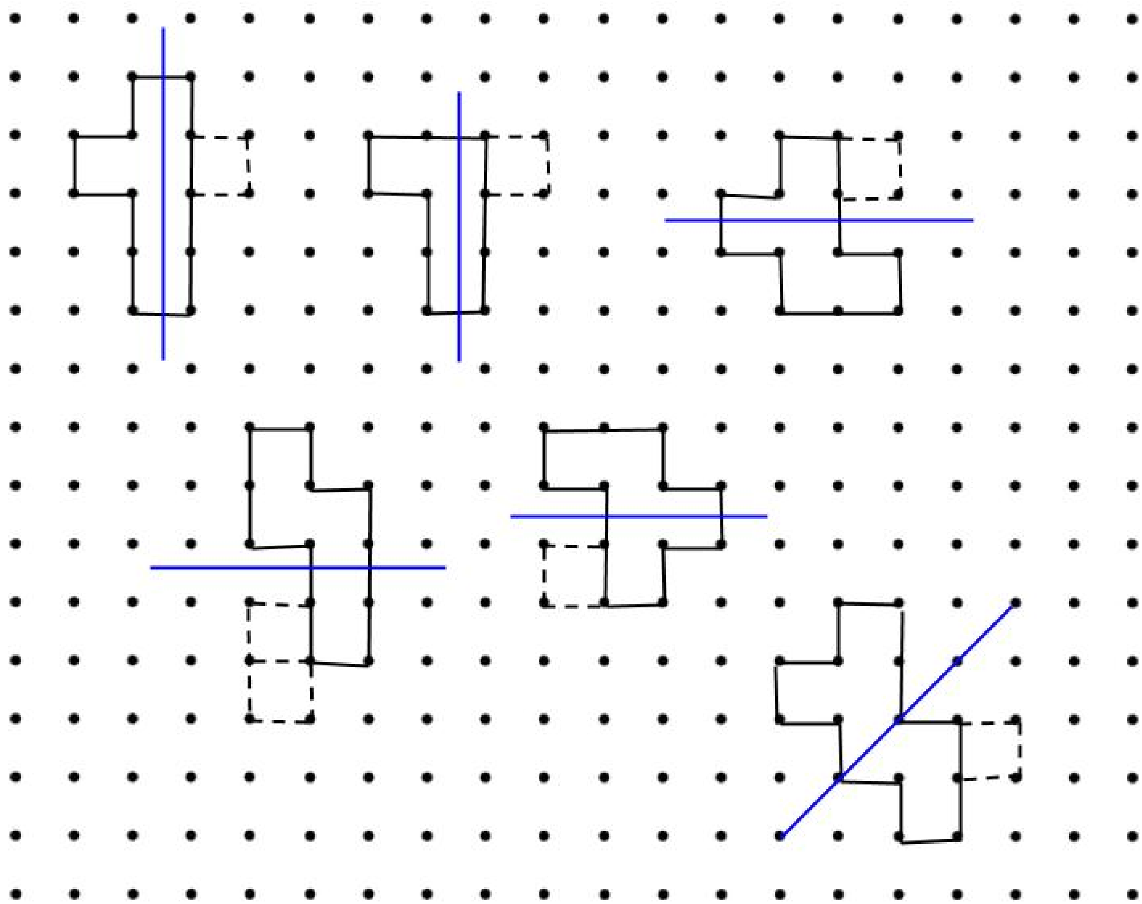
Strand Geometry and Spatial Sense

Problem of the Week
Problem A and Solution
What is Missing?

Problem

Add the fewest number of squares to each figure shown below, so that each has at least one line of symmetry?

Solution



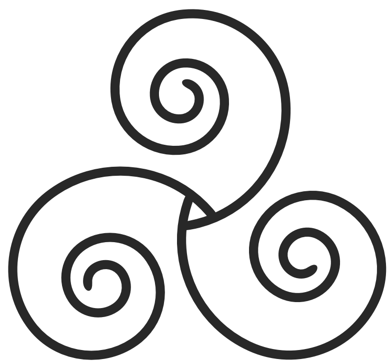
The straight, blue lines show the lines of symmetry. The dotted lines are the boxes that you may add to the figures to make them symmetrical. There may be other ways to add the same number of squares to the figures that produce a line of symmetry.

You may use miras, tracing paper, or cut out and fold to confirm the line of symmetry.

Teacher's Notes

This question asks students to modify the figures so that the results have at least one line of symmetry. When we refer to a figure that has a line of symmetry, this is identifying reflection symmetry, also known as mirror symmetry. We can confirm that a figure has this type of symmetry by making a fold along the line to confirm that that two halves of the image align perfectly.

There are other types of symmetry that can be used to describe images. Point reflection describes a symmetry of an image that, when rotated about a fixed point by 180°, will look exactly the same. The letter Z is symmetric in this way. Rotational symmetry is similar to point reflection, except the image will look exactly the same when rotated by some amount - the rotation is not restricted to exactly 180°. A classic example of this kind of symmetry is the triskelion. Here is an example:



Symmetries can be generated as a result of basic transformations: flips, slides, and turns. We can create mathematical models of the images and write mathematical functions to complete these transformations. Ultimately if we want to manipulate graphic images on a computer, we rely on mathematics to handle the details.

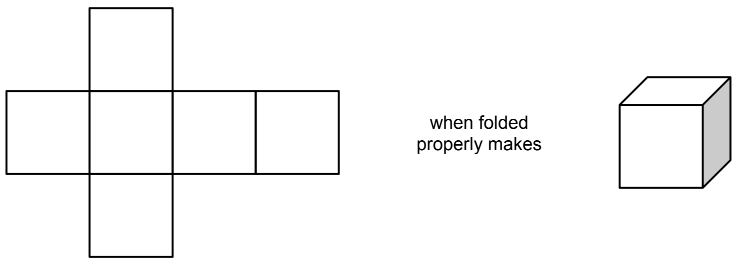
(Main source - Wikipedia)

Problem of the Week

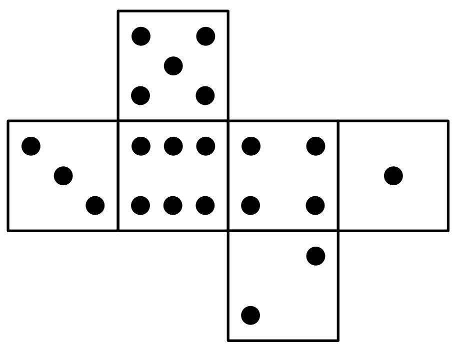
Problem A

Which Way Do I Roll?

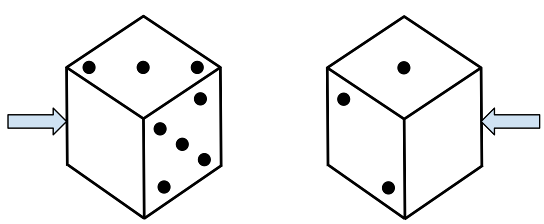
A net is a pattern that can be cut out and then folded together to create a solid shape like a cube. For instance:



Here is a net that can be used to form a single die.



Here are two views of the die that is formed by this net.



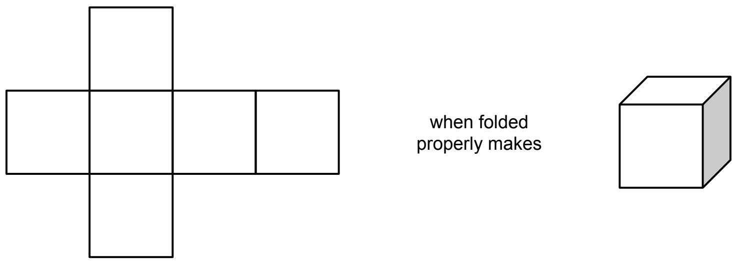
Complete the blank faces of the die, indicated by the arrows. Ensure that you have the correct number of dots, oriented in the right direction.

Strand Geometry and Spatial Sense

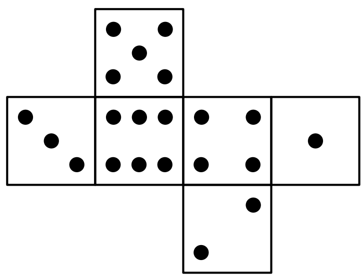
Problem of the Week
Problem A and Solution
Which Way Do I Roll?

Problem

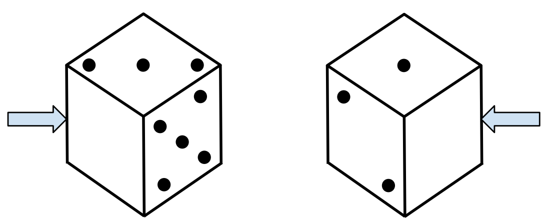
A net is a pattern that can be cut out and then folded together to create a solid shape like a cube. For instance:



Here is a net that can be used to form a single die.

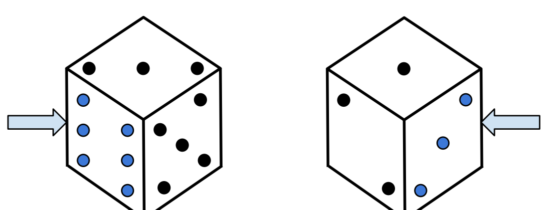


Here are two views of the die that is formed by this net.



Complete the blank faces of the die, indicated by the arrows. Ensure that you have the correct number of dots, oriented in the right direction.

Solution



Teacher's Notes

A net is a 2-dimensional object that can be folded to form a 3-dimesional object. The 3-dimensional objects in this problem are represented by freehand graphical images that are still drawn in 2-dimesional space. It is not easy to visualize 3D transformations of objects that we represent in 2D space.

In mathematics we can use the Cartesian plane with the X-axis and the Y-axis to draw 2D shapes, and we can include the Z-axis to represent the third dimension. This means we can model the 3D objects using coordinates that are written using three numbers. For example (5, 2, 4) represents a point in 3D space that is 5 units along the X-axis, 2 units along the Y-axis and 4 units along the Z-axis.

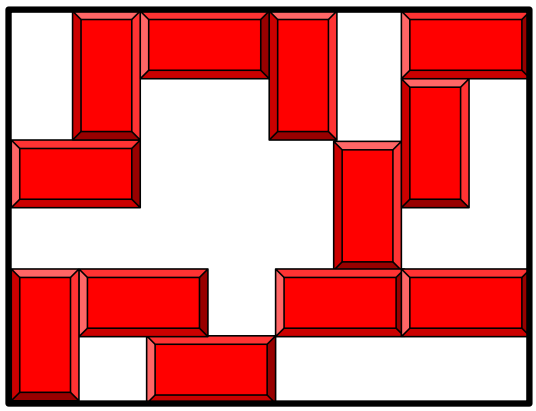
Engineers, architects, animators and others have drawn representations of solid objects on paper or on computer screens for years. A good mathematical understanding of 3D geometry is essential to being able to accurately describe the models that are eventually built or animated. Computer software can help visualize 3D objects, since you can rotate and view your design from different angles to make sure it is correct. With the increased accessibility of 3D printers, the ability to represent solid objects on a 2D surface is even more useful.

Problem of the Week

Problem A

Building Blocks

Johnna Lee loves to build with interlocking building blocks. She starts building on a large, white, flat piece for the base, but the rest of the blocks she uses are red. Looking from overhead, this is what Johnna Lee built.



Each of the red blocks has a length of 2 cm and a width of 1 cm.

A) What is the area of the white base?

B) What fraction of the area of the base is covered by red blocks?

Strands Measurement, Geometry and Spatial Sense, Number Sense and Numeration

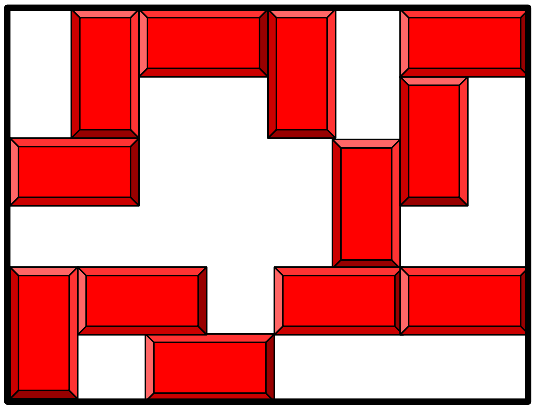
Problem of the Week

Problem A and Solution

Building Blocks

Problem

Johnna Lee loves to build with interlocking building blocks. She starts building on a large, white, flat piece for the base, but the rest of the blocks she uses are red. Looking from overhead, this is what Johnna Lee built.



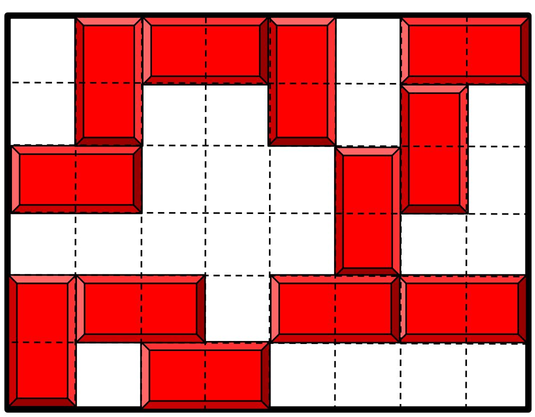
Each of the red blocks has a length of 2 cm and a width of 1 cm.

A) What is the area of the white base?

B) What fraction of the area of the base is covered by red blocks?

Solution

You can draw grid lines that align with the edges of the blocks. The squares formed by the grid lines are 1 cm × 1 cm.



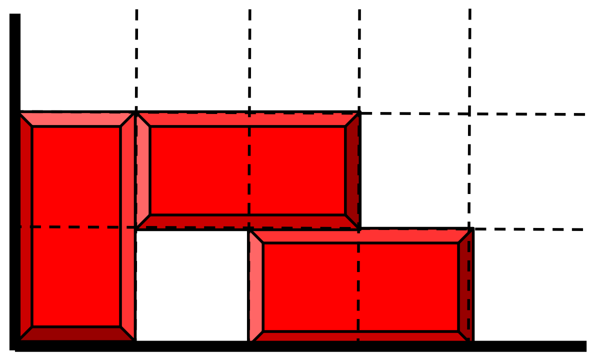
A) If you count the total number of squares formed by the grid lines, you see that there are 48. Alternatively, you can see that the base is 8 cm wide and 6 cm high. The area of the base is 8 cm $\times$ 6 cm $=$ 48 cm^2.

B) Count the number of squares formed by the grid lines that overlap the blocks. The total is 24. The fraction of the area of covered by blocks over the area of the base is $\frac{24}{48}$. Since $48 = 24 \times 2$ we can also write the fraction as $\frac{1}{2}$.

Here is another way to calculate this fraction. The area of one of the blocks is 1 cm $\times$ 2 cm $=$ 2 cm^2. There are 12 blocks showing on the base. The total area they cover is 12×2 cm^2 $=$ 24 cm^2. Since the area of the base is 48 cm^2, the blocks cover $\frac{24}{48}$ or $\frac{1}{2}$ the area of the base.

Teacher's Notes

The second part of this problem could be solved without knowing any specific information about the dimensions of the blocks. Everything you need to determine what fraction of the base is being covered by the blocks is available in the picture. The key is knowing that the ratio of the length to the width of the building block is 2 : 1. This can be determined by examining the bottom left corner of the diagram.



From this part of the picture, we see that the twice the shorter side of the block is equal to the length of the longer side of the block. This relationship, along with the way the blocks are aligned in the rest of the diagram, allows us to add the grid lines to the diagram. These grid lines form unit squares. The actual size of those squares is irrelevant; their measurements could be 1 cm $\times$ 1 cm $=$ 1 cm^2, or 1 m $\times$ 1 m $=$ 1 m^2, or 1 Wiffle $\times$ 1 Wiffle $=$ 1 Wiffle^2. What is important for us, is that they are squares. Since we are computing a fraction, the units will disappear in the calculation.

Ultimately the solution for this problem can be computed based on the number of unit squares that form the base of the structure and the number of unit squares the blocks cover. In this case we have a fraction of:

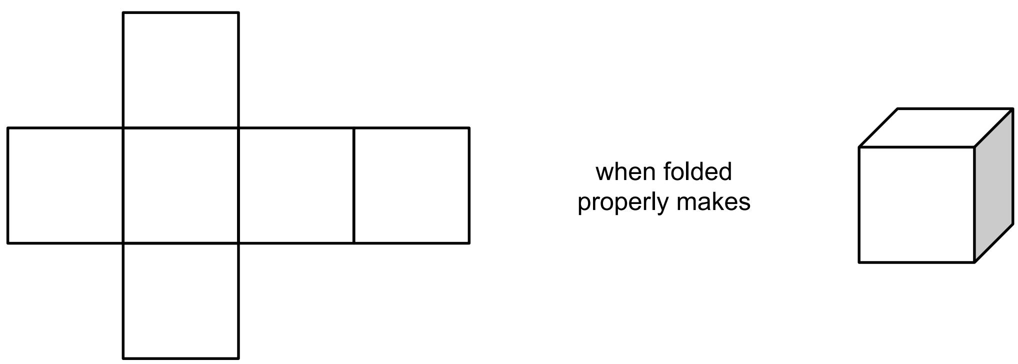
$$\frac{24 \text{ \sout{units}}^2}{48 \text{ \sout{units}}^2} = \frac{1}{2}$$

Problem of the Week

Problem A

Nancy's Nets

When Nancy recycles boxes she flattens them. Since she is learning about nets in school, she notices there are many different ways to flatten a box that is a cube into a net. A net is a pattern that can be cut out and then folded together to create a solid shape.
For example:



Nancy draws all the possible nets for a cube. There are eleven nets. What do they look like?

Strand Geometry and Spatial Sense

Problem of the Week
Problem A and Solution
Nancy's Nets

Problem

When Nancy recycles boxes she flattens them. Since she is learning about nets in school, she notices there are many different ways to flatten a box that is a cube into a net. A net is a pattern that can be cut out and then folded together to create a solid shape.
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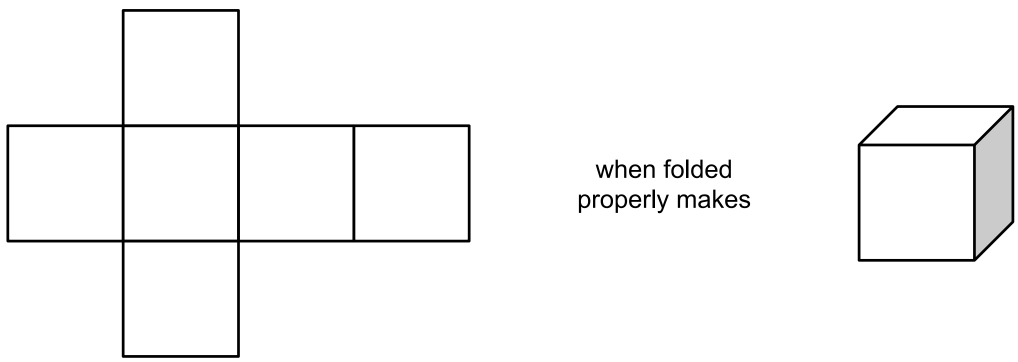
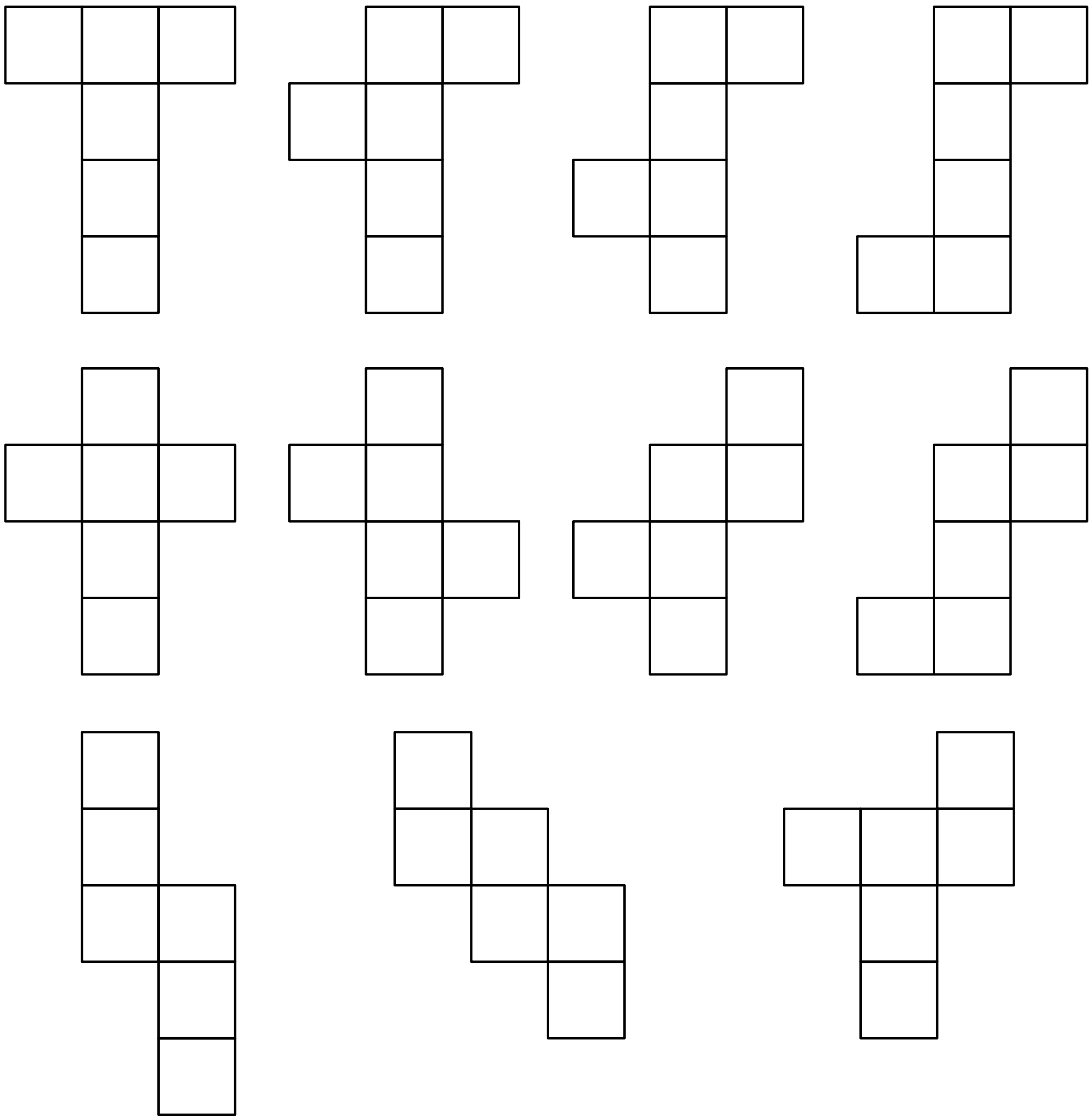


Nancy draws all the possible nets for a cube. There are eleven nets. What do they look like?

Solution

Here are the eleven possible nets for a cube:



Teacher's Notes

Being able to design a 2-dimensional net that can be folded into a 3-dimensional object has obvious advantages when it comes to storing and transportation. Imagine if you were manufacturing cardboard boxes and had to ship them already assembled. This would be an incredible waste of space. This application of geometry can be used for items as simple as cardboard boxes or as complex as elements of the International Space Station.

The CEMC has a series of e-books called "Invitations to Mathematics".

`http://www.cemc.uwaterloo.ca/resources/invitations-to-math.html`

They are available for free online. One of the volumes aimed at grade 6 students focusses on nets. If you have not already seen these, you may find them useful resources.

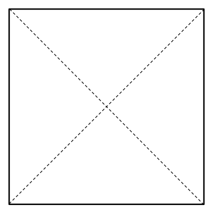
Problem of the Week

Problem A

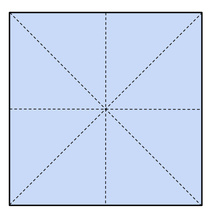
Origami

Laila likes to do origami which is the art of paper folding. Just by folding paper in a particular way, she can make all sorts of different animals. Many of the animals start with the same steps.

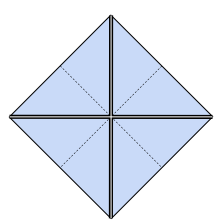
Laila starts with a square piece of paper. She folds it in half to form a triangle. Then she opens up the paper to start with a square again. She folds it in the opposite direction and also forms a triangle. When she opens it up again she can see creases on the paper that look like this:



Then she turns the paper over and folds it in half to form a rectangle. She opens up the paper and folds it in the opposite direction to form another rectangle. When she opens up the paper this time, she sees creases in the paper that look like this:



The centre of the square is the point where all of the creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another square.



What fraction of the area of the original square is the area of the smaller square? Justify your answer.

Strands Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Origami

Problem

Laila likes to do origami which is the art of paper folding. Just by folding paper in a particular way, she can make all sorts of different animals. Many of the animals start with the same steps.

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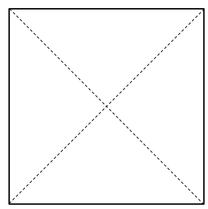
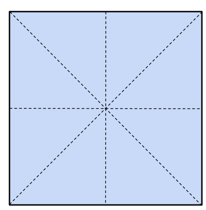
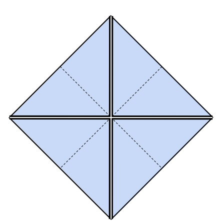


Then she turns the paper over and folds it in half to form a rectangle. She opens up the paper and folds it in the opposite direction to form another rectangle. When she opens up the paper this time, she sees creases in the paper that look like this:



The centre of the square is the point where all of the creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another square.



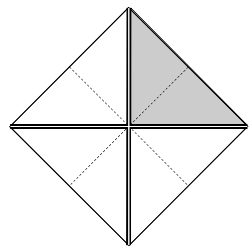
What fraction of the area of the original square is the area of the smaller square? Justify your answer.

Solution

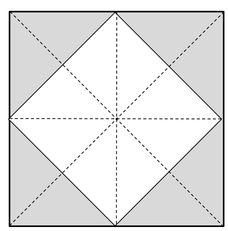
The smaller square has an area that is $\frac{1}{2}$ the area of the original square. There are several ways in which you can justify this.

Solution 1:

One way is to look at the following picture. Notice that the shaded triangle covers an area underneath it that is exactly the same size. That is true for all four of the triangles that have their points meet at the middle.



If we opened up the paper again, we could make the following observations:

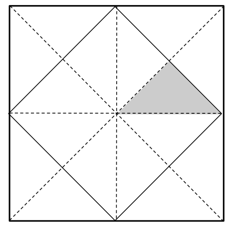


- The shaded parts of the original square each have a matching unshaded part.
- The shaded parts make up the area of the smaller square.
- This means that the area of the original square is 2 times the area of the smaller square.

It follows that the area of the smaller square is $\frac{1}{2}$ the area of the original square.

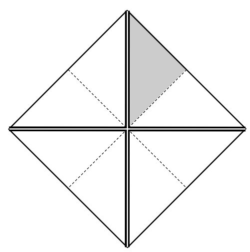
Solution 2:

Another way to show that the area of the smaller square is $\frac{1}{2}$ the area of the original square is to count the number of small triangles that are formed by the creases in the paper. One such triangle is shaded in the following diagram.



Each of these triangles have the same area. One way of showing that they all have the same area would be to cut up the square into the triangles and stack them on top of each other. If you count the number of small triangles in the original square there are 16 of them.

If you count the number of small triangles in the smaller square, there are 8 of them.



Since the smaller square is formed by half the number of small triangles as compared to the original square, then the smaller square has $\frac{1}{2}$ the area of the original square.

Teacher's Notes

Origami can be used to demonstrate many different geometric shapes, especially triangles and quadrilaterals. The instructions in this problem start with a square and then make folds to form a triangle, a rectangle, and then a smaller square.

There are many books and websites that can show you the steps required to create simple and complex origami figures. If you follow the steps to creating complex figures, you may see several different polygons, including a parallelogram, a trapezoid, and a *kite*. A kite is a quadrilateral with adjacent sides that are the same length.

Both the triangle formed in the first diagonal fold, and the smaller triangles formed by the creases are all right-angled, isoceles triangles. It is relatively easy to confirm that the triangle from the first diagonal fold satisfies this condition. Since you start with a square, and the interior angles of a square are all 90 degrees, then the corner of the triangle where the points of the square meet must be 90 degrees. Therefore the triangle is a right-angled triangle. Since the other two sides of the triangle are sides of the original square, those sides must be equal. Therefore it is an isoceles triangle. Also, we can determine the size of the other two interior angles of this triangle with logical deduction. When you fold the square on the diagonal and then open it up again, the crease that you see has bisected the angle in the corner of the square. Since that angle is 90 degrees, then the angle formed by the side of the square and the crease is half that size, which is 45 degrees. It would take a longer argument to show that the 16 smaller triangles are also right-angled, isoceles triangles. You could convince yourself it is true by cutting up the square into the 16 pieces and comparing the sides of a pair of triangles. You could also check that one corner of one of the triangles aligns with the corner of a square or a rectangle to confirm that its angle is 90 degrees.

Being able to make logical deductions in geometry is important. Finding the correct answer to a problem is not the only thing mathematicians and computer scientists care about. They are also concerned with describing the process for finding the correct answer and **proving** that the answer is correct. Problems in geometry can be a good place to practice these skills.

Measurement

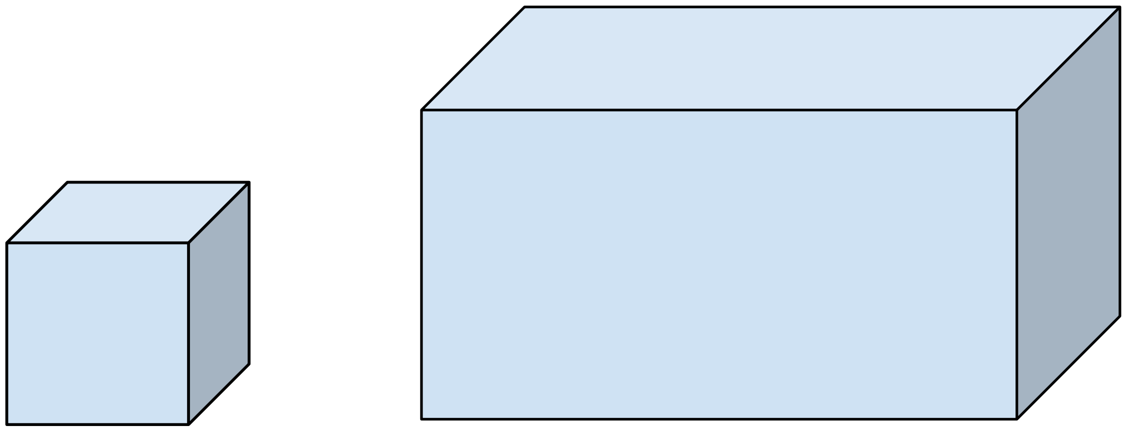
TAKE ME TO THE COVER

Problem of the Week

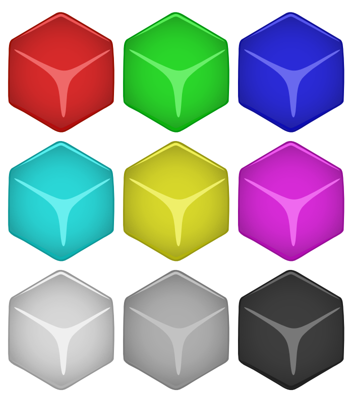
Problem A

Block Builder

Tabia has 18 blocks. Show the number of different solid, rectangular prisms she can build, using all 18 cubes.



Note: A rectangular prism is a 3-D figure with six rectangular faces.

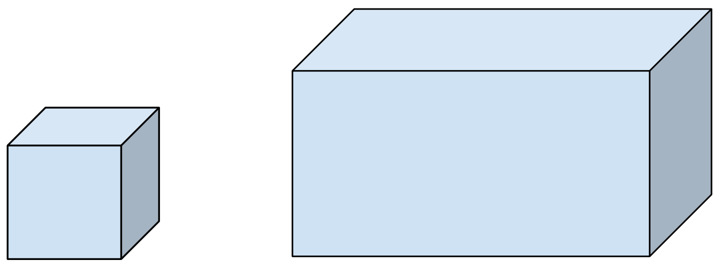


Strands Measurement, Geometry and Spatial Sense

Problem of the Week
Problem A and Solution
Block Builder

Problem

Tabia has 18 blocks. Show the number of different solid, rectangular prisms can she build, using all 18 cubes.



Note: A rectangular prism is a 3-D figure with six rectangular faces.

Solution

Since the rectangular prism is formed by solid objects, each dimension of the prism must be a whole number of blocks. According to the question, the volume of the prism is 18. The volume is calculated by multiplying the three dimensions together. So you need to find the total number of distinct combinations of three whole numbers that have a product of 18.

To find these combinations, look at the numbers that divide exactly into 18. They are: 1, 2, 3, 6, 9, and 18. Pick any two of these numbers and determine if there is a third value from this list so that the product of the three numbers is 18. There are four combinations that will work:

- $1 \times 1 \times 18 = 18$
- $1 \times 2 \times 9 = 18$
- $1 \times 3 \times 6 = 18$
- $2 \times 3 \times 3 = 18$

Note that a rectangular prism with dimensions $1 \times 3 \times 6$ is the same as a rectangular prism with dimensions $3 \times 1 \times 6$. The objects are simply rotations of each other.

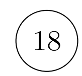
Teacher's Notes

In this problem, you are given 18 blocks and there are four different rectangular prisms you can assemble with those blocks. Interestingly, a larger number of blocks does not necessarily lead to a larger number of prisms. For example, given 35 blocks, there are only two possibilities: $1 \times 1 \times 35 = 35$ and $1 \times 5 \times 7 = 35$.

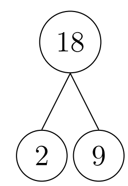
The number of different prisms that can be formed depends on the prime factorization of the number of blocks. A prime factorization is a product formed entirely of prime numbers. A prime number has exactly two factors: 1 and itself, and 2 is the smallest prime number. For example, the prime factorization of 48 is: $2 \times 2 \times 2 \times 2 \times 3$.

One way to find the prime factorization of a number is to use a factor tree. Here is how you can form a factor tree for the number 18.

Start with the number 18.

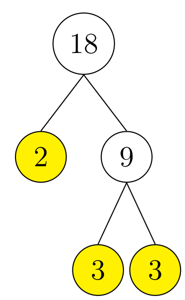


Since 18 is not a prime number, find a factor pair, and add two *branches* that lead to *nodes* (the circles) containing each number of the factor pair.



Look for any other *nodes* in the tree that are not prime numbers. Add two branches from those nodes that lead to *nodes* containing each number of a factor pair. Continue adding new *branches* and *nodes* to the tree as long as there are non-prime number nodes. In this case, the only non-prime number left is 9 with a factor pair 3×3. At this point, the factor tree for 18 is complete.

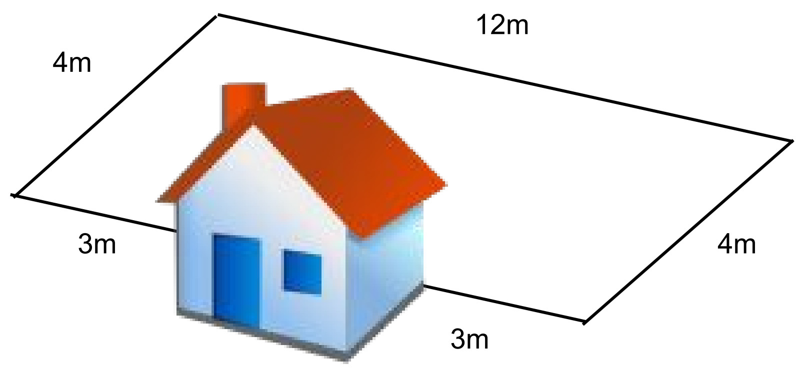
The prime factorization is composed of the nodes in the tree that contain prime numbers. So, the prime factorization of 18 is: $2 \times 3 \times 3$.



More factors in the prime factorization of a number means more possible divisors for that number, which means there are more ways you can write the number as a product of three numbers. *Highly composite numbers* are a special set of numbers with many divisors. They were first described in detail by Srinivasa Ramanujan, a mathematician from India, born in 1887, who unfortunately died at the age of 32. Despite very limited opportunities for higher education early in his life, Ramanujan eventually went on to be a well-respected mathematician. Eventually he went on to continue his research at the University of Cambridge. His life has been dramatized in the film *The Man Who Knew Infinity (2015).*

Problem of the Week
Problem A
The Fence

Agnes and her mother are building a fence around their backyard to keep their puppy safe. Their yard looks like the picture.



They start building the fence by placing one post at the back corner of the house. Then they add one fence post every meter around the yard until they put the final post at the other back corner of the house. Every corner of the yard has a post.

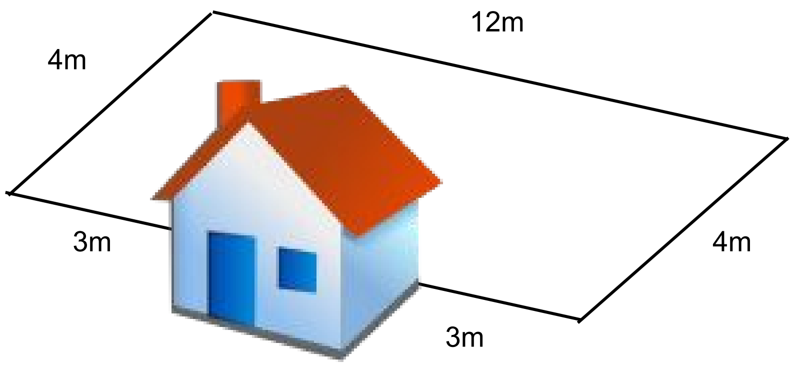
How many fence posts are needed altogether?

Strands Measurement, Number Sense and Numeration

Problem of the Week
Problem A and Solution
The Fence

Problem

Agnes and her mother are building a fence around their backyard to keep their puppy safe. Their yard looks like the picture.

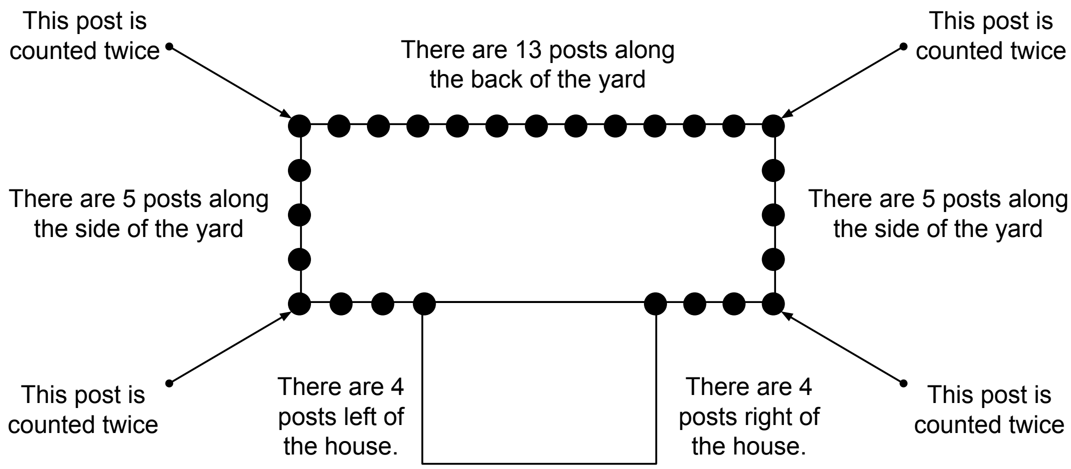


They start building the fence by placing one post at the back corner of the house. Then they add one fence post every meter around the yard until they put the final post at the other back corner of the house. Every corner of the yard has a post.

How many fence posts are needed altogether?

Solution

Here is a diagram of the layout for the fence posts.



There are many ways to count the number of fence posts required. You can simply count the number of dots in the diagram to see that that Agnes and her mother need 27 posts.

Another way to calculate the total is to realize that since the fence posts are 1 m apart, it is easy to determine how many posts are required in each section of the backyard. They will need one more fence post than number of metres in each case. For example, for the 12 m across the back, they need 13 posts, since they need one at the first corner (0 metres), and one every metre after that until the second corner which is 12 metres away.

If they add these numbers up, the total is: $4 + 5 + 13 + 5 + 4 = 31$. However, the posts in each corner of the yard are counted twice using this method. Since 4 posts are counted twice, they need to subtract the 4 duplicates from the total calculated by adding the number of posts in each section. So the total number of posts required to build the fence is: $31 - 4 = 27$.

Teacher's Notes

This problem is a literal example of the fencepost problem, which is an analogy used to describe off-by-one errors in coding. These errors are very common, especially when dealing with a range of values where the beginning and end of the range are variables.

For example, in programming strings consist of a sequence of characters. We can think of each character as having a numbered position in the string. Normally, we start that numbering at 0, and the position is known as the index. So for example, the string `‘‘Problem of the Week’’` would be indexed as follows:

P	r	o	b	l	e	m		o	f		t	h	e		W	e	e	k
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Many programming languages include a function that extracts a substring by identifying the index positions of the characters you want from the original string. For example, you might have an expression like this:

```
substring(‘‘Problem of the Week’’, 15, 17)
```

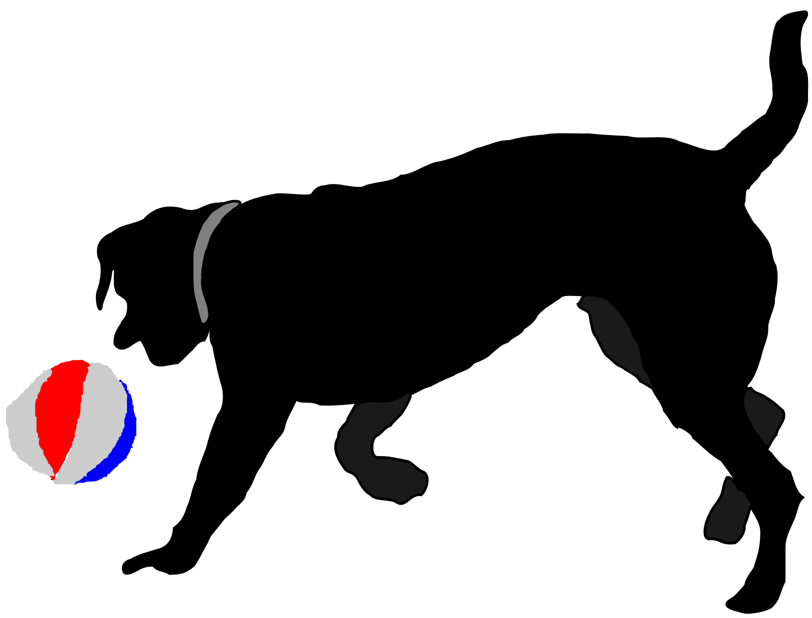
Normally, the first number you provide to the substring function indicates index position of the first character you want and the second number is one more than the index position of the last character you want. So the substring in our example would be `‘‘We’’`. This might seem odd, but this rule actually addresses the fencepost problem. Notice that the difference between the two numbers equals the number characters in the substring. So if we know the starting point of the substring, and the number of characters we want, then we simply add those two numbers together to find the second number for the substring function. Very often we know the starting position and the number of characters, so this can be the easier calculation.

Problem of the Week
Problem A
Playing Fetch

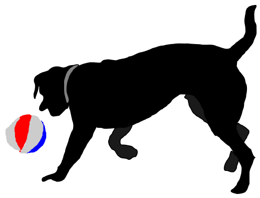
Robbie loves playing fetch with his dog Spencer. Spencer always starts by sitting beside Robbie before Robbie throws the ball. When Robbie throws the ball, Spencer runs to it and brings the ball back to the same spot. Robbie throws the ball three times.

- The first time he throws it 8 metres.
- The second time he throws it twice as far as the first time.
- The third time he throws it 5 metres less than the second time.

How far does Spencer run in total?



Strands Number Sense and Numeration, Measurement



Problem of the Week
Problem A and Solution
Playing Fetch

Problem

Robbie loves playing fetch with his dog Spencer. Spencer always starts by sitting beside Robbie before Robbie throws the ball. When Robbie throws the ball, Spencer runs to it and brings the ball back to the same spot. Robbie throws the ball three times.

- The first time he throws it 8 metres.
- The second time he throws it twice as far as the first time.
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How far does Spencer run in total?

Solution

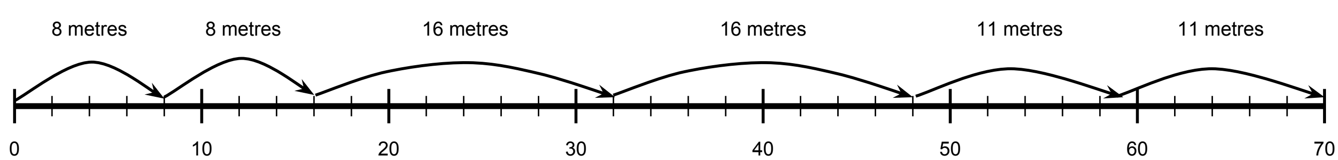
On the first throw, Spencer runs $2 \times 8 = 16$ metres.

The distance of the second throw is $8 \times 2 = 16$ metres.
On the second throw, Spencer runs $2 \times 16 = 32$ metres.

The distance of the third throw is $16 - 5 = 11$ metres.
On the third throw, Spencer runs $2 \times 11 = 22$ metres.

The total distance Spencer runs is: $16 + 32 + 22 = 70$ metres.

We can also show the distance Spencer runs on a number line.



Teacher's Notes

This problem can be described algebraically. We can use a variable d to represent the distance that Robbie throws the ball the first time. Then, the rest of the distances can be described in terms of d.

The distance he throws the ball the second time is $2d$.
The distance he throws the ball the third time is $2d - 5$.

Since Spencer runs to get the ball and returns to the original spot, every time Robbie throws the ball, Spencer will run that distance twice. We can use t to represent the total distance Spencer runs. Here is one way to calculate t:

$$t = d + d + 2d + 2d + 2d - 5 + 2d - 5$$
$$t = 10d - 10$$

We can also think of the total distance that Spencer runs as two times the total distance that Robbie throws the ball. So here is another way to calculate t:

$$t = 2 \times (d + 2d + 2d - 5)$$
$$t = 2 \times (5d - 5)$$
$$t = 10d - 10$$

Using the second approach, we had to use the *distributive law* to calculate that

$$2 \times (5d - 5) \text{ is equal to } 10d - 10$$

In other words, we multiplied 2 by $5d$ and by -5.

There are other ways we could derive the total distance, however when we simplify the equation, the result will always be:

$$t = 10d - 10$$

Now we can calculate the value of t by substituting the value we have for d. So

$$t = 10(8) - 10$$
$$t = 80 - 10$$
$$t = 70$$

This seems like a lot of work to get the answer we could have just counted using a number line. If we are only interested calculating the distance for one set of throws, then creating the equation is not particularly helpful. However, if we saw the same pattern for throwing with different starting distances, then the equation can be helpful. Suppose Robbie always throws in the same pattern, but this time his first throw is 5 metres. The total distance Spencer runs in this case would be:

$$t = 10(5) - 10$$
$$t = 50 - 10$$
$$t = 40$$

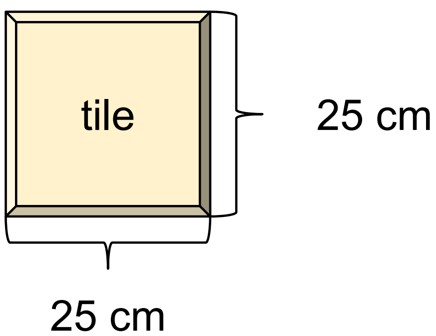
Algebraic equations describe a general case, and they can be very helpful when there are multiple specific cases to consider.

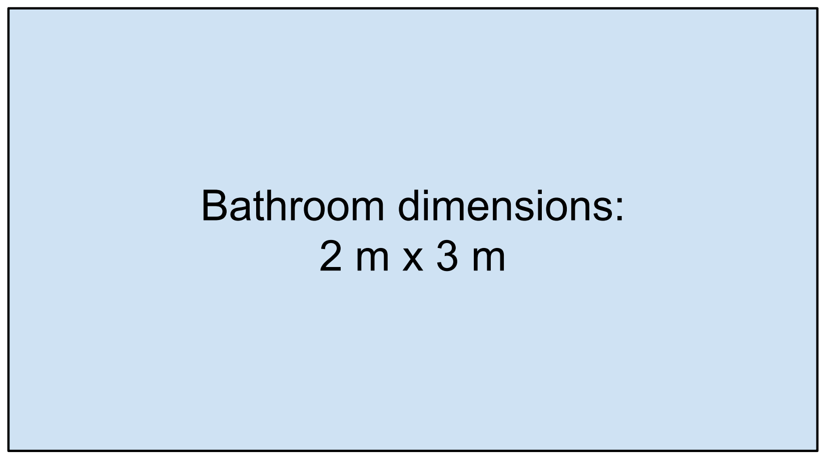
Problem of the Week

Problem A

Mrs. Thomas's Tidy Tiles

Mrs. Thomas wants to tile her 2m × 3m bathroom floor. Each tile is 25cm × 25cm.





A) How many tiles will she need to cover her bathroom floor?

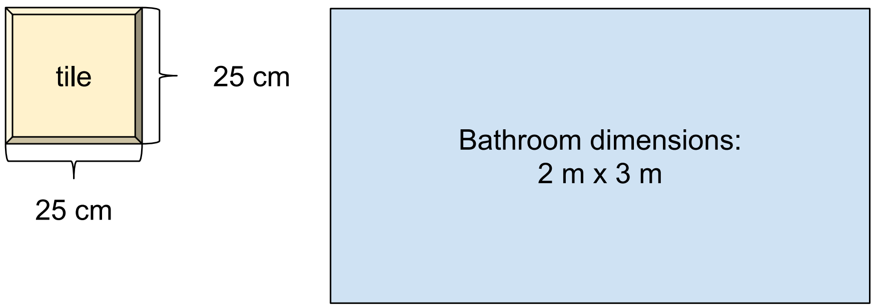
B) If each tile costs $1.50, how much will it cost Mrs. Thomas to tile her bathroom?

Strands Measurement, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Mrs. Thomas's Tidy Tiles

Problem

Mrs. Thomas wants to tile her 2m × 3m bathroom floor. Each tile is 25cm × 25cm.



A) How many tiles will she need to cover her bathroom floor?

B) If each tile costs \$1.50, how much will it cost Mrs. Thomas to tile her bathroom?

Solution

A) Each tile is 25 cm × 25 cm. This means that if we put 4 tiles in a line they would form a rectangle where the length of the long side is 1 m. If we put 16 tiles together (4 × 4) to form a square, the dimensions of that square would be $1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$. The area needed to be covered by tiles is $2 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$. So, Mrs. Thomas would need $16 \times 6 = 96$ tiles to complete her bathroom.

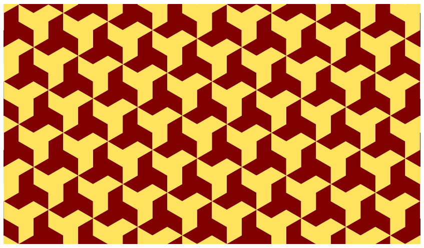
Alternatively, we could look at the bathroom dimensions and convert them into centimetres. The length of the bathroom is $3 \times 100 = 300$ cm. The width of the bathroom is $2 \times 100 = 200$ cm. If we divide the dimension of the bathroom by the width of the tile, then we know how many tiles are required along each edge. For the length, it is $300 \div 25 = 12$ tiles. For the width, it is $200 \div 25 = 8$ tiles. Therefore we need $12 \times 8 = 96$ tiles to cover the area of the bathroom.

B) If she needs 96 tiles and they each cost \$1.50, she would spend $96 \times 1.50 = (96 \times 1) + (96 \times 0.50) = 96 + (96 \div 2) = 96 + 48 = \$144.00$. She would spend \$144 for her tiles.

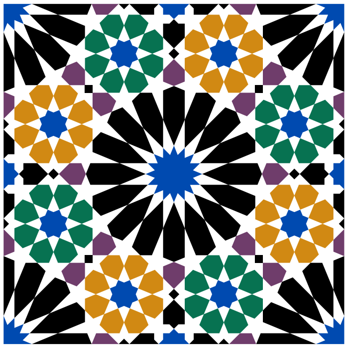
Teacher's Notes

This is a literal example of a tiling problem. In mathematics, tiling describes the process of covering a flat surface (or *plane*) with geometric shapes known as tiles. We imagine that the plane extends infinitely in all directions. Two examples are illustrated below.

The first example illustrates tiling with a single geometric shape.



The second example illustrates tiling with multiple geometric shapes.



You can imagine that these patterns could be continued indefinitely in all four directions. A real world tiling has boundaries. The problem of trying to fit geometric shapes into a bounded area is called a packing problem. Packing problems are studied by mathematicians and computer scientists. This type of problem asks the question, what is the maximum number of objects I can fit into a fixed space. That space can be two or three dimensional. Unlike with tilings, the packing problem allows gaps between the objects. Filling the entire space without any gaps would be optimal. However, there are some situations where you unable to avoid gaps. For example, consider trying to maximize the number of circles you can fit into a square. It is not possible to arrange the circles inside the square without gaps. Another example would be maximizing the number of squares inside a triangle. Again, you are guaranteed to have gaps. Solving packing problems has real life applications such as determining a company's shipping and storage requirements.

Problem of the Week

Problem A

The Wooden Spoon

Thomas wants to make his grandfather's famous wooden spoon drink. He has a list of the ingredients and needs to know if he has a pot that is big enough to fit the entire recipe contents.

Ingredient	Amount
vanilla pudding	0.25 L
apple sauce	150 mL
milk	1 L
lemon juice	1500 mL
pumpkin see oil	350 mL
cream of tartar	100 mL

He has a 3 L container, a 3.25 L container and a 4 L container. Which one should he use? Explain your thinking.

Remember that 1 L equals 1000 mL.

Strands Measurement, Number Sense and Numeration

Problem of the Week

Problem A and Solution

The Wooden Spoon

Problem

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He has a 3 L container, a 3.25 L container and a 4 L container. Which one should he use? Explain your thinking.

Remember that 1 L equals 1000 mL.

Solution

It is probably easier to figure out the solution if all of the amounts are shown in the same units. This table shows everything in mL.

Ingredient	Amount
vanilla pudding	$0.25 \text{ L} = 0.25 \times 1000 = 250 \text{ mL}$
apple sauce	150 mL
milk	$1 \text{ L} = 1 \times 1000 = 1000 \text{ mL}$
lemon juice	1500 mL
pumpkin see oil	350 mL
cream of tartar	100 mL

Now we can calculate the total amount of the recipe in mL:

$$250 + 150 + 1000 + 1600 + 350 + 100 = 3350 \text{ mL}$$

This is equal to $3450 \div 1000 = 3.35$ L.

So, the 3 L and the 3.25 L containers are not big enough. Thomas needs to use the 4 L container to hold all of the ingredients.

Teacher's Notes

The metric system is a system of measurement which allows for easy conversions between different measurement units with the same base. There are a standard set of prefixes that indicate a multiple or fraction of the base unit. When measuring every day items, the prefixes *kilo* meaning 1000 or 10^3, *centi* meaning $\frac{1}{100}$ or 10^{-2}, and *milli* meaning $\frac{1}{1000}$ or 10^{-3}, are very useful.

Since today's technology is so small and so powerful, we often use other metric system prefixes to measure its size and speed. Here are some examples of these very large and very small values.

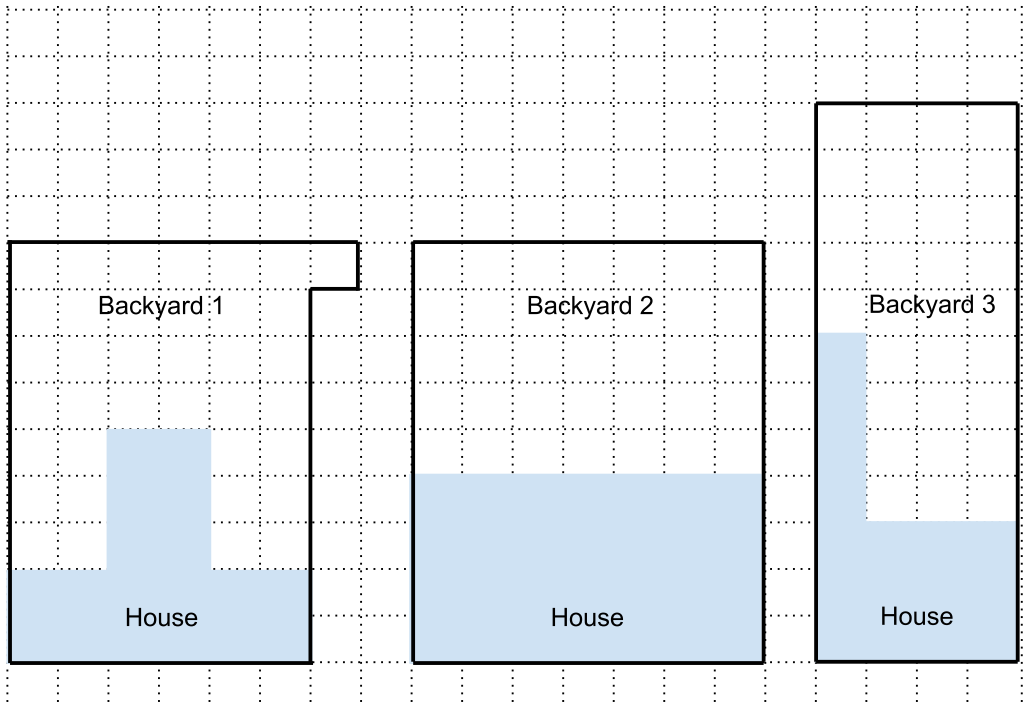
giga 1 000 000 000	In 2017, a common size of SD cards for digital cameras is 32 or 64 gigabytes (GB). An individual pixel of an image would typically require 3 bytes of storage space. The size of an image taken by the camera depends on many factors, but would normally be described in terms of *megapixels*. (The prefix *mega* means 1 000 000.) Suppose you have a camera that takes pictures that take up 32 megabytes of storage space. This means you could save 1000 pictures on a 32 GB card or 2000 pictures on a 64 GB card.
tera 1 000 000 000 000	In 2017, an external hard drive for your home computer would normally be between 1 and 4 terabytes (TB). A single character in text would take up 1 or 2 bytes (depending on how the computer represents the character). According to *Wikipedia*, in 1989 the second edition of the Oxford English Dictionary (OED) was published, containing approximately 59 million words in 20 printed volumes. The second edition can be stored in approximately 540 megabytes (MB). This means a 1 TB external hard drive could contain over 1800 copies of the second edition of the OED. As of 2017, the third edition has not been completed.
nano $\frac{1}{1\,000\,000\,000}$	In a vacuum, light or electricity can travel at a speed of approximately 30 cm in one nanosecond. Remember that it takes approximately 8 minutes for light to travel from the sun to the Earth, and the sun is approximately 150 million kilometres away. Computer scientist Grace Hopper (1906 - 1992) famously carried a bundle of "nanoseconds" with her. These were wires cut to lengths of 30 cm each. She would use these as visual aids to explain, among other things, why it took so long for messages to be sent via satellite.
pico $\frac{1}{1\,000\,000\,000\,000}$	A computer has an internal clock that coordinates all of its processes. That clock beats very fast. The speed of the clock is usually described in gigahertz (GHz). Suppose your computer had a clock speed of 4 GHz. This means the clock beats 4 000 000 000 times per second. Looking at it another way, a single beat of the clock takes 250 picoseconds. Grace Hopper also had a way of visualizing picoseconds. She would take a packet of pepper as an illustration of many picoseconds, where the size of an individual pepper grain is the maximum distance light can travel in 1 picosecond.

Problem of the Week

Problem A

House Hunt

The Ali family is looking for a new house with a large backyard. They are trying to decide between the 3 homes shown below.



Each square in the grid represents one square unit.

A) Which house has the backyard with the largest area? Explain your thinking.

B) A *lot* is the property that contains the house and the backyard. The family wants to install an invisible fence around their property to keep their pets safe. They will need to know the perimeter of the lot that they buy. What is the perimeter of each lot?

Strands Measurement, Number Sense and Numeration

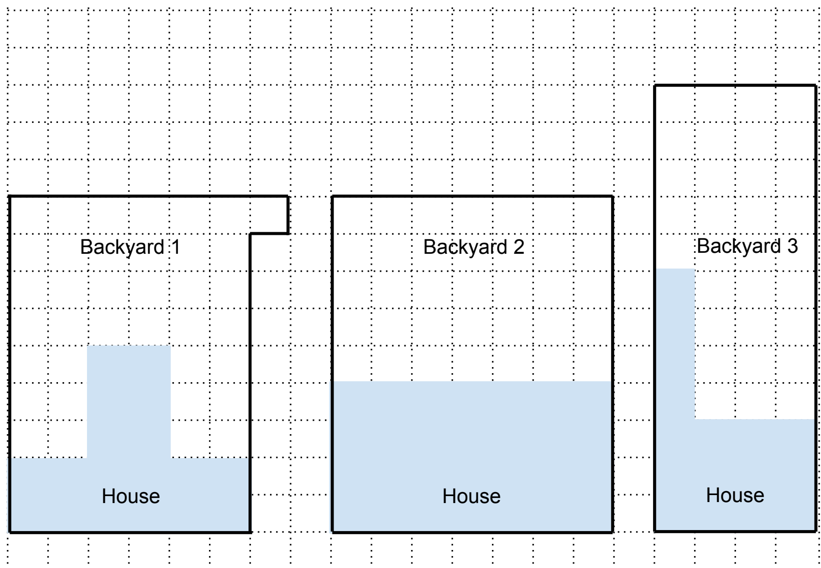
Problem of the Week

Problem A and Solution

House Hunt

Problem

The Ali family is looking for a new house with a large backyard. They are trying to decide between the 3 homes shown below.



Each square in the grid represents one square unit.

A) Which house has the backyard with the largest area? Explain your thinking.

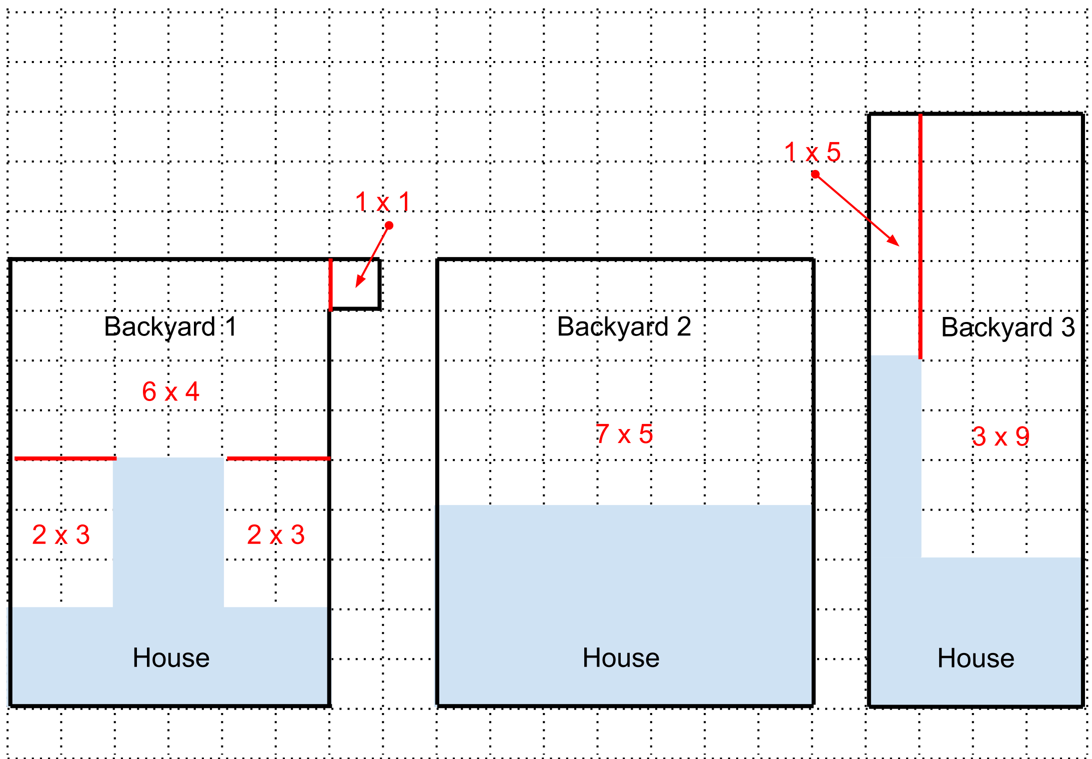
B) A *lot* is the property that contains the house and the backyard. The family wants to install an invisible fence around their property to keep their pets safe. They will need to know the perimeter of the lot that they buy. What is the perimeter of each lot?

Solution

A) There are many ways to solve this problem. One way to determine the area of each yard is to simply count the squares within each outlined backyard.

Some students may decide to break the yards up into smaller, more manageable rectangles, in order to count the units.

Students may also use the formula $l \times w$ to determine area for each of the smaller rectangular components in the backyards, adding the products of each section to determine the overall area.



You can calculate the areas for each of the backyards by looking at the areas of rectangles that make up each yard.

Area of backyard 1:
$(6 \times 4) + (2 \times 3) + (2 \times 3) + (1 \times 1) = 24 + 6 + 6 + 1 = 37$ units^2.
Area of backyard 2: $7 \times 5 = 35$ units^2.
Area of backyard 3: $(1 \times 5) + (3 \times 9) = 5 + 27 = 32$ units^2.

So the first house has the backyard with the biggest area.

B) Looking at the diagrams, you can calculate the perimeters:

Perimeter of lot 1: $6 + 9 + 7 + 1 + 1 + 8 = 32$ units.
Perimeter of lot 2: $7 + 9 + 7 + 9 = 32$ units.
Perimeter of lot 3: $4 + 12 + 4 + 12 = 32$ units.

Teacher's Notes

This question asks about area and perimeter. There is no general relationship between the area of a shape and its perimeter.

In some cases, like regular polygons or circles, the area of the shape has a direct relationship with its perimeter. For example, if you double the perimeter of a square this will increase the area by four times. Similarly, if you increase the circumference of a circle by three times, the area will increase by nine times.

It is possible to draw a shape that has the property that: as the perimeter increases, the area decreases. An example of this would be a shape that includes a concave angle. Consider these two images:

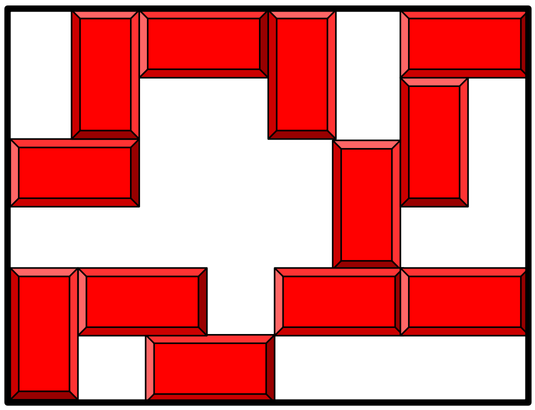
They are the same shape, but the image on the left has a larger area than the image on the right, and the image on the right has a larger perimeter than the image on the left.

Problem of the Week

Problem A

Building Blocks

Johnna Lee loves to build with interlocking building blocks. She starts building on a large, white, flat piece for the base, but the rest of the blocks she uses are red. Looking from overhead, this is what Johnna Lee built.



Each of the red blocks has a length of 2 cm and a width of 1 cm.

A) What is the area of the white base?

B) What fraction of the area of the base is covered by red blocks?

Strands Measurement, Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Building Blocks

Problem

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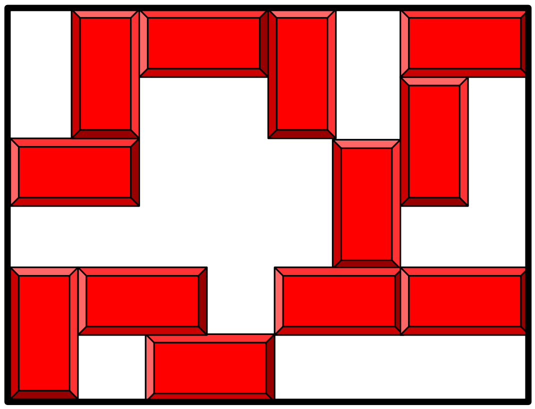


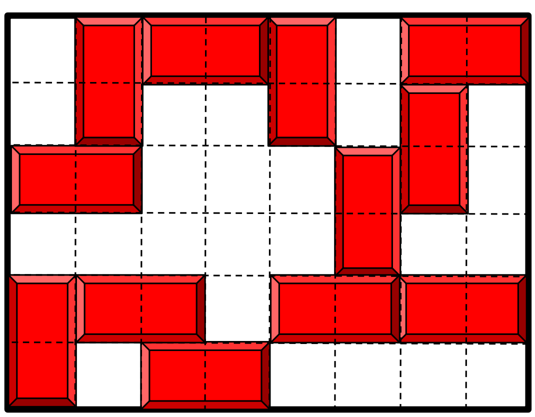
Each of the red blocks has a length of 2 cm and a width of 1 cm.

A) What is the area of the white base?

B) What fraction of the area of the base is covered by red blocks?

Solution

You can draw grid lines that align with the edges of the blocks. The squares formed by the grid lines are 1 cm × 1 cm.



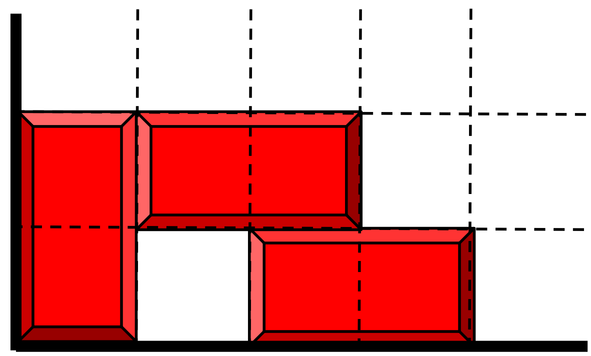
A) If you count the total number of squares formed by the grid lines, you see that there are 48. Alternatively, you can see that the base is 8 cm wide and 6 cm high. The area of the base is 8 cm $\times$ 6 cm $=$ 48 cm^2.

B) Count the number of squares formed by the grid lines that overlap the blocks. The total is 24. The fraction of the area of covered by blocks over the area of the base is $\frac{24}{48}$. Since $48 = 24 \times 2$ we can also write the fraction as $\frac{1}{2}$.

   Here is another way to calculate this fraction. The area of one of the blocks is 1 cm $\times$ 2 cm $=$ 2 cm^2. There are 12 blocks showing on the base. The total area they cover is 12×2 cm^2 $=$ 24 cm^2. Since the area of the base is 48 cm^2, the blocks cover $\frac{24}{48}$ or $\frac{1}{2}$ the area of the base.

Teacher's Notes

The second part of this problem could be solved without knowing any specific information about the dimensions of the blocks. Everything you need to determine what fraction of the base is being covered by the blocks is available in the picture. The key is knowing that the ratio of the length to the width of the building block is 2 : 1. This can be determined by examining the bottom left corner of the diagram.



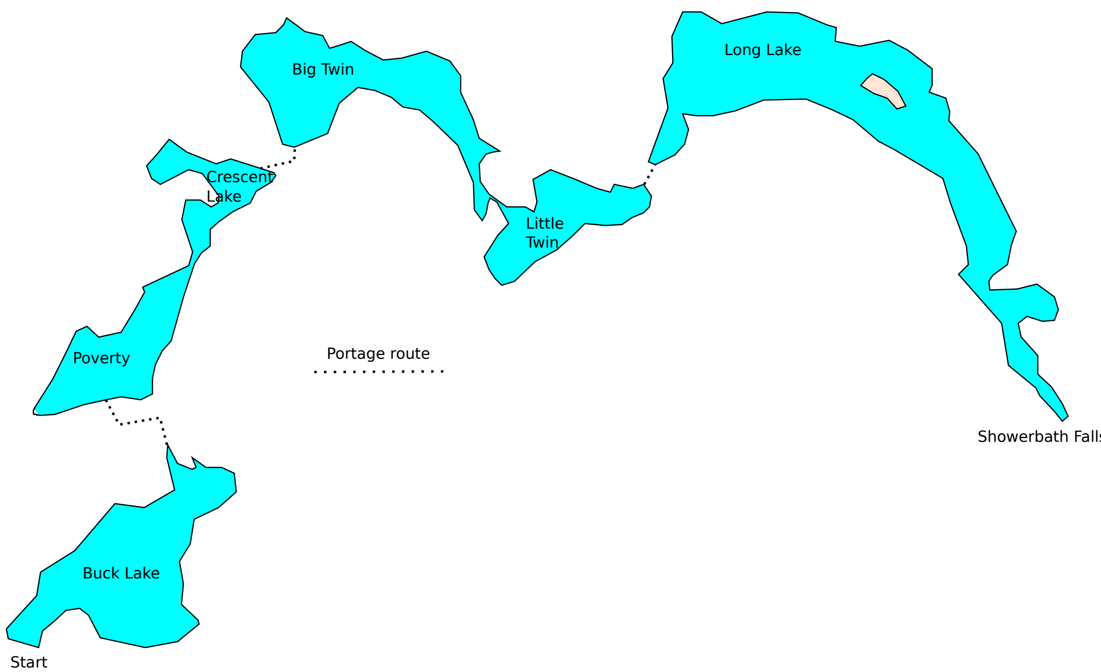
From this part of the picture, we see that the twice the shorter side of the block is equal to the length of the longer side of the block. This relationship, along with the way the blocks are aligned in the rest of the diagram, allows us to add the grid lines to the diagram. These grid lines form unit squares. The actual size of those squares is irrelevant; their measurements could be 1 cm $\times$ 1 cm $=$ 1 cm^2, or 1 m $\times$ 1 m $=$ 1 m^2, or 1 Wiffle $\times$ 1 Wiffle $=$ 1 Wiffle^2. What is important for us, is that they are squares. Since we are computing a fraction, the units will disappear in the calculation.

Ultimately the solution for this problem can be computed based on the number of unit squares that form the base of the structure and the number of unit squares the blocks cover. In this case we have a fraction of:

$$\frac{24 \cancel{\text{units}^2}}{48 \cancel{\text{units}^2}} = \frac{1}{2}$$

Problem of the Week
Problem A
Canoeing to Showerbath Falls

James and Katie decide to paddle their canoe to Showerbath Falls. There are six lakes to paddle through to get from the starting point of the trip to the end. The lakes are Buck, Poverty, Crescent, Big Twin, Little Twin, and Long Lake. Some of the lakes are connected by narrows, where you can paddle straight through, and some lakes are connected by portages, which means James and Katie have to get out of the canoe and carry it across a path to the next lake. Poverty and Crescent are connected by narrows, and Big Twin and Little Twin are connected by narrows. There are portages between Buck and Poverty, Crescent and Big Twin, and Little Twin and Long Lake.



They start in Buck Lake and it takes them 30 minutes to paddle across it. The portage between Buck and Poverty takes 10 minutes to cross. It takes 20 minutes to paddle across Poverty and Crescent. The portage between Crescent and Big Twin takes 15 minutes. From Big Twin to the final portage is a 25 minute paddle. This last portage is only 5 minutes long. Long Lake is well named and it takes 50 minutes to get to the end. Showerbath Falls is at the end of Long Lake.

A) How much more time do they spend paddling rather than portaging the canoe during the trip from Buck Lake to Showerbath Falls?

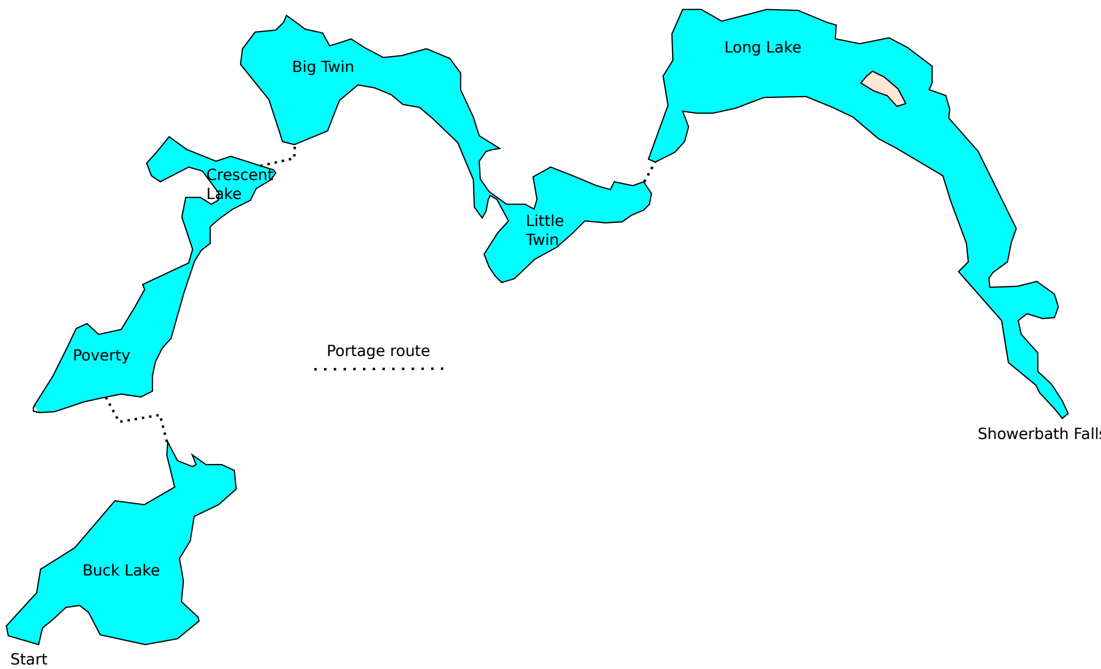
B) Katie and James spent an hour at Showerbath Falls eating lunch and then returned home. It took them the same amount of time to travel back. If they left at 9:00 in the morning, what time did they get home?

Strands Number Sense and Numeration, Measurement

Problem of the Week
Problem A and Solution
Canoeing to Showerbath Falls

Problem

James and Katie decide to paddle their canoe to Showerbath Falls. There are six lakes to paddle through to get from the starting point of the trip to the end. The lakes are Buck, Poverty, Crescent, Big Twin, Little Twin, and Long Lake. Some of the lakes are connected by narrows, where you can paddle straight through, and some lakes are connected by portages, which means James and Katie have to get out of the canoe and carry it across a path to the next lake. Poverty and Crescent are connected by narrows, and Big Twin and Little Twin are connected by narrows. There are portages between Buck and Poverty, Crescent and Big Twin, and Little Twin and Long Lake.



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A) How much more time do they spend paddling rather than portaging the canoe during the trip from Buck Lake to Showerbath Falls?

B) Katie and James spent an hour at Showerbath Falls eating lunch and then returned home. It took them the same amount of time to travel back. If they left at 9:00 in the morning, what time did they get home?

Solution

A) The total time paddling is: $30 + 20 + 25 + 50 = 125$ minutes.

The total time portaging is: $10 + 15 + 5 = 30$ minutes.

Katie and James spent $125 - 30 = 95$ minutes more paddling than portaging.. Since there are 60 minutes in 1 hour, we can do repeated subtraction to convert minutes into a combination of minutes and hours. In this case, $95 - 60 = 35$. Since $35 < 60$, we cannot subtract 60 any more. So 95 minutes is equal to 1 hour and 35 minutes.

B) The total time for the trip one way is: $125 + 30 = 155$ minutes. The total time for the round trip including lunch is: $155 + 60 + 155 = 370$ minutes. Using repeated subtraction we calculate

$$370 - 60 = 310$$

$$310 - 60 = 250$$

$$250 - 60 = 190$$

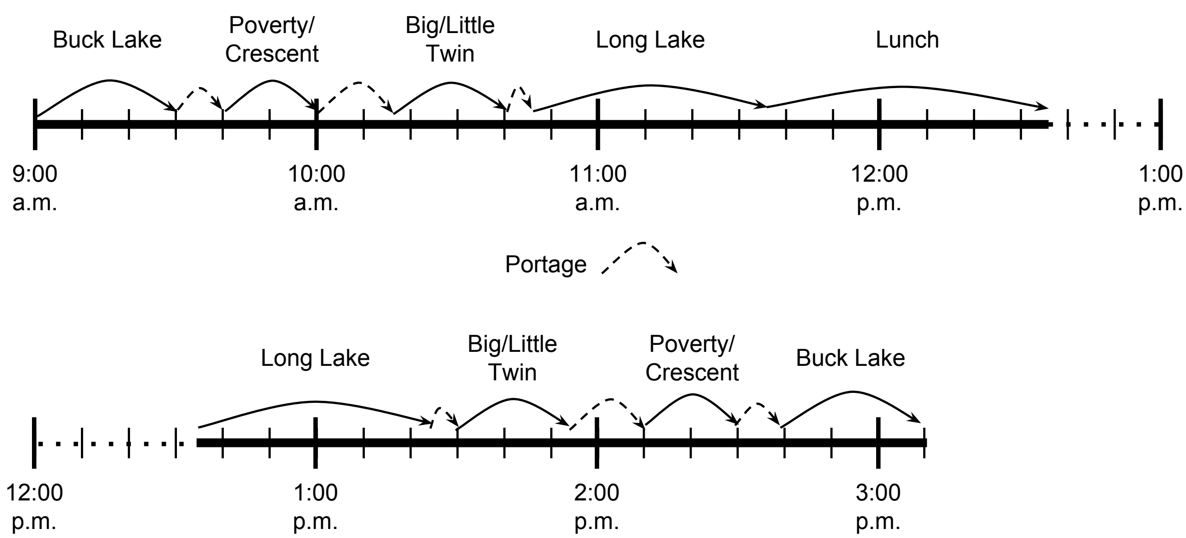
$$190 - 60 = 130$$

$$130 - 60 = 70$$

$$70 - 60 = 10$$

Since 60 is repeatedly subtracted 6 times before the difference is less than 60, and last difference is 10, then 370 minutes is equal to 6 hours and 10 minutes. If James and Katie left at 9:00 a.m. then they would get back at 3:10 p.m.

You can also find the time it takes for James and Katie to complete the trip using a timeline.



Teacher's Notes

Converting a total number of minutes for the trip to the format using hours and minutes is another good example of why it is important to be able to calculate the quotient and the remainder when dividing. The *modulo* or *mod* operation that is used in mathematics and in many programming languages computes the same value as the remainder for positive integers. This operation is also part of an area of mathematics known as *modular arithmetic*.

Modular arithmetic describes relationships between integers where the numbers repeat themselves after reaching a certain value. It is often referred to "clock arithmetic" since our use of a 12-hour clock is a very good example of this idea. When we describe time using a 12-hour clock, the maximum number of minutes is 59 and the maximum number of hours is 12. If it is 3:14 right now, then after 60 minutes, it will be 4:14. The number of minutes in both those times is the same, even though 60 minutes has elapsed. If it is 3:14 right now, then in exactly five days, it will also be 3:14. In this example the elapsed time is 120 hours or 7200 minutes, and yet we refer to the time using exactly the same number of minutes and hours. That happens because when we deal with time this way, every 60 minutes we repeat the number of minutes, and every 12 hours we repeat the number of hours.

The CEMC has an activity known as Math Circles which is presented at the University of Waterloo. The materials from past years of this activity are available online at:

`http://www.cemc.uwaterloo.ca/events/mathcircle_presentations.html`

The materials from Fall 2016 on November 8-9 for the Junior Grade 6 students introduce the idea of modular, or clock, arithmetic.

Number Sense & Numeration

TAKE ME TO THE COVER

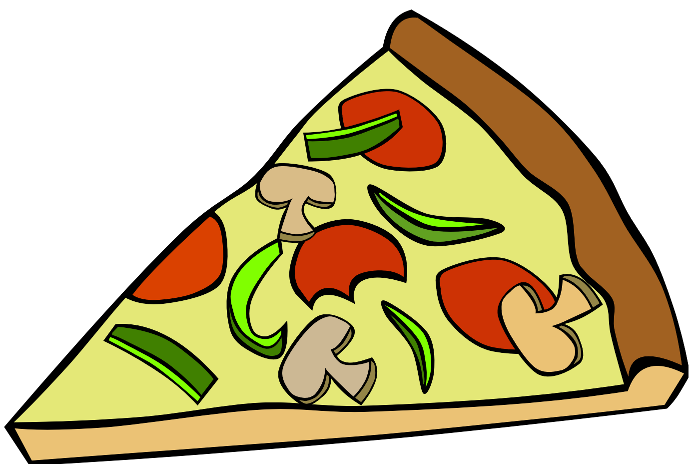
Problem of the Week
Problem A
Fair Share

A class wins a pizza party for reading the most books in September. There are 28 students in the class and each student will get 2 slices of pizza. Each pizza has 6 slices.

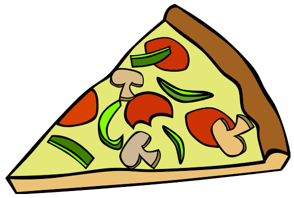
A) How many pizzas will the school need to buy so that each student can have 2 slices?

Show your thinking.

B) Will there be any slices left over? If so, how many?



Strand Number Sense and Numeration



Problem of the Week

Problem A and Solution

Fair Share

Problem

A class wins a pizza party for reading the most books in September. There are 28 students in the class and each student will get 2 slices of pizza. Each pizza has 6 slices.

A) How many pizzas will the school need to buy so that each student can have 2 slices? Show your thinking.

B) Will there be any slices left over? If so, how many?

Solution

A) If each student in the class gets 2 pieces of pizza, then we will need to order:

28 students $\times$ 2 slices $=$ 56 slices of pizza

Since the pizzas are divided into 6 slices each, we notice

$$6+6+6+6+6+6+6+6+6 = 54 \textit{ or } 6 \times 9 = 54$$

and

$$6+6+6+6+6+6+6+6+6+6 = 60 \textit{ or } 6 \times 10 = 60.$$

Therefore, the school will need to order 10 pizzas so that the 28 students can have 2 slices of pizza each.

B) If there are 60 pieces in 10 pizzas and giving each student 2 pieces of pizza means that 56 pieces are eaten, then the number of pieces left over is:

$$60 - 56 = 4.$$

Therefore, there would be 4 slices of pizza left over.

Teacher's Notes

Here is another way to think about the problem. Since each student wants 2 slices of pizza and there are 28 students in the class they need a total of $2 \times 28 = 56$ slices of pizza. Also, since each pizza has 6 slices, we can calculate the number of pizzas required by dividing $56 \div 6 = 9$ remainder 2. This means that we need 9 full pizzas and 2 extra slices. Since we can only buy full pizzas, we will need to get 10 pizzas in total

The exact result of the division is $9\frac{2}{6}$ or $9\frac{1}{3}$. However the answer that we need is a whole number of pizzas. There are a few different mathematical operations that will convert a number with a fractional part into an integer result. One way is to round off the number to the nearest integer. If we round off $9\frac{1}{3}$ we get 9; if we round off the number 2.71 to the nearest integer we get 3. We can also round up the number to the nearest integer or round down to the nearest integer. The mathematical function that rounds a number up to the nearest integer is called the *ceiling* function. The *floor* function is equivalent to rounding a number down to the nearest integer. These functions are used both in mathematical expressions as well as many programming languages.

In mathematics, we use special notation for these rounding functions.
The notation for *floor* looks like this: $\lfloor 2.71 \rfloor = 2$,
and the notation for *ceiling* looks like this: $\lceil 2.71 \rceil = 3$.

For this problem, we would want to calculate the *ceiling* of the result we get from the division, since we need to round up. So the number of pizzas required is:
$\lceil 56 \div 6 \rceil = \lceil 9\frac{1}{3} \rceil = 10$.

Note that the *floor* and *ceiling* of an integer are equal to each other and to the integer itself. For example, $\lfloor 3 \rfloor = \lceil 3 \rceil = 3$.

Problem of the Week

Problem A

Friendship Bracelets

Naomi is making bracelets to raise money for the hospital in her town. On her first day of bracelet making, Naomi makes 7 bracelets. Each day after, Naomi makes one more bracelet than she did the day before.

A) How many bracelets has she made after 7 days?

B) If the materials for a single bracelet cost \$2.50, and she sells each bracelet for \$4.50, how much money will she be able to donate to the hospital if she sells all of the bracelets?

C) If she wants to raise at least \$200.00 for the hospital, and she continued to make bracelets at the same rate (making one more bracelet each day), how many more days does she have to make bracelets?

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Friendship Bracelets

Problem

Naomi is making bracelets to raise money for the hospital in her town. On her first day of bracelet making, Naomi makes 7 bracelets. Each day after, Naomi makes one more bracelet than she did the day before.

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c) If she wants to raise at least \$200.00 for the hospital, and she continued to make bracelets at the same rate (making one more bracelet each day), how many more days does she have to make bracelets?

Solution

a) We can make a table of the pattern:

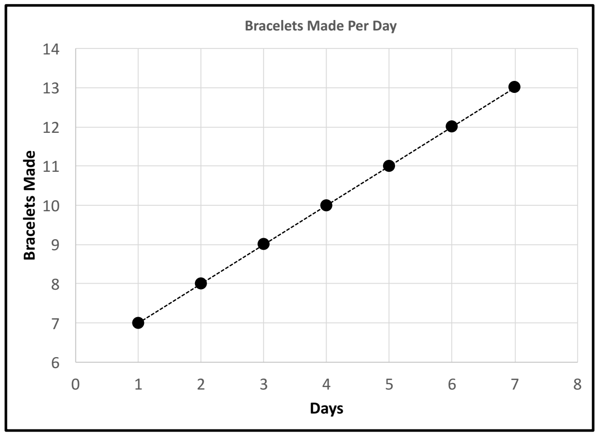
Day	1		2		3		4		5		6		7	
Bracelets	7	+	8	+	9	+	10	+	11	+	12	+	13	= 70 bracelets

b) Naomi makes a profit of $\$4.50 - \$2.50 = \$2.00$ for each bracelet. This means for 70 bracelets, her profit is $70 \times \$2.00 = \$140.00$.

c) To get to \$200.00, Naomi needs to earn $\$200.00 - \$140.00 = \$60.00$ more after day 7. If the pattern continues, on day 8 Naomi would make 14 bracelets for a profit of $14 \times \$2.00 = \$28.00$. Then on day 9 Naomi would make 15 bracelets for a profit of $15 \times \$2.00 = \$30.00$. This is a total of $\$28.00 + \$30.00 = \$58.00$. That is not quite enough to make a total of \$200.00. Naomi needs to make at least one more bracelet. So it would take her 10 days to earn a profit of at least \$200.00.

Teacher's Notes

The pattern seen in part (a) represents a linear relationship between the number of days and the number of bracelets made on that day. Using the table we could make a chart where the horizontal axis marks the days, and the vertical axis marks the number of bracelets made that day, and plot a point for each entry in the table.

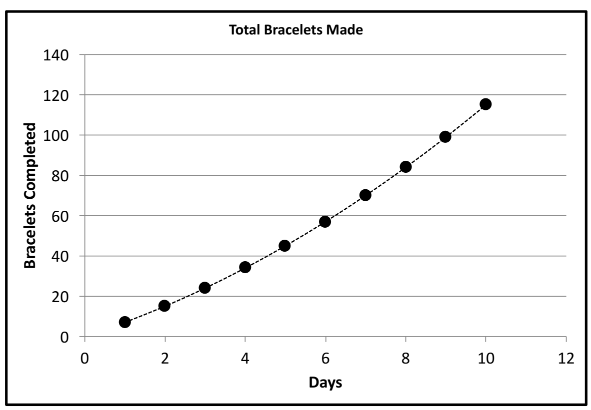


If we connect the points, they form a straight line showing a *linear* relationship. We can even write an equation representing the line:

$$b = 6 + d$$

where d represents the day, and b represents the number of bracelets made on day d.

We could also calculate the total number of bracelets made at the end of each day. For example, after the first day Naomi made 7 bracelets. At the end of the second day she made $7 + 8 = 15$ bracelets. If we calculate the totals for each day and then plot the points on a chart and connect them, we see that they form a curve rather than a straight line.



The result shows a *quadratic* relationship between the number of days and the total number of bracelets made. Again, we can write an equation representing the curve:

$$t = \frac{d^2 + 13d}{2}$$

where d represents the day, and t represents the total number of bracelets made up to day d. This relationship is quadratic because we can describe the relationship using an equation that includes the number of days squared (d^2) and the curved line that is formed is part of a shape known as a parabola.

Problem of the Week
Problem A
Watermelons for Sale

This week's flyer advertises watermelons on sale. Jared notices that the advertisement says: "One watermelon for \$4.50 or three watermelons for \$13.50."

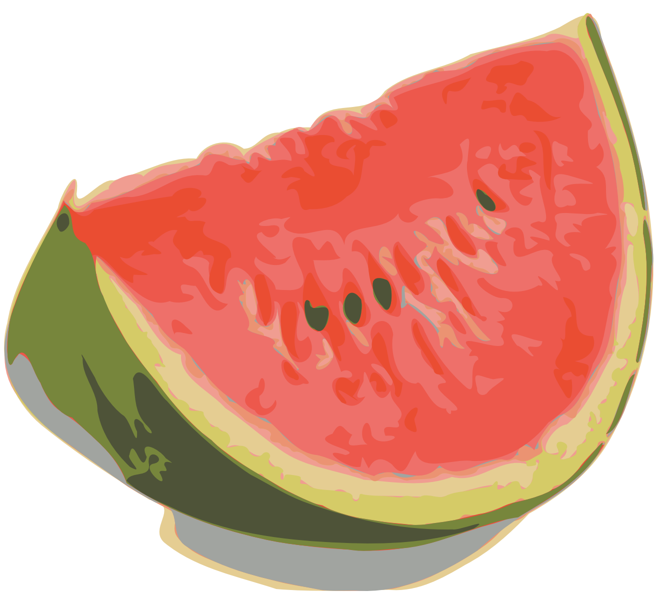
A) Explain which is the better deal for watermelons.

B) Why do you think the advertisement is written this way?

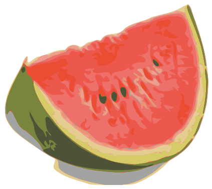
The flyer also advertised: "2 L of milk for \$2.15 or 4 L of milk for \$4.70". He needs 12 L of milk for the week.

C) Which is the better deal for milk?

D) How much money does Jared save if he buys the better size of milk? Explain your answer.



Strand Number Sense and Numeration



Problem of the Week
Problem A and Solution
Watermelons for Sale

Problem

This week's flyer advertises watermelons on sale. Jared notices that the advertisement says: "One watermelon for \$4.50 or three watermelons for \$13.50."

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C) Which is the better deal for milk?

D) How much money does Jared save if he buys the better size of milk? Explain your answer.

Solution

A) If Jared bought three watermelons individually it would cost: $\$4.50 + \$4.50 + \$4.50 = \$13.50$. There is no savings buying three at once.

B) The store might advertise the cost of three watermelons as a way to get buyers to purchase more than one item.

C) The cost for 4 L of milk if Jared bought two 2 L cartons is $\$2.15 + \$2.15 = \$4.30$. So, buying the 2 L size is a better deal.

D) Based on the calculation in part C), it would be $4.70 - 4.30 = \$0.40$ (or 40 cents) cheaper to buy two 2 L cartons rather than one 4 L carton of milk. Since 3×4 L $= 12$ L, then Jared would save $3 \times \$0.40 = \$1.20$.

Teacher's Notes

Shopping for groceries provides a wealth of mathematical problems, from careful study of the "deals" in flyers, to the various ways items are priced, to the ingredients lists on products, and more.

Stores use various marketing techniques to try to get the public to buy more. One of the most popular is to advertise *"Buy One Get One Free"*. Of course you are not actually getting anything for free, because you must spend money to get them both. The best way to confirm that you getting the best price for a purchase is to calculate the unit cost of the items that you are buying. The units you are comparing may be individual items such as whole watermelons, or units of measurement such as litres or grams.

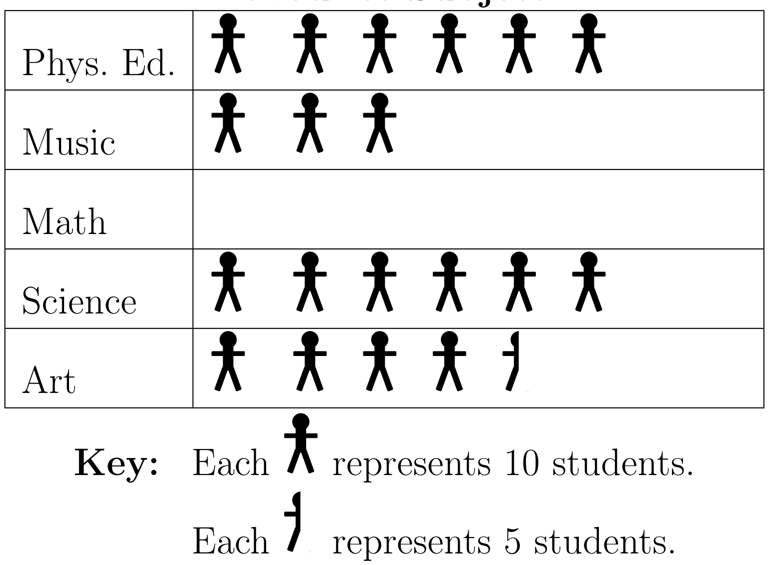
Problem of the Week

Problem A

Mathematical Mystery

A school of 270 students completed a survey about their favourite subject in school. The results for all subjects except Math are shown in the pictograph below.

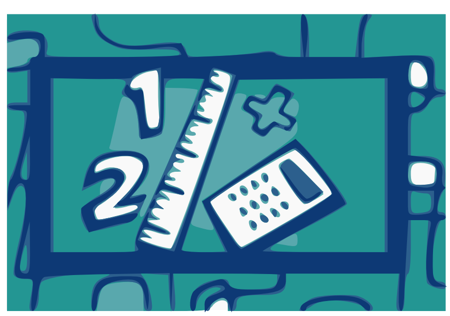
Favourite Subject



A) How many students selected Math as their favourite subject? Explain your reasoning.

B) Complete the pictograph.

C) What is the mode and median for this set of data?



Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Mathematical Mystery

Problem

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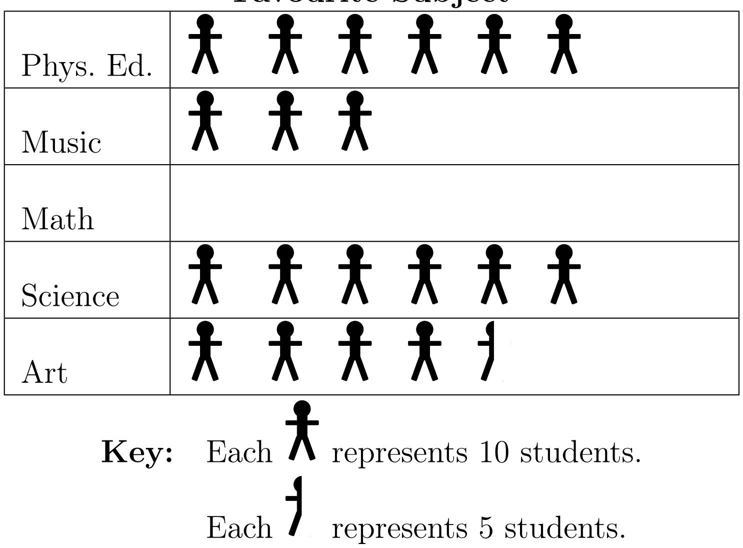


A) How many students selected Math as their favourite subject? Explain your reasoning.

B) Complete the pictograph.

C) What is the mode and median for this set of data?

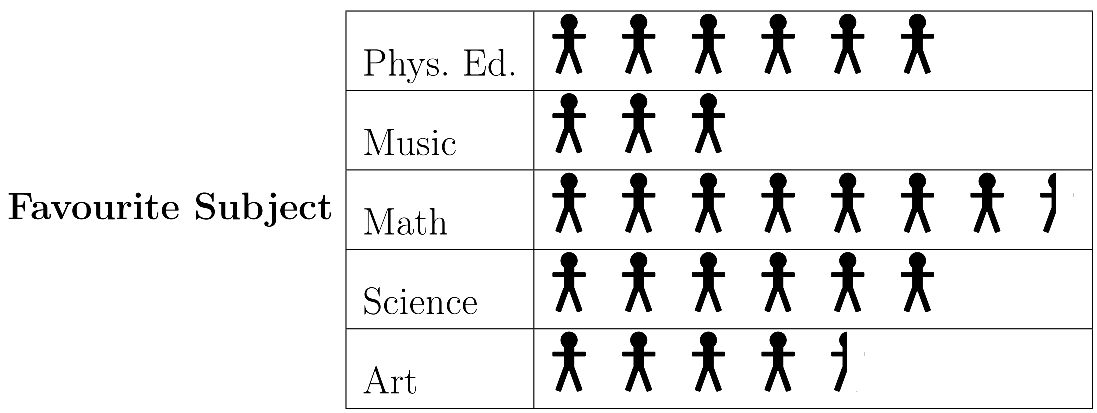
Solution

A) If each represents 10 students, and a half represents 5 then;

- $6 \times 10 = 60$ students chose Phys. Ed. as their favourite subject
- $3 \times 10 = 30$ students chose Music as their favourite subject
- $5 \times 10 = 50$ students chose Science as their favourite subject
- $(4 \times 10) + 5 = 45$ students chose Art as their favourite subject

The total of the known data is: $60 + 30 + 60 + 45 = 195$. To determine how many students voted for Math, you must remove the votes from the 195 students that have been counted, from the total 270 who voted. So, $270 - 195 = 75$ students voted for Math as their favourite subject.

B) Completed pictograph:



Key: Each represents 10 students.

Each represents 5 students.

C) In order to find the mode and the median for the data set, you should arrange the data from least to greatest. In this case, the data values are:
30, 45, 60, 60, 75

The mode is the value that occurs most often in the data set, so the mode is 60.

The median is the middle value in the data set, so the median is 60.

Teacher's Notes

The word average is often used in general conversation, but it is an ambiguous term. Many people equate *average* to *mean*, but statisticians use several different measurements of central tendency to describe averages of data. Central tendency is a formal way of saying "the typical values in a set of numbers", and *mean*, *median* and *mode* are three standard measurement tools. The *mean* tends to be easier to calculate, since the data does not have to be sorted. However, the *median* is often a better descriptor of an average value. The *mean* weighs all values in the set equally, so small numbers of extreme values can shift the calculated average. The *median* will not be as affected by a small number of outlining values.

Statisticians measure the data in many other ways. For example, *standard deviation* is a measurement tool that describes how clustered the data is around its *mean*. If a set of data has a low standard deviation, that indicates that the individual data values tend to be close to the *mean*. If the *standard deviation* is high, then the data values are much more spread out. If you are using data to predict the future, then a low standard deviation will generally lead to better predictions in applications such as weather, finance, or polling. However, there are never any guarantees.

Problem of the Week

Problem A

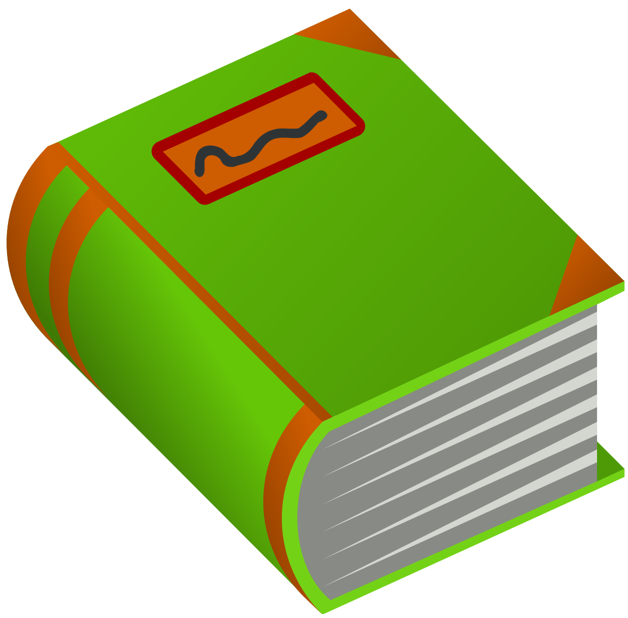
Money Management

Adam only saves quarters and dimes. He has already saved 13 quarters and 5 dimes. In Canada, one quarter equals 25 cents, one dime equals 10 cents, and 100 cents equals $1 (one dollar).

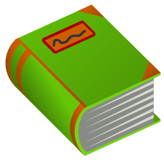
He wants to purchase a book that costs $7.75.

Not including the money he already has, what combinations of quarters and dimes could Adam save so he has exactly enough money to buy the book?

Show your thinking.



Strands Number Sense and Numeration, Patterning and Algebra



Problem of the Week
Problem A and Solution
Money Management

Problem

Adam only saves quarters and dimes. He has already saved 13 quarters and 5 dimes. In Canada, one quarter equals 25 cents, one dime equals 10 cents, and 100 cents equals \$1 (one dollar).

He wants to purchase a book that costs \$7.75.

Not including the money he already has, what combinations of quarters and dimes could Adam save so he has exactly enough money to buy the book?

Show your thinking.

Solution

Adam already has:

$$\begin{array}{llcr} & \text{13 quarters} & = & \$3.25 \\ + & \text{5 dimes} & = & \$0.50 \\ \hline & & & \$3.75 \end{array}$$

So Adam needs $\$7.75 - \$3.75 = \$4.00$ to buy the book. Since the total number of cents required (400) is an even number, and Adam only has dimes and quarters, if he uses quarters to pay for the book he must use an even number of quarters. Note that 5 dimes and 2 quarters both equal 50¢. The first combination calculates the total using all quarters. As we move down the table, we can remove two quarters each time and replace them with five dimes. This keeps the total the same, and is a good way to make sure all possible combinations have been considered.

Combination	10¢ (dime)	25¢ (quarter)
1	0	16
2	5	14
3	10	12
4	15	10
5	20	8
6	25	6
7	30	4
8	35	2
9	40	0

Therefore, there are 9 different combinations of quarters and dimes that Adam could collect to equal four dollars.

Teacher's Notes
The solution to this problem looks very much like a table of values for an equation that uses two variables. We can describe relationships between two or more values using equations. For example:

$$y = 5x + 1$$

describes a relationship between the variables x and y. Essentially, for any value we choose for x, the value of y will be one more than five times the value chosen. From this equation we can generate a table that contains a sample of the values for x and y that satisfy the relationship. You can create the table by picking a random value for one of the variables (known as the *independent* variable) and that will determine the value of the other variable (known as the *dependent* variable). If we pick the values 1, 2, 3, 4, and 5 for x, we get the following table:

x	y
1	6
2	11
3	16
4	21
5	26

We can write the relationship between the number of dimes and number of quarters needed to have enough money to purchase the book as an equation as well. If we use a variable d to represent the number of dimes Adam may use, and a variable q to represent the number of quarters he may use, we can write the following equation:

$$10d + 25q = 400$$

since each dime is worth 10¢, each quarter is worth 25¢, and Adam needs 400¢ more to buy the book.

Looking at each combination row in the table, we have values for d and q that make this equation true. For example, using combination 4, we can substitute 15 for d and 10 for q to get:

$$10(15) + 25(10) = 150 + 250 = 400$$

The equation describing this problem has a couple of restrictions that are not necessarily part of all equations. Since the values of the variables are representing physical coins, then the values can not be negative numbers, nor can they be fractions. The possible values for d are between 0 and 40. The possible values for q are between 0 and 16. The possible values for variables can be described as the *domain* and *range*. Since all values for these variables must be whole numbers, then this equation is describing a *discrete function*.

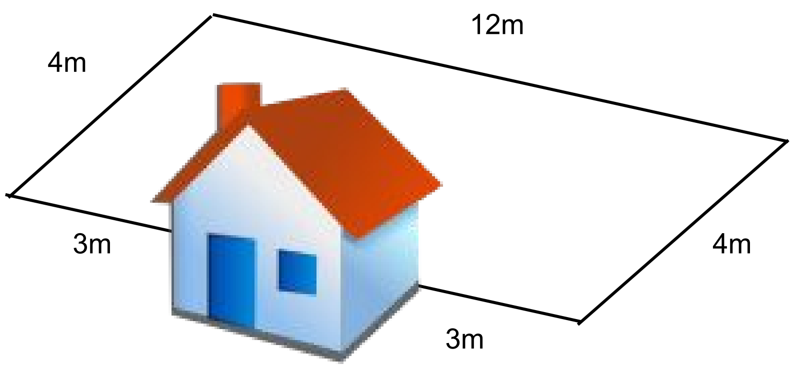
All of these concepts: independent variable, dependent variable, domain, range, and discrete functions are investigated in mathematics courses students may study in the future.

Problem of the Week

Problem A

The Fence

Agnes and her mother are building a fence around their backyard to keep their puppy safe. Their yard looks like the picture.



They start building the fence by placing one post at the back corner of the house. Then they add one fence post every meter around the yard until they put the final post at the other back corner of the house. Every corner of the yard has a post.

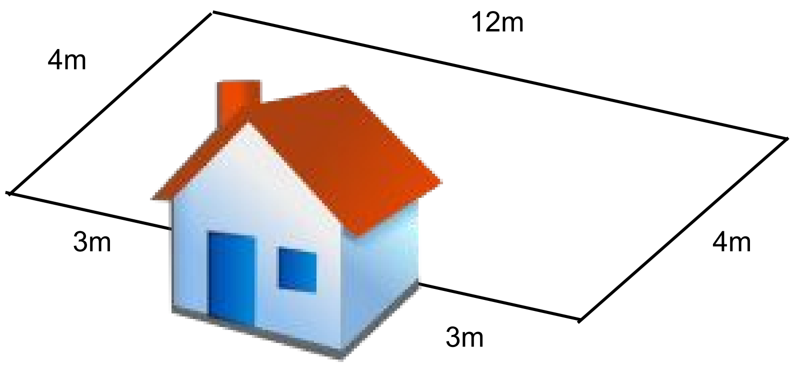
How many fence posts are needed altogether?

Strands Measurement, Number Sense and Numeration

Problem of the Week
Problem A and Solution
The Fence

Problem

Agnes and her mother are building a fence around their backyard to keep their puppy safe. Their yard looks like the picture.

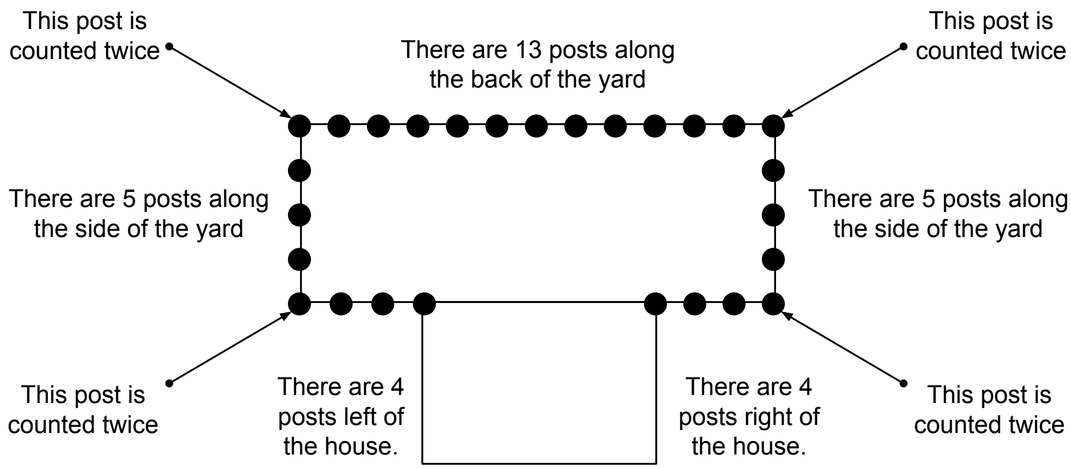


They start building the fence by placing one post at the back corner of the house. Then they add one fence post every meter around the yard until they put the final post at the other back corner of the house. Every corner of the yard has a post.

How many fence posts are needed altogether?

Solution

Here is a diagram of the layout for the fence posts.



There are many ways to count the number of fence posts required. You can simply count the number of dots in the diagram to see that that Agnes and her mother need 27 posts.

Another way to calculate the total is to realize that since the fence posts are 1 m apart, it is easy to determine how many posts are required in each section of the backyard. They will need one more fence post than number of metres in each case. For example, for the 12 m across the back, they need 13 posts, since they need one at the first corner (0 metres), and one every metre after that until the second corner which is 12 metres away.

If they add these numbers up, the total is: $4 + 5 + 13 + 5 + 4 = 31$. However, the posts in each corner of the yard are counted twice using this method. Since 4 posts are counted twice, they need to subtract the 4 duplicates from the total calculated by adding the number of posts in each section. So the total number of posts required to build the fence is: $31 - 4 = 27$.

Teacher's Notes

This problem is a literal example of the fencepost problem, which is an analogy used to describe off-by-one errors in coding. These errors are very common, especially when dealing with a range of values where the beginning and end of the range are variables.

For example, in programming strings consist of a sequence of characters. We can think of each character as having a numbered position in the string. Normally, we start that numbering at 0, and the position is known as the index. So for example, the string `‘‘Problem of the Week’’` would be indexed as follows:

P	r	o	b	l	e	m		o	f		t	h	e		W	e	e	k
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Many programming languages include a function that extracts a substring by identifying the index positions of the characters you want from the original string. For example, you might have an expression like this:

```
substring(‘‘Problem of the Week’’, 15, 17)
```

Normally, the first number you provide to the substring function indicates index position of the first character you want and the second number is one more than the index position of the last character you want. So the substring in our example would be `‘‘We’’`. This might seem odd, but this rule actually addresses the fencepost problem. Notice that the difference between the two numbers equals the number characters in the substring. So if we know the starting point of the substring, and the number of characters we want, then we simply add those two numbers together to find the second number for the substring function. Very often we know the starting position and the number of characters, so this can be the easier calculation.

Problem of the Week

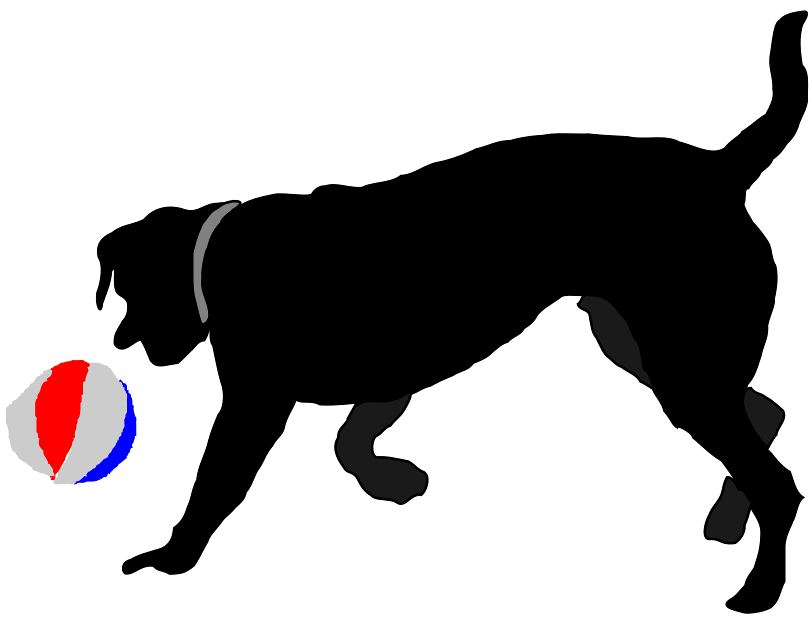
Problem A

Playing Fetch

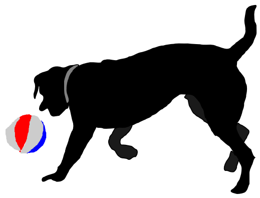
Robbie loves playing fetch with his dog Spencer. Spencer always starts by sitting beside Robbie before Robbie throws the ball. When Robbie throws the ball, Spencer runs to it and brings the ball back to the same spot. Robbie throws the ball three times.

- The first time he throws it 8 metres.
- The second time he throws it twice as far as the first time.
- The third time he throws it 5 metres less than the second time.

How far does Spencer run in total?



Strands Number Sense and Numeration, Measurement



Problem of the Week
Problem A and Solution
Playing Fetch

Problem

Robbie loves playing fetch with his dog Spencer. Spencer always starts by sitting beside Robbie before Robbie throws the ball. When Robbie throws the ball, Spencer runs to it and brings the ball back to the same spot. Robbie throws the ball three times.

- The first time he throws it 8 metres.
- The second time he throws it twice as far as the first time.
- The third time he throws it 5 metres less than the second time.

How far does Spencer run in total?

Solution

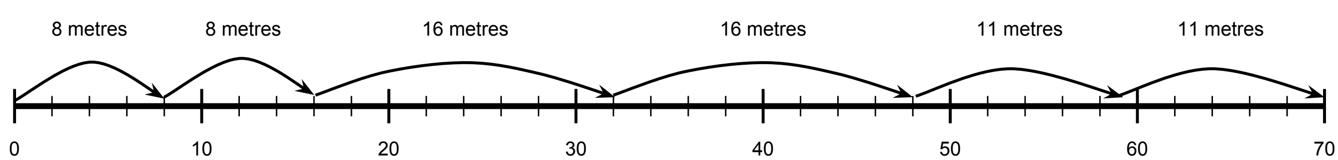
On the first throw, Spencer runs $2 \times 8 = 16$ metres.

The distance of the second throw is $8 \times 2 = 16$ metres.
On the second throw, Spencer runs $2 \times 16 = 32$ metres.

The distance of the third throw is $16 - 5 = 11$ metres.
On the third throw, Spencer runs $2 \times 11 = 22$ metres.

The total distance Spencer runs is: $16 + 32 + 22 = 70$ metres.

We can also show the distance Spencer runs on a number line.



Teacher's Notes

This problem can be described algebraically. We can use a variable d to represent the distance that Robbie throws the ball the first time. Then, the rest of the distances can be described in terms of d.

The distance he throws the ball the second time is $2d$.
The distance he throws the ball the third time is $2d - 5$.

Since Spencer runs to get the ball and returns to the original spot, every time Robbie throws the ball, Spencer will run that distance twice. We can use t to represent the total distance Spencer runs. Here is one way to calculate t:

$$t = d + d + 2d + 2d + 2d - 5 + 2d - 5$$
$$t = 10d - 10$$

We can also think of the total distance that Spencer runs as two times the total distance that Robbie throws the ball. So here is another way to calculate t:

$$t = 2 \times (d + 2d + 2d - 5)$$
$$t = 2 \times (5d - 5)$$
$$t = 10d - 10$$

Using the second approach, we had to use the *distributive law* to calculate that

$$2 \times (5d - 5) \text{ is equal to } 10d - 10$$

In other words, we multiplied 2 by $5d$ and by -5.

There are other ways we could derive the total distance, however when we simplify the equation, the result will always be:

$$t = 10d - 10$$

Now we can calculate the value of t by substituting the value we have for d. So

$$t = 10(8) - 10$$
$$t = 80 - 10$$
$$t = 70$$

This seems like a lot of work to get the answer we could have just counted using a number line. If we are only interested calculating the distance for one set of throws, then creating the equation is not particularly helpful. However, if we saw the same pattern for throwing with different starting distances, then the equation can be helpful. Suppose Robbie always throws in the same pattern, but this time his first throw is 5 metres. The total distance Spencer runs in this case would be:

$$t = 10(5) - 10$$
$$t = 50 - 10$$
$$t = 40$$

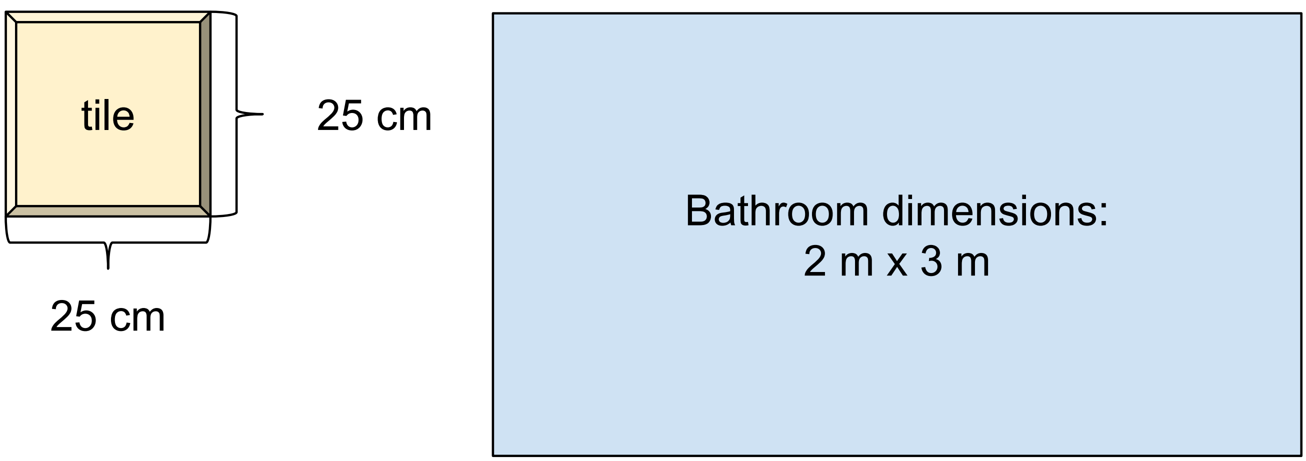
Algebraic equations describe a general case, and they can be very helpful when there are multiple specific cases to consider.

Problem of the Week

Problem A

Mrs. Thomas's Tidy Tiles

Mrs. Thomas wants to tile her 2m × 3m bathroom floor. Each tile is 25cm × 25cm.



A) How many tiles will she need to cover her bathroom floor?

B) If each tile costs $1.50, how much will it cost Mrs. Thomas to tile her bathroom?

Strands Measurement, Number Sense and Numeration

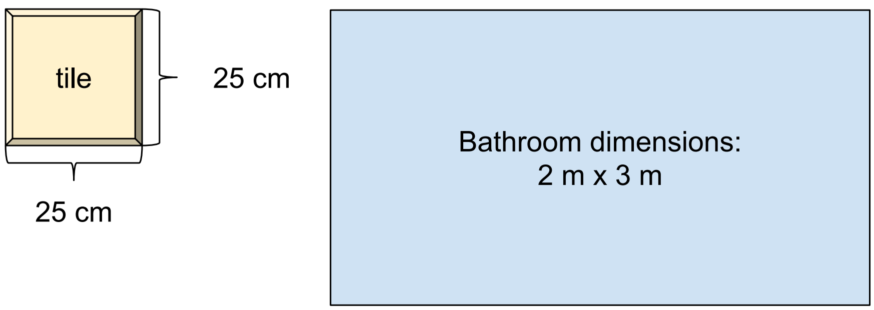
Problem of the Week

Problem A and Solution

Mrs. Thomas's Tidy Tiles

Problem

Mrs. Thomas wants to tile her 2m $\times$ 3m bathroom floor. Each tile is 25cm $\times$ 25cm.



A) How many tiles will she need to cover her bathroom floor?

B) If each tile costs \$1.50, how much will it cost Mrs. Thomas to tile her bathroom?

Solution

A) Each tile is 25 cm $\times$ 25 cm. This means that if we put 4 tiles in a line they would form a rectangle where the length of the long side is 1 m. If we put 16 tiles together (4×4) to form a square, the dimensions of that square would be $1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$. The area needed to be covered by tiles is $2 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$. So, Mrs. Thomas would need $16 \times 6 = 96$ tiles to complete her bathroom.

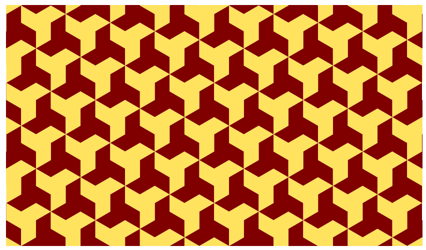
Alternatively, we could look at the bathroom dimensions and convert them into centimetres. The length of the bathroom is $3 \times 100 = 300$ cm. The width of the bathroom is $2 \times 100 = 200$ cm. If we divide the dimension of the bathroom by the width of the tile, then we know how many tiles are required along each edge. For the length, it is $300 \div 25 = 12$ tiles. For the width, it is $200 \div 25 = 8$ tiles. Therefore we need $12 \times 8 = 96$ tiles to cover the area of the bathroom.

B) If she needs 96 tiles and they each cost \$1.50, she would spend $96 \times 1.50 = (96 \times 1) + (96 \times 0.50) = 96 + (96 \div 2) = 96 + 48 = \$144.00$. She would spend \$144 for her tiles.

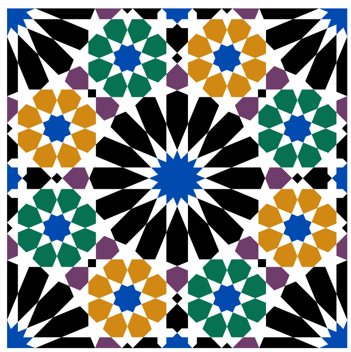
Teacher's Notes

This is a literal example of a tiling problem. In mathematics, tiling describes the process of covering a flat surface (or *plane*) with geometric shapes known as tiles. We imagine that the plane extends infinitely in all directions. Two examples are illustrated below.

The first example illustrates tiling with a single geometric shape.



The second example illustrates tiling with multiple geometric shapes.



You can imagine that these patterns could be continued indefinitely in all four directions. A real world tiling has boundaries. The problem of trying to fit geometric shapes into a bounded area is called a packing problem. Packing problems are studied by mathematicians and computer scientists. This type of problem asks the question, what is the maximum number of objects I can fit into a fixed space. That space can be two or three dimensional. Unlike with tilings, the packing problem allows gaps between the objects. Filling the entire space without any gaps would be optimal. However, there are some situations where you unable to avoid gaps. For example, consider trying to maximize the number of circles you can fit into a square. It is not possible to arrange the circles inside the square without gaps. Another example would be maximizing the number of squares inside a triangle. Again, you are guaranteed to have gaps. Solving packing problems has real life applications such as determining a company's shipping and storage requirements.

Problem of the Week

Problem A

Jasmine's Loot Bags

Jasmine is making loot bags of candy for 12 friends coming to her birthday party. She buys 60 gooey bears, 93 fluffy peaches and 75 jelly beans.

In order to be fair, Jasmine would like to put the same number of each candy into each of the 12 bags. She wants to give everyone as much candy as possible.

How many of each candy will be in one friend's loot bag?

Strand Number Sense and Numeration

Problem of the Week
Problem A and Solution
Jasmine's Loot Bags

Problem

Jasmine is making loot bags of candy for 12 friends coming to her birthday party. She buys 60 gooey bears, 93 fluffy peaches and 75 jelly beans.

In order to be fair, Jasmine would like to put the same number of each candy into each of the 12 bags. She wants to give everyone as much candy as possible.

How many of each candy will be in one friend's loot bag?

Solution

One possible way to determine the number of candies in each bag is to divide. We calculate the quotient and remainder when we take the number of candies of each type and divide by the number of friends at the party.

Number of gooey bears per bag:
$60 \div 12 = 5$ remainder 0.

Number of fluffy peaches per bag:
$93 \div 12 = 7$ remainder 9.

Number of jelly beans per bag:
$75 \div 12 = 6$ remainder 3.

Another way to calculate these results is to look for the largest multiple of 12 that is less than or equal to the number of candies of each type. This can be done with trial and error, or it can be done with repeated addition.

Number of gooey bears per bag:
$5 \times 12 = 12 + 12 + 12 + 12 + 12 = 60$

Number of fluffy peaches per bag:
$7 \times 12 = 12 + 12 + 12 + 12 + 12 + 12 + 12 = 84$ (with 9 unused)

Number of jelly beans per bag:
$6 \times 12 = 12 + 12 + 12 + 12 + 12 + 12 = 72$ (with 3 unused)

Either way it is calculated, Jasmine will need to put 5 gooey bears, 7 fluffy peaches, and 6 jelly beans into each loot bag.

Teacher's Notes

One of the solutions for this problem uses division to determine how many candies each friend receives. When you are dividing whole numbers (or any non-zero integers), the answer can always be calculated precisely. An exact answer can be described with the quotient and remainder. For example, $93 \div 12$ is exactly 7 with a remainder of 9. This means that we can write: $(12 \times 7) + 9 = 93$.

Another way of precisely representing the result of the division is by using a fraction. For example, $93 \div 12$ is exactly $7\frac{3}{4}$ or $\frac{31}{4}$.

We can show that this is correct by multiplying $\dfrac{31}{4} \times 12 = \dfrac{372}{4} = 93$.

If the answer is computed using a calculator, the result may not be precise. In the case of $93 \div 12$, the calculator will show the answer 7.75. This is an example of a terminating decimal, since there is a finite number of digits after the decimal point. All fractions can be written as terminating or repeating decimals. A repeating decimal has an infinite number of digits after the decimal point, but there is a pattern of one or more digits that repeats.

For example, if you compute $95 \div 12$ on a calculator, you will see a result like 7.9166666. This answer is not exact. This is a close approximation of the correct answer which is $7\frac{11}{12}$. If we wanted to represent the result exactly using a decimal point, then we would need to show that the digit 6 is repeated forever. We can do that by putting a line over the repeated pattern. For example, $95 \div 12$ is exactly $7.91\overline{6}$.

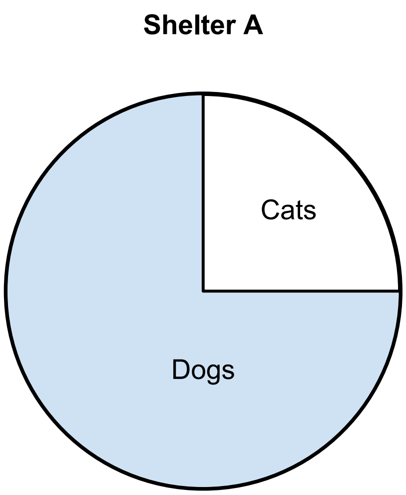
Knowing and keeping the exact answer when possible, and having access to the quotient and remainder when dividing can be useful. Loss of precision when dividing two numbers is something to be considered when using a calculator or a computer. Unfortunately, students often forget how to compute the exact result of division once they use calculators for their computations.

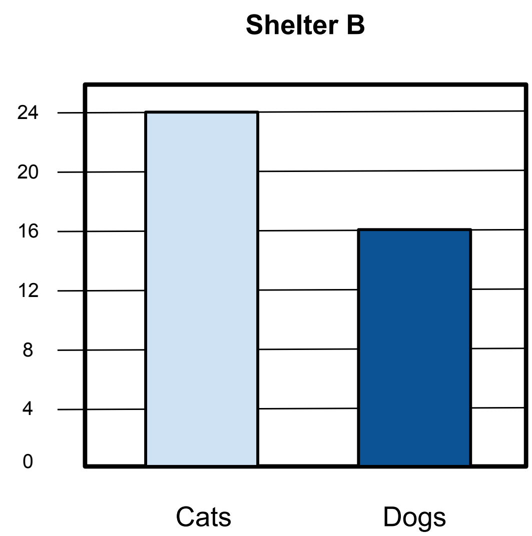
Problem of the Week

Problem A

More Dogs

The graphs below represent the number of cats and dogs in two local animal shelters. Shelter A and Shelter B have the same number of animals.





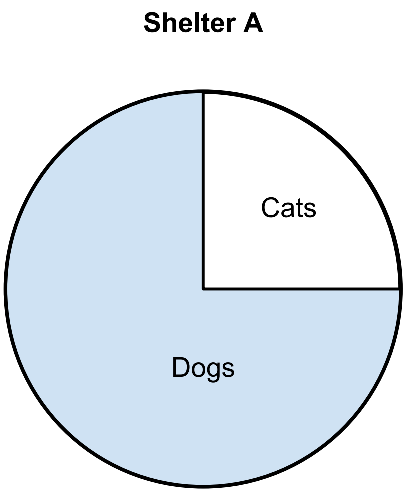
There are more dogs in Shelter A than in Shelter B. How many more? Justify your answer.

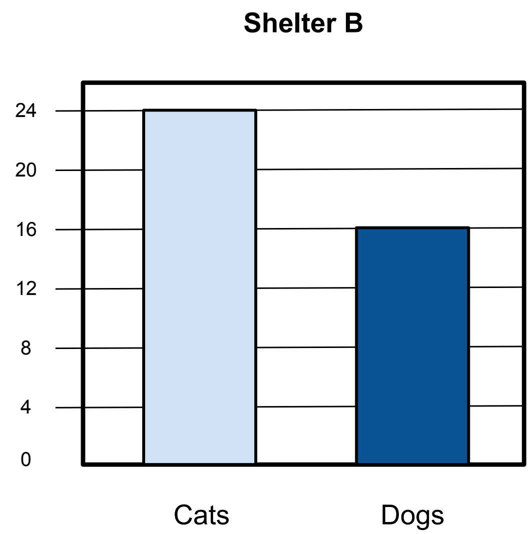
Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
More Dogs

Problem

The graphs below represent the number of cats and dogs in two local animal shelters. Shelter A and Shelter B have the same number of animals.





There are more dogs in Shelter A than in Shelter B. How many more? Justify your answer.

Solution

Looking at Shelter B, we see that there are 24 cats and 16 dogs. This means there is a total of $24 + 16 = 40$ animals in each of the two shelters.

It appears that in Shelter A, $\frac{3}{4}$ of the shelter is filled with dogs, and $\frac{1}{4}$ are cats. If the total number of animals at this shelter is 40, then we need to calculate $\frac{1}{4}$ of 40.

Since $10 + 10 + 10 + 10 = 40$, then $\frac{1}{4}$ of $40 = 10$. So there are 10 cats in Shelter A. This means that there are $40 - 10 = 30$ dogs in Shelter A.

Since there are 30 dogs at Shelter A and 16 dogs at Shelter B, then there are $30 - 16 = 14$ more dogs at Shelter A than Shelter B.

Teacher's Notes

Data can be visualized in many ways. A tool like a spreadsheet can automatically convert numeric data into a chart. The same data can be used to generate different styles of charts. Consider the bar chart showing the data for Shelter B. The spreadsheet automatically calculated the maximum value on the y-axis and chose the distance between the horizontal lines in the chart. Most spreadsheeets would give the user the option of changing the maximum value and changing the distance between each of the horizontal lines. Once those decisions are made, the spreadsheet will automatically recalculate the size of the chart and its elements.

If students want to create their own charts, they would need to do all of those calculations themselves. The work to determine the values of regular intervals from the minimum to the maximum as the locations of the horizontal lines is not trivial. Suppose you use graph paper to draw the chart. You need to determine a scale, such as 1 square represents 2 dogs. This is setting up a *ratio*, which is a fixed relationship between the number of squares on the paper and the number of dogs that distance represents.

The calculations involved in producing a pie chart can also be tricky. It is easy to divide a pie in half or in quarters, but other fractions can be more difficult. The size of a pie slice that represents some data can be determined by equivalent fractions. A whole circle contains 360 degrees. A slice of a circle is called a *sector*. The size of the sector can be described by the angle from one edge of the slice to the other. So a sector that is one quarter of the circle has an angle of 90 degrees, since 90 is one quarter of 360. To determine the size of a sector for any data value, you need to find a fraction with a denominator of 360 that is equivalent to the fraction of the data value divided by the total number in your set. For example, suppose we had a total of 40 animals at the shelter, and 4 of them are birds. If we wanted to show a pie slice representing the birds in this example, we need to find the value of x when $\frac{x}{360} = \frac{4}{40}$. In this case $x = 36$. So the sector will have an angle of 36 degrees.

Problem of the Week
Problem A
Colin's Coloured Blocks

Colin arranges blocks in a 12×12 grid. He starts in position A1 then follows a snake pattern to fill the grid. When he reaches the end of the first row, he continues filling up the second row from right to left. When he reaches the third row, he fills from left to right and so on. All the odd numbered rows are filled from left to right and the even numbered rows are filled from right to left.

Colin uses yellow, blue, red and green blocks to fill the grid. He starts with 1 yellow block, then 3 blue blocks, then 5 red blocks, then 7 green blocks. He continues the pattern starting with yellow blocks again. Each change of colour has 2 more blocks than the previous colour. The beginning of the pattern is shown here:

	A	B	C	D	E	F	G	H	I	J	K	L
1	Y	B	B	B	R	R	R	R	R	G	G	G
2	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
3	Y											
4												
5												
6												
7												
8												
9												
10												
11												
12												

A) What is the colour of the block at position F8 on the grid?

B) What is the colour of the block at position J9 on the grid?

C) What is the colour of the block at position B11 on the grid?

D) If Colin starts with 50 blocks of each colour, how many yellow, blue, red, and green blocks does he have left over after filling the grid?

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Colin's Coloured Blocks

Problem

Colin arranges blocks in a 12×12 grid. He starts in position A1 then follows a snake pattern to fill the grid. When he reaches the end of the first row, he continues filling up the second row from right to left. When he reaches the third row, he fills from left to right and so on. All the odd numbered rows are filled from left to right and the even numbered rows are filled from right to left.

Colin uses yellow, blue, red and green blocks to fill the grid. He starts with 1 yellow block, then 3 blue blocks, then 5 red blocks, then 7 green blocks. He continues the pattern starting with yellow blocks again. Each change of colour has 2 more blocks than the previous colour.

A) What is the colour of the block at position F8 on the grid?

B) What is the colour of the block at position J9 on the grid?

C) What is the colour of the block at position B11 on the grid?

D) If Colin starts with 50 blocks of each colour, how many yellow, blue, red, and green blocks does he have left over after filling the grid?

Solution

After filling up the grid it looks like this:

	A	B	C	D	E	F	G	H	I	J	K	L
1	Y	B	B	B	R	R	R	R	R	G	G	G
2	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
3	Y	B	B	B	B	B	B	B	B	B	B	B
4	R	R	R	R	R	R	R	R	R	R	R	R
5	R	G	G	G	G	G	G	G	G	G	G	G
6	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
7	Y	Y	Y	Y	Y	Y	Y	Y	Y	B	B	B
8	B	B	B	B	B	B	B	B	B	B	B	B
9	B	B	B	B	R	R	R	R	R	R	R	R
10	R	R	R	R	R	R	R	R	R	R	R	R
11	R	G	G	G	G	G	G	G	G	G	G	G
12	G	G	G	G	G	G	G	G	G	G	G	G

A) F8 is blue.

B) J9 is red.

C) B11 is green.

D) The total number of yellow blocks used is $1 + 9 + 17 = 27$.
The total number of blue blocks used is $3 + 11 + 19 = 33$.
The total number of red blocks used is $5 + 13 + 21 = 39$.
The total number of green blocks used is $7 + 15 + 23 = 45$.

If Colin starts with 50 blocks of each colour, he will have:

$50 - 27 = 23$ yellow blocks left over,
$50 - 33 = 17$ blue blocks left over,
$50 - 39 = 11$ red blocks left over, and
$50 - 45 = 5$ green blocks left over.

Teacher's Notes

Calculating the number of blocks for each colour is an example of determining the value of an *arithmetic series*. An arithmetic series is a sum, where the terms in the sum have a common difference. For example, in this arithmetic series $1 + 9 + 17 = 27$, the difference between the first and second terms is 8, and the difference between the second and third terms is 8. In fact, all of the series used to determine the total number of blocks of each colour have a common difference of 8. This common difference can be explained by the fact that we repeat the colours in a regular pattern. In between repeating the colours, we increase the number of blocks in the pattern by 2. We switch colours 4 times before returning to the same colour in the pattern. Since $2 \times 4 = 8$, the difference in the number of blocks we will use the next time with the same colour will be 8.

There is a formula for calculating the result of an arithmetic sequence.

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

where n is the number of terms in the sequence, a_1 is the value of the first term, and d is the common difference. Using this formula we could calculate the total number of red blocks for example. In this case, we would have to figure out that Colin uses 5 red blocks the first time (a_1), and that red blocks are used 3 times (n). We know that the common difference is 8 (d). Then we calculate the sum as:

$$\begin{aligned} S_n &= \frac{(3)}{2}[2(5) + ((3) - 1)(8)] \\ &= \frac{3}{2}[10 + (2)(8)] \\ &= \frac{3}{2}[26] \\ &= (3)(13) \\ &= 39 \end{aligned}$$

Adding up three terms is probably easier than using the formula to calculate the number of blocks in this problem. However, if we wanted to know how many block are used if the pattern continues for another 50 cycles, the formula would be useful.

Problem of the Week

Problem A

The Wooden Spoon

Thomas wants to make his grandfather's famous wooden spoon drink. He has a list of the ingredients and needs to know if he has a pot that is big enough to fit the entire recipe contents.

Ingredient	**Amount**
vanilla pudding	0.25 L
apple sauce	150 mL
milk	1 L
lemon juice	1500 mL
pumpkin see oil	350 mL
cream of tartar	100 mL

He has a 3 L container, a 3.25 L container and a 4 L container. Which one should he use? Explain your thinking.

Remember that 1 L equals 1000 mL.

Strands Measurement, Number Sense and Numeration

Problem of the Week
Problem A and Solution
The Wooden Spoon

Problem

Thomas wants to make his grandfather's famous wooden spoon drink. He has a list of the ingredients and needs to know if he has a pot that is big enough to fit the entire recipe contents.

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He has a 3 L container, a 3.25 L container and a 4 L container. Which one should he use? Explain your thinking.

Remember that 1 L equals 1000 mL.

Solution

It is probably easier to figure out the solution if all of the amounts are shown in the same units. This table shows everything in mL.

Ingredient	**Amount**
vanilla pudding	$0.25 \text{ L} = 0.25 \times 1000 = 250 \text{ mL}$
apple sauce	150 mL
milk	$1 \text{ L} = 1 \times 1000 = 1000 \text{ mL}$
lemon juice	1500 mL
pumpkin see oil	350 mL
cream of tartar	100 mL

Now we can calculate the total amount of the recipe in mL:

$$250 + 150 + 1000 + 1600 + 350 + 100 = 3350 \text{ mL}$$

This is equal to $3450 \div 1000 = 3.35$ L.

So, the 3 L and the 3.25 L containers are not big enough. Thomas needs to use the 4 L container to hold all of the ingredients.

Teacher's Notes

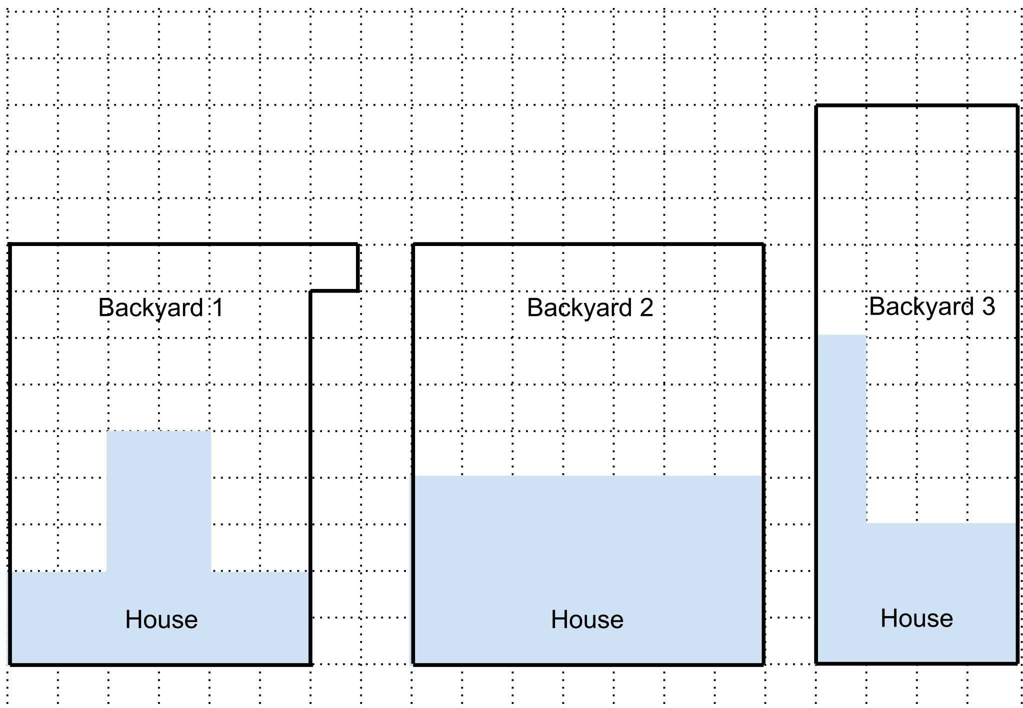
The metric system is a system of measurement which allows for easy conversions between different measurement units with the same base. There are a standard set of prefixes that indicate a multiple or fraction of the base unit. When measuring every day items, the prefixes *kilo* meaning 1000 or 10^3, *centi* meaning $\frac{1}{100}$ or 10^{-2}, and *milli* meaning $\frac{1}{1000}$ or 10^{-3}, are very useful.

Since today's technology is so small and so powerful, we often use other metric system prefixes to measure its size and speed. Here are some examples of these very large and very small values.

giga 1 000 000 000	In 2017, a common size of SD cards for digital cameras is 32 or 64 gigabytes (GB). An individual pixel of an image would typically require 3 bytes of storage space. The size of an image taken by the camera depends on many factors, but would normally be described in terms of *megapixels*. (The prefix *mega* means 1 000 000.) Suppose you have a camera that takes pictures that take up 32 megabytes of storage space. This means you could save 1000 pictures on a 32 GB card or 2000 pictures on a 64 GB card.
tera 1 000 000 000 000	In 2017, an external hard drive for your home computer would normally be between 1 and 4 terabytes (TB). A single character in text would take up 1 or 2 bytes (depending on how the computer represents the character). According to *Wikipedia*, in 1989 the second edition of the Oxford English Dictionary (OED) was published, containing approximately 59 million words in 20 printed volumes. The second edition can be stored in approximately 540 megabytes (MB). This means a 1 TB external hard drive could contain over 1800 copies of the second edition of the OED. As of 2017, the third edition has not been completed.
nano $\frac{1}{1\,000\,000\,000}$	In a vacuum, light or electricity can travel at a speed of approximately 30 cm in one nanosecond. Remember that it takes approximately 8 minutes for light to travel from the sun to the Earth, and the sun is approximately 150 million kilometres away. Computer scientist Grace Hopper (1906 - 1992) famously carried a bundle of "nanoseconds" with her. These were wires cut to lengths of 30 cm each. She would use these as visual aids to explain, among other things, why it took so long for messages to be sent via satellite.
pico $\frac{1}{1\,000\,000\,000\,000}$	A computer has an internal clock that coordinates all of its processes. That clock beats very fast. The speed of the clock is usually described in gigahertz (GHz). Suppose your computer had a clock speed of 4 GHz. This means the clock beats 4 000 000 000 times per second. Looking at it another way, a single beat of the clock takes 250 picoseconds. Grace Hopper also had a way of visualizing picoseconds. She would take a packet of pepper as an illustration of many picoseconds, where the size of an individual pepper grain is the maximum distance light can travel in 1 picosecond.

Problem of the Week
Problem A
House Hunt

The Ali family is looking for a new house with a large backyard. They are trying to decide between the 3 homes shown below.



Each square in the grid represents one square unit.

A) Which house has the backyard with the largest area? Explain your thinking.

B) A *lot* is the property that contains the house and the backyard. The family wants to install an invisible fence around their property to keep their pets safe. They will need to know the perimeter of the lot that they buy. What is the perimeter of each lot?

Strands Measurement, Number Sense and Numeration

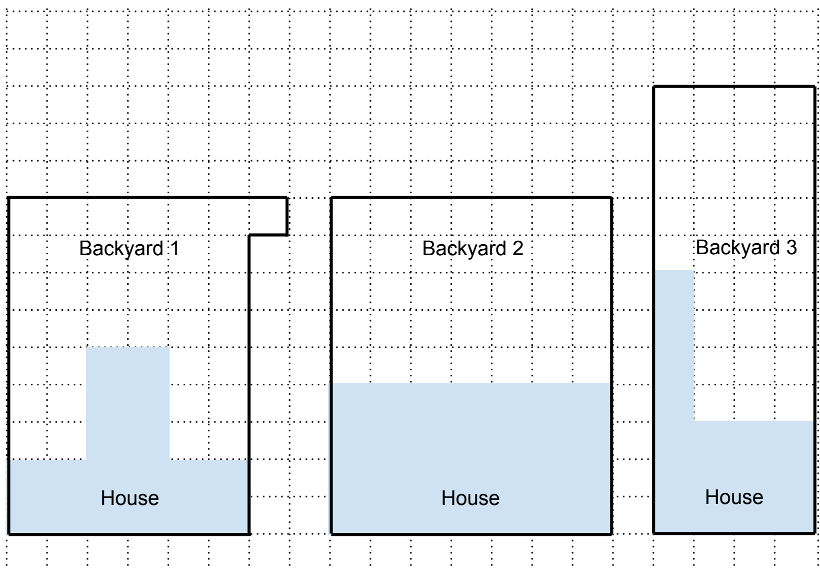
Problem of the Week

Problem A and Solution

House Hunt

Problem

The Ali family is looking for a new house with a large backyard. They are trying to decide between the 3 homes shown below.



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A) Which house has the backyard with the largest area? Explain your thinking.

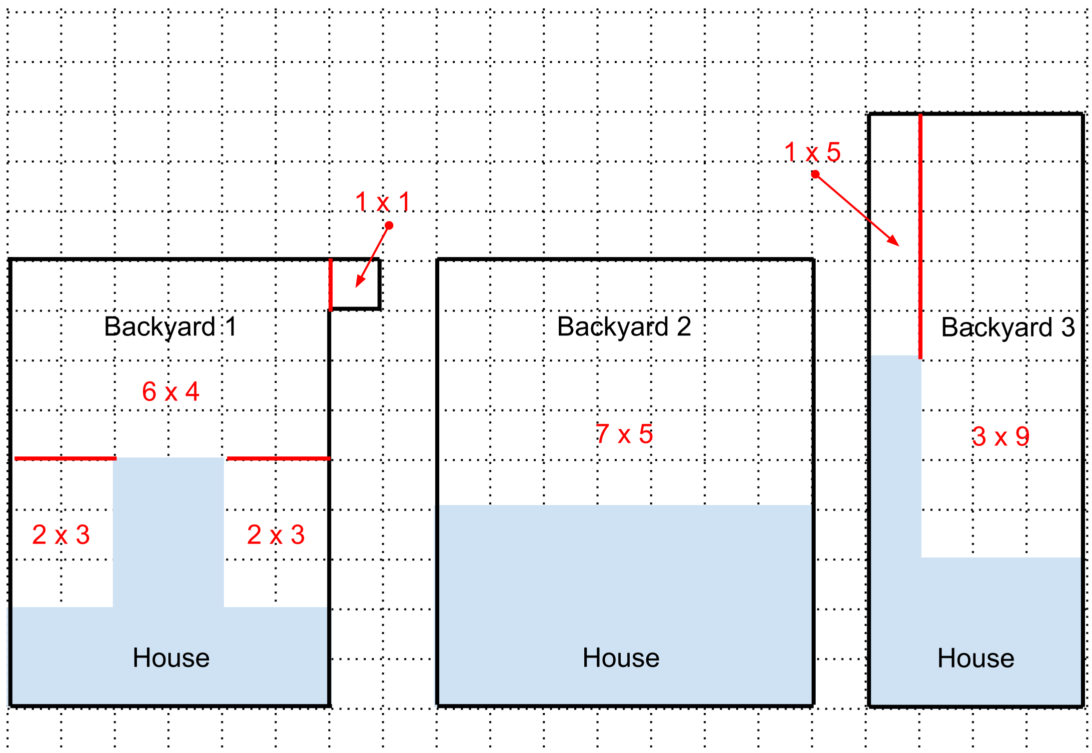
B) A *lot* is the property that contains the house and the backyard. The family wants to install an invisible fence around their property to keep their pets safe. They will need to know the perimeter of the lot that they buy. What is the perimeter of each lot?

Solution

A) There are many ways to solve this problem. One way to determine the area of each yard is to simply count the squares within each outlined backyard.

Some students may decide to break the yards up into smaller, more manageable rectangles, in order to count the units.

Students may also use the formula $l \times w$ to determine area for each of the smaller rectangular components in the backyards, adding the products of each section to determine the overall area.



You can calculate the areas for each of the backyards by looking at the areas of rectangles that make up each yard.

Area of backyard 1:
$(6 \times 4) + (2 \times 3) + (2 \times 3) + (1 \times 1) = 24 + 6 + 6 + 1 = 37$ units^2.
Area of backyard 2: $7 \times 5 = 35$ units^2.
Area of backyard 3: $(1 \times 5) + (3 \times 9) = 5 + 27 = 32$ units^2.

So the first house has the backyard with the biggest area.

B) Looking at the diagrams, you can calculate the perimeters:

Perimeter of lot 1: $6 + 9 + 7 + 1 + 1 + 8 = 32$ units.
Perimeter of lot 2: $7 + 9 + 7 + 9 = 32$ units.
Perimeter of lot 3: $4 + 12 + 4 + 12 = 32$ units.

Teacher's Notes

This question asks about area and perimeter. There is no general relationship between the area of a shape and its perimeter.

In some cases, like regular polygons or circles, the area of the shape has a direct relationship with its perimeter. For example, if you double the perimeter of a square this will increase the area by four times. Similarly, if you increase the circumference of a circle by three times, the area will increase by nine times.

It is possible to draw a shape that has the property that: as the perimeter increases, the area decreases. An example of this would be a shape that includes a concave angle. Consider these two images:

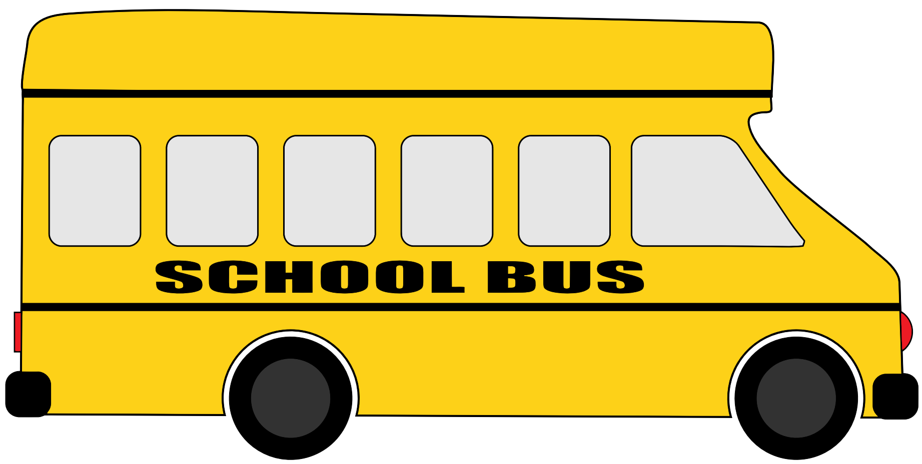
They are the same shape, but the image on the left has a larger area than the image on the right, and the image on the right has a larger perimeter than the image on the left.

Problem of the Week
Problem A
Getting to School

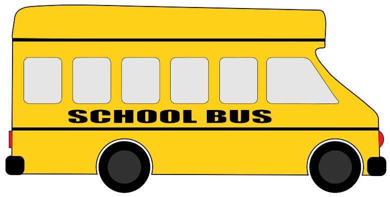
Twenty grade 3 students and 25 grade 4 students attend the School of Math. Twelve grade 3 students take the bus, and the rest walk to school. Out of all the students in grades 3 and 4, there are 23 who walk to school. At an assembly for grade 3 and 4 students, a name is drawn for a prize.

A) What is the probability that the prize winner is in grade 3 and walks to school?

B) What is the probability that the prize winner is in grade 4 and takes the bus to school?



Strands Data Management and Probability, Number Sense and Numeration



Problem of the Week

Problem A and Solution

Getting to School

Problem

Twenty grade 3 students and 25 grade 4 students attend the School of Math. Twelve grade 3 students take the bus, and the rest walk to school. Out of all the students in grades 3 and 4, there are 23 who walk to school. At an assembly for grade 3 and 4 students, a name is drawn for a prize.

A) What is the probability that the prize winner is in grade 3 and walks to school?

B) What is the probability that the prize winner is in grade 4 and takes the bus to school?

Solution

The solution for both parts is based on determining the total number of students at the assembly. There are a total of $20 + 25 = 45$ students at the assembly.

A) Since there are 20 students in grade 3, and 12 of them take the bus, then $20 - 12 = 8$ grade 3 students walk to school.

The probability that the prize winner is in grade 3 and walks to school is 8 out of 45 or $\frac{8}{45}$.

B) Since 23 students walk to school, and 8 of them are grade 3 students, then $23 - 8 = 15$ grade 4 students walk to school.

Since there are 25 students in grade 4 and 15 of them walk to school, then $25 - 15 = 10$ grade 4 students take the bus to school.

The probability that the prize winner is in grade 4 and takes the bus to school is 10 out of 45 or $\frac{10}{45}$ or $\frac{2}{9}$.

Teacher's Notes

The probability of some outcome can be described in many different ways. The solution uses the words "out of" essentially as a replacement for the line between the numerator and the denominator of a fraction. The solution also uses fractions to describe the probability.

We expect the numerator and denominator of a fraction describing probability to have specific characteristics. The numerator must be a non-negative number (i.e. greater than or equal to 0), and the denominator must be a positive number that is greater than or equal to the numerator. Since there are infinitely many equivalent fractions, there are many fractions we could use to describe the same probability. In this problem, the probability of the prize winner being in grade 4 was described as $\frac{10}{45}$ or $\frac{2}{9}$. But we could also describe the probability as $\frac{100}{450}$ or $\frac{12}{54}$.

If we compare all of these fractions by doing division with a calculator, the result is the same:

$$\frac{10}{45} = \frac{2}{9} = \frac{100}{450} = \frac{12}{54} = 0.\overline{2}$$

As shown here, all probabilities can also be described as a number between 0 and 1. So in this problem, we could describe the probability as approximately 0.22222.

(Note that since dividing the numerator and denominator of any of the fractions produces a repeating decimal, we have to describe it as *approximately* the same.)

Problem of the Week

Problem A

Doing Chores

Oliver can earn money by doing chores each week. His parents made up the following chart:

Chore	**Money Earned**
Cleaning his room	\$1.00
Walking the dog	\$1.25
Clearing the table	\$0.50
Shovel snow from the deck	\$1.50

Over four weeks he did some of the chores. The first week he did everything except shovel the snow. The second week he cleaned his room and walked the dog. The third week he did everything except clean his room. The fourth week, he cleaned his room and shovelled the snow.

How much money did Oliver earn over these four weeks?

Strand Number Sense and Numeration


Problem of the Week
Problem A and Solution
Doing Chores

Problem

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Over four weeks he did some of the chores. The first week he did everything except shovel the snow. The second week he cleaned his room and walked the dog. The third week he did everything except clean his room. The fourth week, he cleaned his room and shovelled the snow.

How much money did Oliver earn over these four weeks?

Solution

The solution can be computed in different ways. One way is to determine how much Oliver earned each week.

Week 1: $\$1.00 + \$1.25 + \$0.50 = \$2.75$
Week 2: $\$1.00 + \$1.25 = \$2.25$
Week 3: $\$1.25 + \$0.50 + \$1.50 = \$3.25$
Week 4: $\$1.00 + \$1.50 = \$2.50$
Total earnings: $\$2.75 + \$2.25 + \$3.25 + \$2.50 = \$10.75$

Another way to calculate it is to multiply each chore value by the number of weeks that the chore was done. Oliver cleaned his room three out of the four weeks. He walked the dog three out of the four weeks. He cleared the table two out the four weeks. He shovelled snow two out of the four weeks.

Amount earned from cleaning his room: $3 \times \$1.00 = \$3.00$
Amount earned from walking the dog: $3 \times \$1.25 = \$3.75$
Amount earned from clearing the table: $2 \times \$0.50 = \$1.00$
Amount earned from shovelling snow: $2 \times \$1.50 = \$3.00$
Total earnings: $\$3.00 + \$3.75 + \$1.00 + \$3.00 = \$10.75$

Teacher's Notes

In their early introduction to mathematics, students learn simple operations: $+$, $-$, $\times$, and $\div$. Later, they learn more complex operations such as square root, logarithms, and modulus. In more advanced mathematics, students are introduced to an operation called *dot product*. Mathematicians use • to represent this operation. A big difference between a dot product and the other operations mentioned is that, the operands of a dot product are not single values; they are collections of values known as *vectors*.

A vector is an ordered list of values. A dot product combines two vectors of the same length, where the values in the the same position within each vector are logically connected. To calculate the dot product you compute the products of each pair of numbers from the same relative position in each vector, and then add those products together. So for example, the dot product of the vectors $[a, b, c]$ and $[x, y, z]$ is:

$$[a, b, c] \cdot [x, y, z] = (a \times x) + (b \times y) + (c \times z).$$

The second solution to this problem could be written as a dot product. We could create vectors where the position of the values in each vector corresponds to a particular chore. So, we have two vectors in this problem.

Vector representing how many weeks each chore was done: $[3, 3, 2, 2]$.
Vector representing the money earned for each chore: $[1, 1.25, 0.5, 1.5]$.

To determine how much money Oliver earned overall, we calculate the dot product of these two vectors.

$$[3, 3, 2, 2] \cdot [1, 1.25, 0.5, 1.5] = (3 \times 1) + (3 \times 1.25) + (2 \times 0.5) + (2 \times 1.5) = 10.75$$

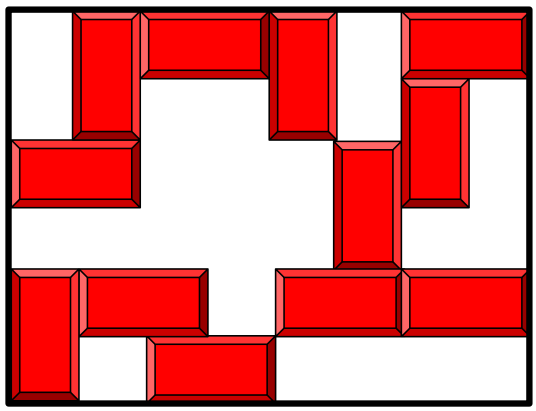
This is essentially the same calculation shown in the solution.

Problem of the Week

Problem A

Building Blocks

Johnna Lee loves to build with interlocking building blocks. She starts building on a large, white, flat piece for the base, but the rest of the blocks she uses are red. Looking from overhead, this is what Johnna Lee built.



Each of the red blocks has a length of 2 cm and a width of 1 cm.

A) What is the area of the white base?

B) What fraction of the area of the base is covered by red blocks?

Strands Measurement, Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Building Blocks

Problem

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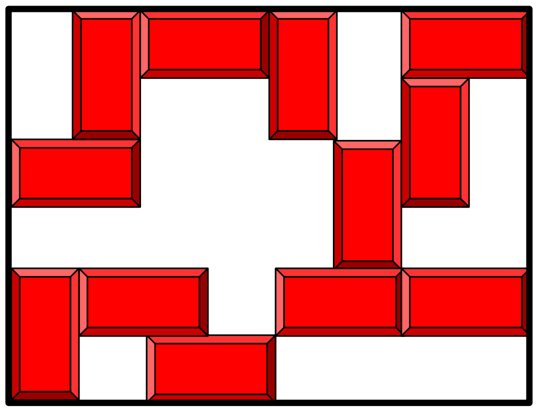


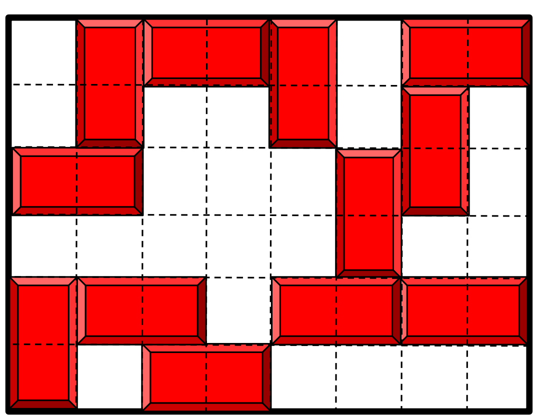
Each of the red blocks has a length of 2 cm and a width of 1 cm.

A) What is the area of the white base?

B) What fraction of the area of the base is covered by red blocks?

Solution

You can draw grid lines that align with the edges of the blocks. The squares formed by the grid lines are 1 cm $\times$ 1 cm.



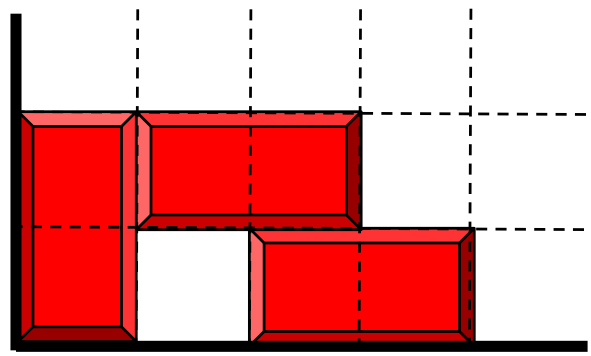
A) If you count the total number of squares formed by the grid lines, you see that there are 48. Alternatively, you can see that the base is 8 cm wide and 6 cm high. The area of the base is 8 cm $\times$ 6 cm $=$ 48 cm^2.

B) Count the number of squares formed by the grid lines that overlap the blocks. The total is 24. The fraction of the area of covered by blocks over the area of the base is $\frac{24}{48}$. Since $48 = 24 \times 2$ we can also write the fraction as $\frac{1}{2}$.

Here is another way to calculate this fraction. The area of one of the blocks is 1 cm $\times$ 2 cm $=$ 2 cm^2. There are 12 blocks showing on the base. The total area they cover is 12×2 cm^2 $=$ 24 cm^2. Since the area of the base is 48 cm^2, the blocks cover $\frac{24}{48}$ or $\frac{1}{2}$ the area of the base.

Teacher's Notes

The second part of this problem could be solved without knowing any specific information about the dimensions of the blocks. Everything you need to determine what fraction of the base is being covered by the blocks is available in the picture. The key is knowing that the ratio of the length to the width of the building block is 2 : 1. This can be determined by examining the bottom left corner of the diagram.



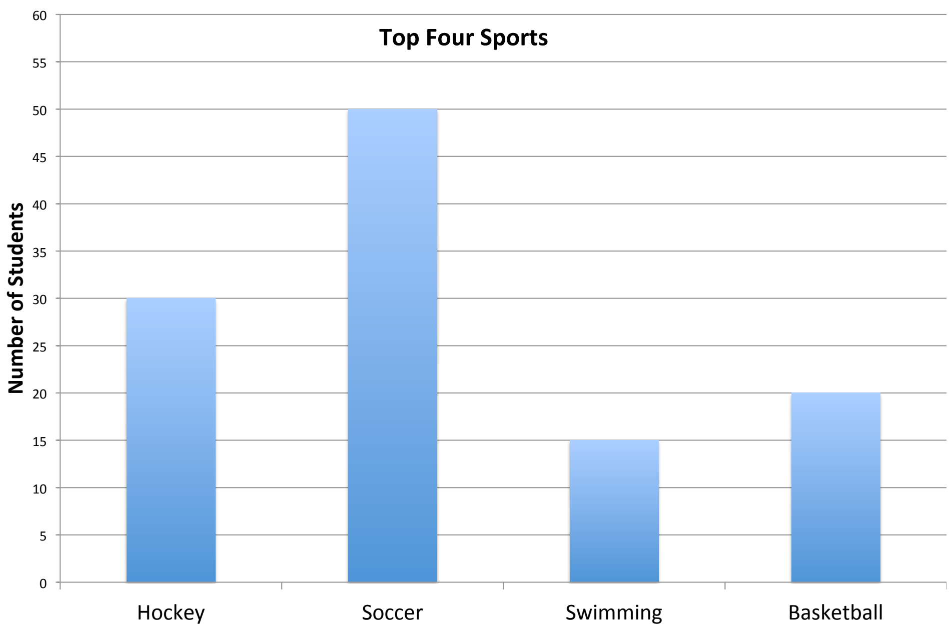
From this part of the picture, we see that the twice the shorter side of the block is equal to the length of the longer side of the block. This relationship, along with the way the blocks are aligned in the rest of the diagram, allows us to add the grid lines to the diagram. These grid lines form unit squares. The actual size of those squares is irrelevant; their measurements could be 1 cm $\times$ 1 cm $=$ 1 cm^2, or 1 m $\times$ 1 m $=$ 1 m^2, or 1 Wiffle $\times$ 1 Wiffle $=$ 1 Wiffle^2. What is important for us, is that they are squares. Since we are computing a fraction, the units will disappear in the calculation.

Ultimately the solution for this problem can be computed based on the number of unit squares that form the base of the structure and the number of unit squares the blocks cover. In this case we have a fraction of:

$$\frac{24 \text{ \sout{units}}^2}{48 \text{ \sout{units}}^2} = \frac{1}{2}$$

Problem of the Week
Problem A
Sporty Choices

Mr. King's class did a survey with the students of their school to determine favourite sports. They determined that the top four sports were soccer, basketball, swimming and hockey. They surveyed the students in the school again to have them choose from the four favourite sports. Here are the results:



A) How many students at the school were surveyed concerning the four sports?

B) Of the top four most popular sports at the school, what is the difference between the number of students selecting the most popular sport and the number of students selecting the least favourite sport?

Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Sporty Choices

Problem

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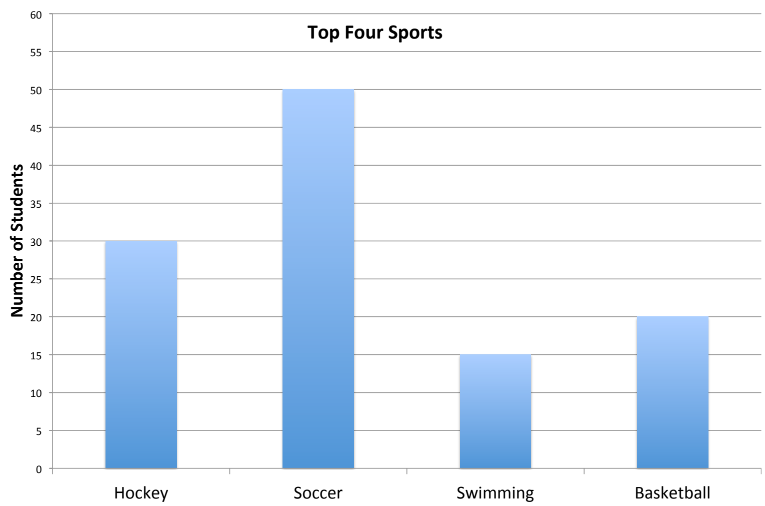


A) How many students at the school were surveyed concerning the four sports?

B) Of the top four most popular sports at the school, what is the difference between the number of students selecting the most popular sport and the number of students selecting the least favourite sport?

Solution

A) Based on the chart we can determine that 30 students chose hockey, 50 students chose soccer, 15 students chose swimming, and 20 students chose basketball.

Therefore, the total number of students in the school that were surveyed is: $30 + 50 + 15 + 20 = 115$.

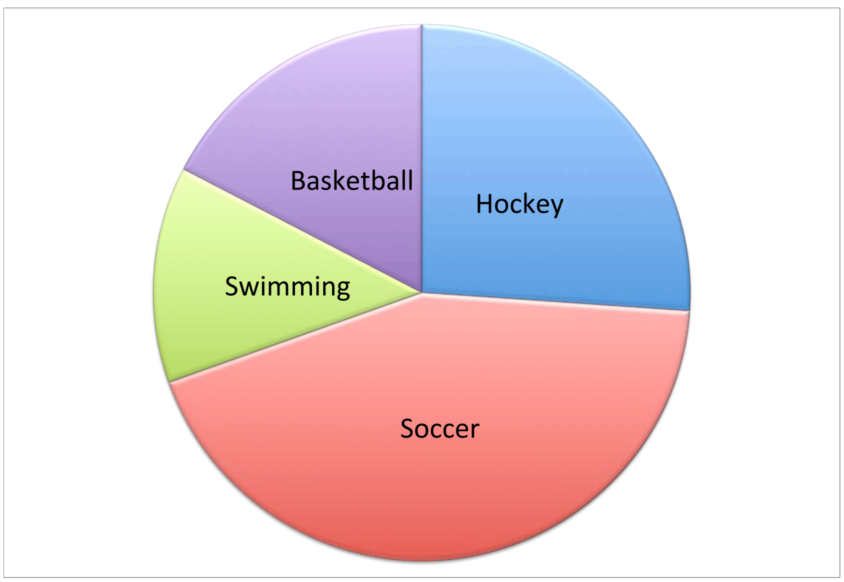
B) The most popular sport is soccer with 50 students. The least popular sport is swimming with 15 students.

The difference between most and least popular sports is: $50 - 15 = 35$ students.

Teacher's Notes

We use charts to visualize numeric data. The same data can be used to generate different types of charts. A column chart is a good way to identify relative sizes. It is immediately clear from the column chart that soccer is the most popular sport and that swimming is the least popular sport in this survey. Since the chart includes clearly marked grid lines, and the top of all the columns align with the grid lines, then it is also easy to determine the numbers that are being represented by the picture. Even if the columns do not align with the gridlines, we can reasonably estimate the values that the columns represent.

Other types of charts can emphasize different aspects of the data. A line chart is a good way to identify trends over time. A pie chart is a good visualization of proportion or the percentage each data point represents in the whole. Here is the same data from the original problem shown in the form of a pie chart:

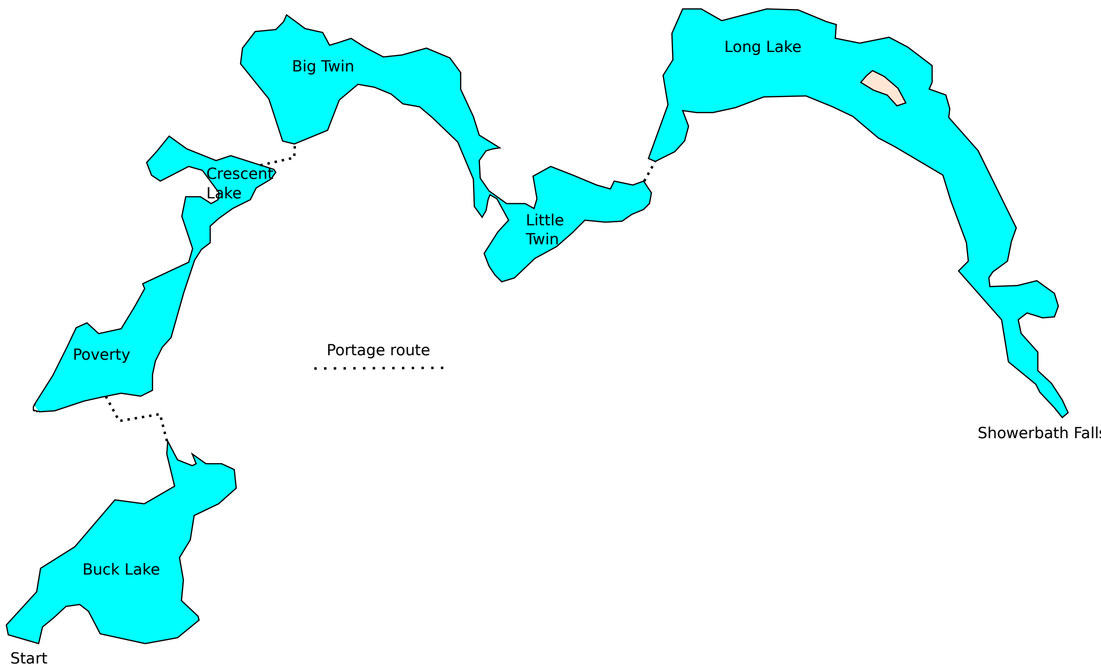


Using this format, it is easy to see that, of the students surveyed in this school, close to half of them say that soccer is their favourite sport and approximately a quarter of them picked hockey.

Spreadsheet programs make it easy to generate different charts depending on how you want to present the data.

Problem of the Week
Problem A
Canoeing to Showerbath Falls

James and Katie decide to paddle their canoe to Showerbath Falls. There are six lakes to paddle through to get from the starting point of the trip to the end. The lakes are Buck, Poverty, Crescent, Big Twin, Little Twin, and Long Lake. Some of the lakes are connected by narrows, where you can paddle straight through, and some lakes are connected by portages, which means James and Katie have to get out of the canoe and carry it across a path to the next lake. Poverty and Crescent are connected by narrows, and Big Twin and Little Twin are connected by narrows. There are portages between Buck and Poverty, Crescent and Big Twin, and Little Twin and Long Lake.



They start in Buck Lake and it takes them 30 minutes to paddle across it. The portage between Buck and Poverty takes 10 minutes to cross. It takes 20 minutes to paddle across Poverty and Crescent. The portage between Crescent and Big Twin takes 15 minutes. From Big Twin to the final portage is a 25 minute paddle. This last portage is only 5 minutes long. Long Lake is well named and it takes 50 minutes to get to the end. Showerbath Falls is at the end of Long Lake.

A) How much more time do they spend paddling rather than portaging the canoe during the trip from Buck Lake to Showerbath Falls?

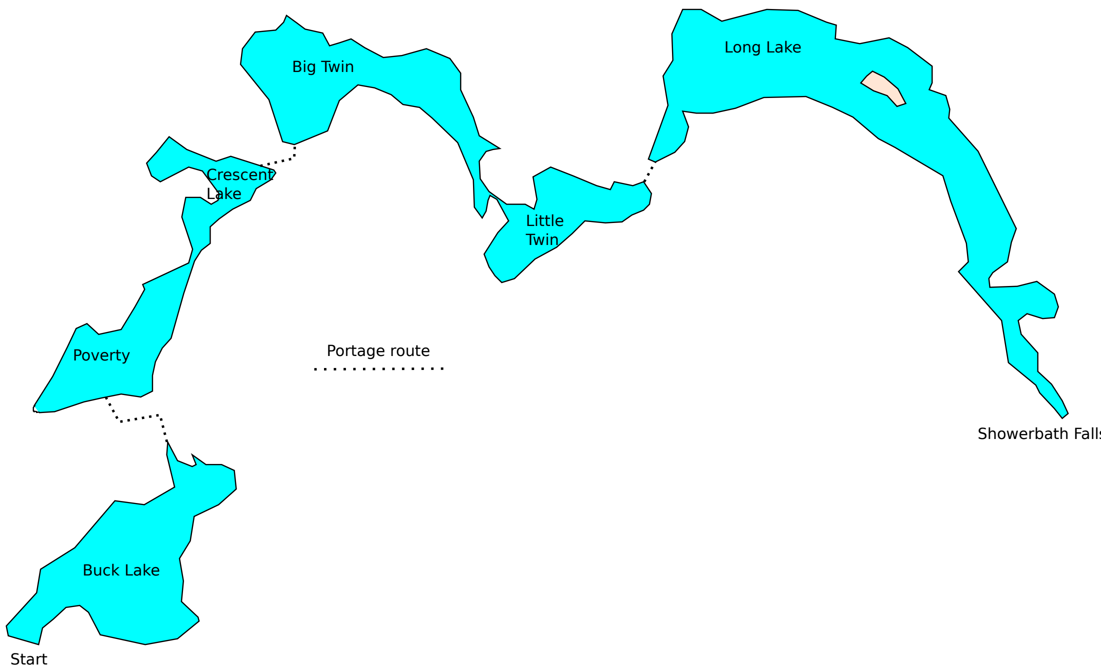
B) Katie and James spent an hour at Showerbath Falls eating lunch and then returned home. It took them the same amount of time to travel back. If they left at 9:00 in the morning, what time did they get home?

Strands Number Sense and Numeration, Measurement

Problem of the Week
Problem A and Solution
Canoeing to Showerbath Falls

Problem

James and Katie decide to paddle their canoe to Showerbath Falls. There are six lakes to paddle through to get from the starting point of the trip to the end. The lakes are Buck, Poverty, Crescent, Big Twin, Little Twin, and Long Lake. Some of the lakes are connected by narrows, where you can paddle straight through, and some lakes are connected by portages, which means James and Katie have to get out of the canoe and carry it across a path to the next lake. Poverty and Crescent are connected by narrows, and Big Twin and Little Twin are connected by narrows. There are portages between Buck and Poverty, Crescent and Big Twin, and Little Twin and Long Lake.



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A) How much more time do they spend paddling rather than portaging the canoe during the trip from Buck Lake to Showerbath Falls?

B) Katie and James spent an hour at Showerbath Falls eating lunch and then returned home. It took them the same amount of time to travel back. If they left at 9:00 in the morning, what time did they get home?

Solution

A) The total time paddling is: $30 + 20 + 25 + 50 = 125$ minutes.

The total time portaging is: $10 + 15 + 5 = 30$ minutes.

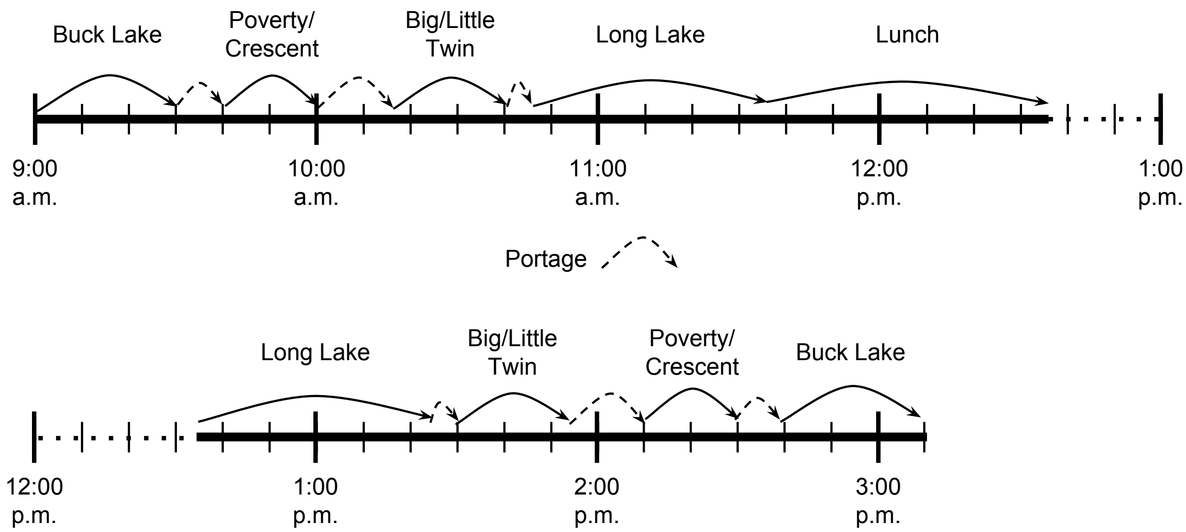
Katie and James spent $125 - 30 = 95$ minutes more paddling than portaging.. Since there are 60 minutes in 1 hour, we can do repeated subtraction to convert minutes into a combination of minutes and hours. In this case, $95 - 60 = 35$. Since $35 < 60$, we cannot subtract 60 any more. So 95 minutes is equal to 1 hour and 35 minutes.

B) The total time for the trip one way is: $125 + 30 = 155$ minutes. The total time for the round trip including lunch is: $155 + 60 + 155 = 370$ minutes. Using repeated subtraction we calculate

$$370 - 60 = 310$$
$$310 - 60 = 250$$
$$250 - 60 = 190$$
$$190 - 60 = 130$$
$$130 - 60 = 70$$
$$70 - 60 = 10$$

Since 60 is repeatedly subtracted 6 times before the difference is less than 60, and last difference is 10, then 370 minutes is equal to 6 hours and 10 minutes. If James and Katie left at 9:00 a.m. then they would get back at 3:10 p.m.

You can also find the time it takes for James and Katie to complete the trip using a timeline.



Teacher's Notes

Converting a total number of minutes for the trip to the format using hours and minutes is another good example of why it is important to be able to calculate the quotient and the remainder when dividing. The *modulo* or *mod* operation that is used in mathematics and in many programming languages computes the same value as the remainder for positive integers. This operation is also part of an area of mathematics known as *modular arithmetic*.

Modular arithmetic describes relationships between integers where the numbers repeat themselves after reaching a certain value. It is often referred to "clock arithmetic" since our use of a 12-hour clock is a very good example of this idea. When we describe time using a 12-hour clock, the maximum number of minutes is 59 and the maximum number of hours is 12. If it is 3:14 right now, then after 60 minutes, it will be 4:14. The number of minutes in both those times is the same, even though 60 minutes has elapsed. If it is 3:14 right now, then in exactly five days, it will also be 3:14. In this example the elapsed time is 120 hours or 7200 minutes, and yet we refer to the time using exactly the same number of minutes and hours. That happens because when we deal with time this way, every 60 minutes we repeat the number of minutes, and every 12 hours we repeat the number of hours.

The CEMC has an activity known as Math Circles which is presented at the University of Waterloo. The materials from past years of this activity are available online at:

`http://www.cemc.uwaterloo.ca/events/mathcircle_presentations.html`

The materials from Fall 2016 on November 8-9 for the Junior Grade 6 students introduce the idea of modular, or clock, arithmetic.

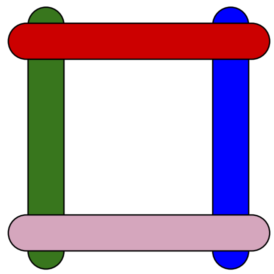
Problem of the Week

Problem A

Crafty Construction

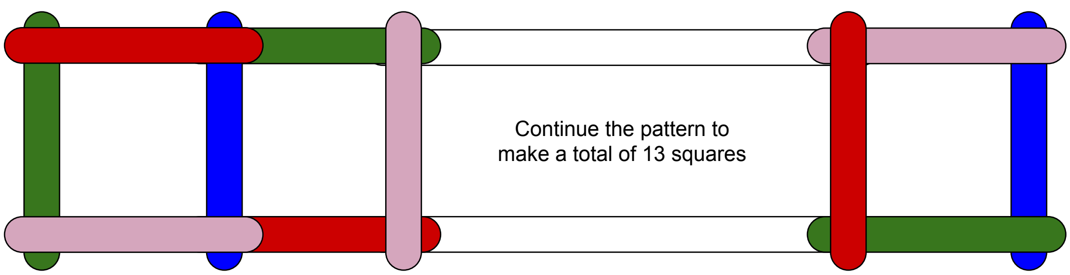
Sam is making square picture frames for his friends and family. He has one box of popsicle sticks and there are 50 sticks in a box. He needs to make 13 frames.

A) If Sam builds individual frames like this:



will he have enough material to build 13 individual frames? Explain your answer.

B) Instead of individual frames, he decides to connect the frames in a line so that any two frames share at most one popsicle stick.



If he built the frames this way, would he have enough sticks in a box of 50 to make connected frames for 13 pictures?

C) Can you draw another layout of the 13 frames where any two frames share at most one popsicle stick?

D) What is the fewest number of popsicle sticks you would need to make connected frames for 13 pictures?

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Crafty Construction

Problem

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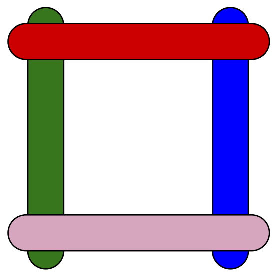
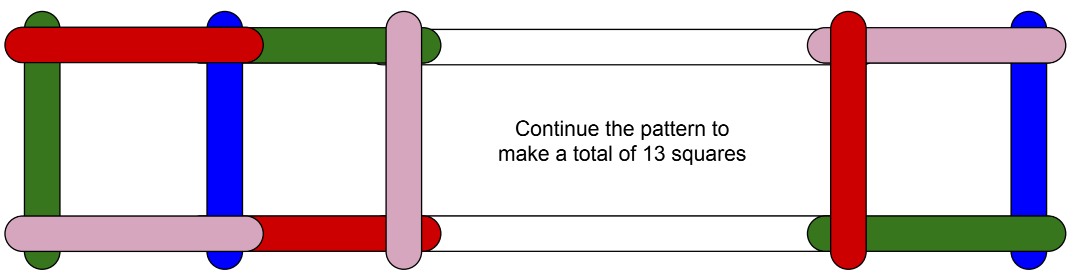


will he have enough material to build 13 individual frames? Explain your answer.

B) Instead of individual frames, he decides to connect the frames in a line so that any two frames share at most one popsicle stick.



If he built the frames this way, would he have enough sticks in a box of 50 to make connected frames for 13 pictures?

C) Can you draw another layout of the 13 frames where any two frames share at most one popsicle stick?

D) Can you find a way of making 13 connected frames that uses less than 35 popsicle sticks?

Solution

A) Since each frame requires 4 sticks, we can calculate the total number of sticks required this way: $13 \times 4 = 52$

We could also make a table showing the number of frames and the number of sticks required and see the following pattern:

# of Frames	# of Sticks
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36
10	40
11	44
12	48
13	52

Either way, since it will take 52 popsicle sticks to build 13 individual frames, Sam does not have enough popsicle sticks in the box.

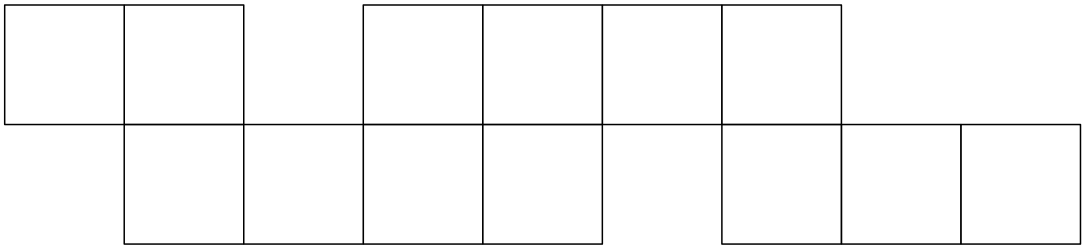
B) Since the first frame takes 4 sticks and the other 12 frames take 3 sticks each, we can calculate the total number of sticks required by adding 4 to 12×3. This gives a total of $4 + 36 = 40$.

Another way to look at this is to make a table to see the following pattern:

# of Frames	# of Sticks
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28
10	31
11	34
12	37
13	40

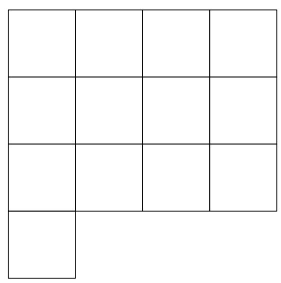
Either way, since it will take 40 popsicle sticks to build 13 connected frames, Sam does have enough popsicle sticks in the box.

C) There are many designs you could make to hold 13 pictures. Here is one:



This one takes only 39 popsicle sticks.

D) The most efficient use of popsicle sticks is to share as many sides as possible with other frames. If you connect the frames to form a shape as close as possible to a square, you can make 13 frames using only 34 popsicle sticks. For example:



Teacher's Notes

Optimization problems appear everywhere in real life, and mathematicians have many techniques to help maximize or minimize some process. Businesses try to minimize costs, farmers try to maximize yield, people try to minimize the time they spend doing things they do not like and maximize the time they spend doing things they do like. Areas of advanced mathematical study such as calculus and linear programming can be used to solve some optimization problems. The key to being able to use these techniques is to translate the real world problem into a mathematical form. This process is called abstraction, and is an essential element of mathematics.

At the end of this particular problem students are asked to minimize the number of popsicle sticks necessary to frame 13 pictures. Another similar optimization problem is to find the maximum area of a shape given a fixed perimeter. For example, what would the maximum area of a quadrilateral be if its perimeter is 100 cm? It can be proven (although not easily) that the solution in this case will be a square. You could try drawing different rectangles that have a perimeter of 100 cm to see that none of them have an area greater than the square that has sides with length 25 cm each. In the general case, to maximize the area of a shape that has n sides and a fixed perimeter you must form a *regular polygon*. A shape where the lengths of all of the sides are equal is known as a regular polygon.

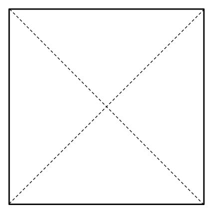
Problem of the Week

Problem A

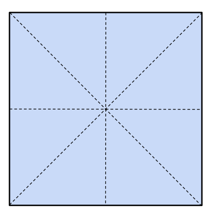
Origami

Laila likes to do origami which is the art of paper folding. Just by folding paper in a particular way, she can make all sorts of different animals. Many of the animals start with the same steps.

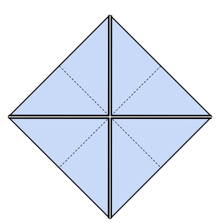
Laila starts with a square piece of paper. She folds it in half to form a triangle. Then she opens up the paper to start with a square again. She folds it in the opposite direction and also forms a triangle. When she opens it up again she can see creases on the paper that look like this:



Then she turns the paper over and folds it in half to form a rectangle. She opens up the paper and folds it in the opposite direction to form another rectangle. When she opens up the paper this time, she sees creases in the paper that look like this:



The centre of the square is the point where all of the creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another square.



What fraction of the area of the original square is the area of the smaller square? Justify your answer.

Strands Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Origami

Problem

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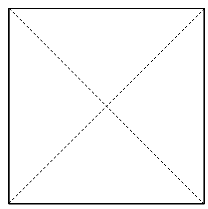
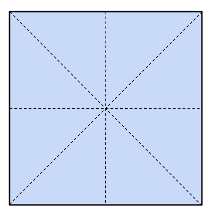
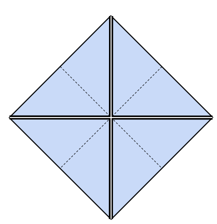


Then she turns the paper over and folds it in half to form a rectangle. She opens up the paper and folds it in the opposite direction to form another rectangle. When she opens up the paper this time, she sees creases in the paper that look like this:



The centre of the square is the point where all of the creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another square.



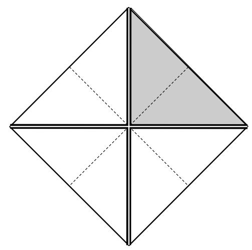
What fraction of the area of the original square is the area of the smaller square? Justify your answer.

Solution

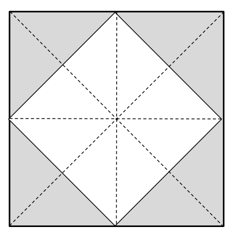
The smaller square has an area that is $\frac{1}{2}$ the area of the original square. There are several ways in which you can justify this.

Solution 1:

One way is to look at the following picture. Notice that the shaded triangle covers an area underneath it that is exactly the same size. That is true for all four of the triangles that have their points meet at the middle.



If we opened up the paper again, we could make the following observations:

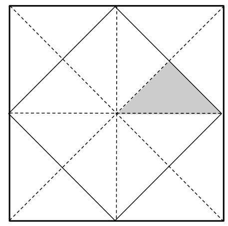


- The shaded parts of the original square each have a matching unshaded part.
- The shaded parts make up the area of the smaller square.
- This means that the area of the original square is 2 times the area of the smaller square.

It follows that the area of the smaller square is $\frac{1}{2}$ the area of the original square.

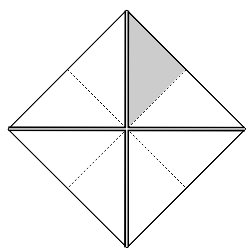
Solution 2:

Another way to show that the area of the smaller square is $\frac{1}{2}$ the area of the original square is to count the number of small triangles that are formed by the creases in the paper. One such triangle is shaded in the following diagram.



Each of these triangles have the same area. One way of showing that they all have the same area would be to cut up the square into the triangles and stack them on top of each other. If you count the number of small triangles in the original square there are 16 of them.

If you count the number of small triangles in the smaller square, there are 8 of them.



Since the smaller square is formed by half the number of small triangles as compared to the original square, then the smaller square has $\frac{1}{2}$ the area of the original square.

Teacher's Notes

Origami can be used to demonstrate many different geometric shapes, especially triangles and quadrilaterals. The instructions in this problem start with a square and then make folds to form a triangle, a rectangle, and then a smaller square.

There are many books and websites that can show you the steps required to create simple and complex origami figures. If you follow the steps to creating complex figures, you may see several different polygons, including a parallelogram, a trapezoid, and a *kite*. A kite is a quadrilateral with adjacent sides that are the same length.

Both the triangle formed in the first diagonal fold, and the smaller triangles formed by the creases are all right-angled, isoceles triangles. It is relatively easy to confirm that the triangle from the first diagonal fold satisfies this condition. Since you start with a square, and the interior angles of a square are all 90 degrees, then the corner of the triangle where the points of the square meet must be 90 degrees. Therefore the triangle is a right-angled triangle. Since the other two sides of the triangle are sides of the original square, those sides must be equal. Therefore it is an isoceles triangle. Also, we can determine the size of the other two interior angles of this triangle with logical deduction. When you fold the square on the diagonal and then open it up again, the crease that you see has bisected the angle in the corner of the square. Since that angle is 90 degrees, then the angle formed by the side of the square and the crease is half that size, which is 45 degrees. It would take a longer argument to show that the 16 smaller triangles are also right-angled, isoceles triangles. You could convince yourself it is true by cutting up the square into the 16 pieces and comparing the sides of a pair of triangles. You could also check that one corner of one of the triangles aligns with the corner of a square or a rectangle to confirm that its angle is 90 degrees.

Being able to make logical deductions in geometry is important. Finding the correct answer to a problem is not the only thing mathematicians and computer scientists care about. They are also concerned with describing the process for finding the correct answer and **proving** that the answer is correct. Problems in geometry can be a good place to practice these skills.

Problem of the Week
Problem A
What Would You Buy?

School Council donates \$75.00 to your school to purchase soccer balls, basketballs, skipping ropes, and frisbees. In order to be fair, you must purchase at least 2 of each type of equipment, but no more than 8 of each type.
The prices are:

- Soccer balls: \$4.90
- Basketballs: \$5.85
- Skipping ropes: \$3.50
- Frisbees: \$2.00

A) What would you buy to use as much of the \$75.00 donation as possible?

B) How much money would be left over?

Strand Number Sense and Numeration

Problem of the Week
Problem A and Solution
What Would You Buy?

Problem

School Council donates \$75.00 to your school to purchase soccer balls, basketballs, skipping ropes, and frisbees. In order to be fair, you must purchase at least 2 of each type of equipment, but no more than 8 of each type.
The prices are:

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- Basketballs: \$5.85
- Skipping ropes: \$3.50
- Frisbees: \$2.00

A) What would you buy to use as much of the \$75.00 donation as possible?

B) How much money would be left over?

Solution

Since you need to buy at least two of each piece of equipment, start by calculating the cost of two soccer balls, two basketballs, two skipping ropes, and two frisbees. There are several ways of calculating this total. Here is one way:

- Two soccer balls cost: $2 \times 4.90 = \$9.80$
- Two basketballs cost: $2 \times 5.85 = \$11.70$
- Two skipping ropes cost: $2 \times 3.50 = \$7.00$
- Two frisbees cost: $2 \times 2.00 = \$4.00$

So the total spent on two of each item is: $9.80 + 11.70 + 7.00 + 4.00 = \$32.50$

Another way to calculate this total is to figure out how much it costs for one of each item: $4.90 + 5.85 + 3.50 + 2.00 = \$16.25$
If we want to buy two of each, then the cost will double: $2 \times 16.25 = \$32.50$

Either way, the total cost of two of each type of equipment is \$32.50.
So, there is $75.00 - 32.50 = \$42.50$ left to spend.

One way to spend exactly \$42.50 is to buy two more soccer balls, two more basketballs, and six more skipping ropes, since:

- Two soccer balls cost: $2 \times 4.90 = \$9.80$
- Two basketballs cost: $2 \times 5.85 = \$11.70$
- Six skipping ropes cost: $6 \times 3.50 = \$21.00$

So the total spent on these extra items is: $9.80 + 11.70 + 21.00 = \$42.50$
So, if we bought four soccer balls, four basketballs, eight skipping ropes, and two frisbees, we would spend exactly \$75.00. No money would be left over.

Another way to spend exactly \$42.50 is to purchase five more soccer balls, four more skipping ropes, and two more frisbees, since:

- Five soccer balls cost: $5 \times 4.90 = \$24.50$
- Four skipping ropes cost: $4 \times 3.50 = \$14.00$
- Two frisbees cost: $2 \times 2.00 = \$4.00$

So the total spent on these extra items is: $24.50 + 14.00 + 4.00 = \$42.50$
In this case we would buy seven soccer balls, two basketballs, six skipping ropes, and four frisbees, and spend exactly \$75.00.

Teacher's Notes

There is no simple way find the solutions to this problem. One possibility would be to try all combinations showing how many of each item you might buy, and then find the values that would have a total purchase cost that is as close to \$75.00 as possible without going over that amount. However, since you can buy between 2 and 8 of each item, there are $7^4 = 2401$ different combinations to check. This is a very typical type of problem in the real world, where you have many variables and some restrictions on those variables, and you need to optimize your resources. Advanced mathematical techniques such as linear programming can be used to solve these kinds of problems.

Another way solve this problem is to guess and check. If you make educated guesses, you can often reduce the number of combinations that need to be checked. When we do not have a clear, deterministic way of solving a problem, we can use *heuristic* methods. A heuristic is any problem solving technique that is not necessarily precise, but will produce a solution that is acceptable in a reasonable amount of time.

Let's try to reduce the number of guesses we have to make. We can hope that there is a solution where we spend exactly \$75.00. This would mean that after buying the first two of each item, we need to spend exactly \$42.50. Now we can check only the combinations that would give us a total cost that ends with 0.50. Let's investigate the multiples of each of our prices.

- Any multiple of 2.00 will be a whole number.
- The even multiples of 3.50 will be whole numbers; the odd multiples will be values that end with 0.50. If we buy an odd number of skipping ropes, then we can look for a total cost of the rest of the items we purchase that is a whole number of dollars.
- The multiples of 4.90 to consider are: 4.90, 9.80, 14.70, 19.60, 24.50, and 29.40
- The multiples of 5.85 to consider are: 5.85, 11.70, 17.55, 23.40, 29.25, and 35.10

The maximum amount we can spend on skipping ropes and frisbees is:

$$(6 \times 3.50) + (6 \times 2.00) = 21.00 + 12.00 = 33.00$$

since the most we can buy is six more of each item. This is much less than we have to spend, so we need to buy at least one soccer ball or basketball. Now we can focus on combinations of multiples of these two items that total a whole number of dollars or have a total that ends with 0.50. This is a much smaller number of combinations to check. We see that:

- $5 \times 4.90 = 24.50$, which comes from 5 soccer balls.
- $9.80 + 11.70 = 21.50$, which comes from 2 soccer balls and 2 basketballs.
- $4.90 + 35.10 = 40.00$ which comes from 1 soccer ball and 6 basketballs.
- $19.60 + 23.40 = 43.00$, which is more than we have to spend.
- $29.40 + 35.10 = 64.50$, which is more than we have to spend.

So now we know that any combination that gets us to exactly \$42.50 will involve either 5 soccer balls, a combination of 2 soccer balls and 2 basketballs, or a combination of 1 soccer ball and 6 basketballs. We can eliminate the last combination by observing that after making that purchase, we only have \$2.50 left to spend. This means we could only buy a skipping rope which cost \$2.00. This total would be \$74.50.

When we look for a solution that involves one of the remaining combinations of soccer balls and basketballs we have a new problem to solve.
In the first case, we still have to spend $42.50 - 24.50 = \$18.00$
In the second case, we still have to spend $42.50 - 21.50 = \$21.00$
So in both cases we are looking at differences that are a whole number of dollars, and we only have skipping ropes and frisbees left to buy. A whole number total can only come from purchasing an even number of skipping ropes and some number frisbees. This is a relatively small number of combinations to check.

Using a combination of educated guesses, and a process that leads to smaller problems that need to be solved, we can find our two solutions.

Patterning & Algebra

TAKE ME TO THE COVER

Problem of the Week

Problem A

Friendship Bracelets

Naomi is making bracelets to raise money for the hospital in her town. On her first day of bracelet making, Naomi makes 7 bracelets. Each day after, Naomi makes one more bracelet than she did the day before.

A) How many bracelets has she made after 7 days?

B) If the materials for a single bracelet cost \$2.50, and she sells each bracelet for \$4.50, how much money will she be able to donate to the hospital if she sells all of the bracelets?

C) If she wants to raise at least \$200.00 for the hospital, and she continued to make bracelets at the same rate (making one more bracelet each day), how many more days does she have to make bracelets?

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Friendship Bracelets

Problem

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c) If she wants to raise at least \$200.00 for the hospital, and she continued to make bracelets at the same rate (making one more bracelet each day), how many more days does she have to make bracelets?

Solution

a) We can make a table of the pattern:

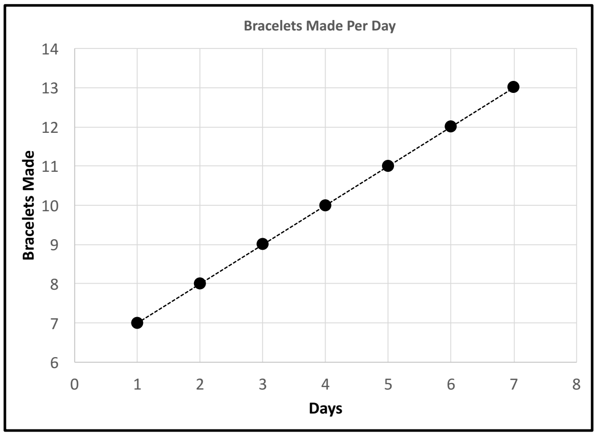
Day	1		2		3		4		5		6		7	
Bracelets	7	+	8	+	9	+	10	+	11	+	12	+	13	= 70 bracelets

b) Naomi makes a profit of $\$4.50 - \$2.50 = \$2.00$ for each bracelet. This means for 70 bracelets, her profit is $70 \times \$2.00 = \$140.00$.

c) To get to \$200.00, Naomi needs to earn $\$200.00 - \$140.00 = \$60.00$ more after day 7. If the pattern continues, on day 8 Naomi would make 14 bracelets for a profit of $14 \times \$2.00 = \$28.00$. Then on day 9 Naomi would make 15 bracelets for a profit of $15 \times \$2.00 = \$30.00$. This is a total of $\$28.00 + \$30.00 = \$58.00$. That is not quite enough to make a total of \$200.00. Naomi needs to make at least one more bracelet. So it would take her 10 days to earn a profit of at least \$200.00.

Teacher's Notes

The pattern seen in part (a) represents a linear relationship between the number of days and the number of bracelets made on that day. Using the table we could make a chart where the horizontal axis marks the days, and the vertical axis marks the number of bracelets made that day, and plot a point for each entry in the table.

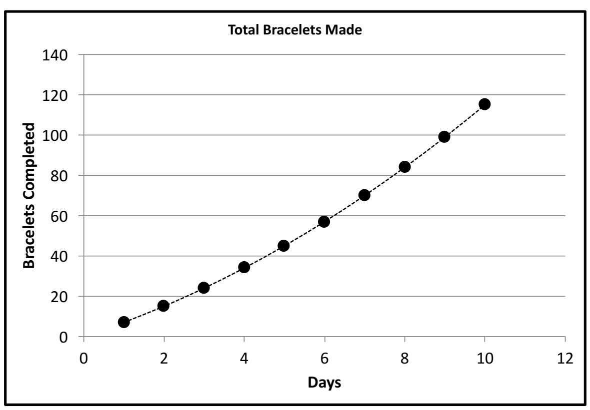


If we connect the points, they form a straight line showing a *linear* relationship. We can even write an equation representing the line:

$$b = 6 + d$$

where d represents the day, and b represents the number of bracelets made on day d.

We could also calculate the total number of bracelets made at the end of each day. For example, after the first day Naomi made 7 bracelets. At the end of the second day she made $7 + 8 = 15$ bracelets. If we calculate the totals for each day and then plot the points on a chart and connect them, we see that they form a curve rather than a straight line.



The result shows a *quadratic* relationship between the number of days and the total number of bracelets made. Again, we can write an equation representing the curve:

$$t = \frac{d^2 + 13d}{2}$$

where d represents the day, and t represents the total number of bracelets made up to day d. This relationship is quadratic because we can describe the relationship using an equation that includes the number of days squared (d^2) and the curved line that is formed is part of a shape known as a parabola.

Problem of the Week

Problem A

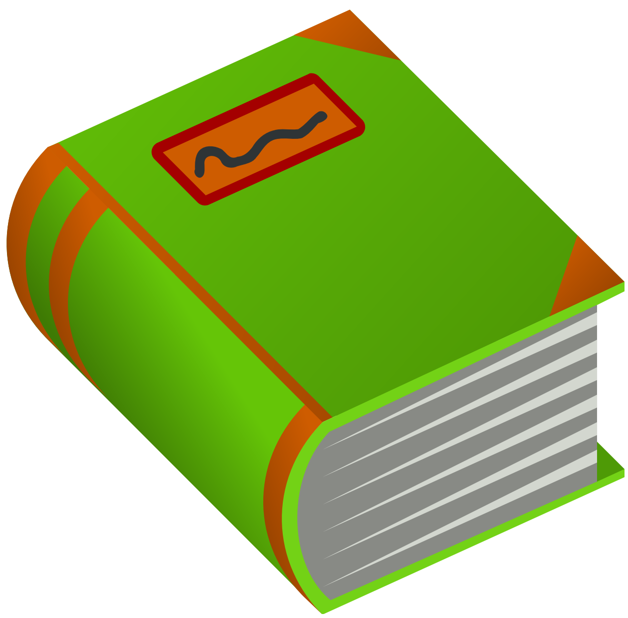
Money Management

Adam only saves quarters and dimes. He has already saved 13 quarters and 5 dimes. In Canada, one quarter equals 25 cents, one dime equals 10 cents, and 100 cents equals $1 (one dollar).

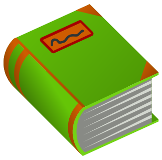
He wants to purchase a book that costs $7.75.

Not including the money he already has, what combinations of quarters and dimes could Adam save so he has exactly enough money to buy the book?

Show your thinking.



Strands Number Sense and Numeration, Patterning and Algebra



Problem of the Week
Problem A and Solution
Money Management

Problem

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He wants to purchase a book that costs \$7.75.

Not including the money he already has, what combinations of quarters and dimes could Adam save so he has exactly enough money to buy the book?

Show your thinking.

Solution

Adam already has:

$$\begin{array}{llcr} & \text{13 quarters} & = & \$3.25 \\ + & \text{5 dimes} & = & \$0.50 \\ \hline & & & \$3.75 \end{array}$$

So Adam needs $\$7.75 - \$3.75 = \$4.00$ to buy the book. Since the total number of cents required (400) is an even number, and Adam only has dimes and quarters, if he uses quarters to pay for the book he must use an even number of quarters. Note that 5 dimes and 2 quarters both equal 50¢. The first combination calculates the total using all quarters. As we move down the table, we can remove two quarters each time and replace them with five dimes. This keeps the total the same, and is a good way to make sure all possible combinations have been considered.

Combination	10¢ (dime)	25¢ (quarter)
1	0	16
2	5	14
3	10	12
4	15	10
5	20	8
6	25	6
7	30	4
8	35	2
9	40	0

Therefore, there are 9 different combinations of quarters and dimes that Adam could collect to equal four dollars.

Teacher's Notes

The solution to this problem looks very much like a table of values for an equation that uses two variables. We can describe relationships between two or more values using equations. For example:

$$y = 5x + 1$$

describes a relationship between the variables x and y. Essentially, for any value we choose for x, the value of y will be one more than five times the value chosen. From this equation we can generate a table that contains a sample of the values for x and y that satisfy the relationship. You can create the table by picking a random value for one of the variables (known as the *independent* variable) and that will determine the value of the other variable (known as the *dependent* variable). If we pick the values 1, 2, 3, 4, and 5 for x, we get the following table:

x	y
1	6
2	11
3	16
4	21
5	26

We can write the relationship between the number of dimes and number of quarters needed to have enough money to purchase the book as an equation as well. If we use a variable d to represent the number of dimes Adam may use, and a variable q to represent the number of quarters he may use, we can write the following equation:

$$10d + 25q = 400$$

since each dime is worth 10¢, each quarter is worth 25¢, and Adam needs 400¢ more to buy the book.

Looking at each combination row in the table, we have values for d and q that make this equation true. For example, using combination 4, we can substitute 15 for d and 10 for q to get:

$$10(15) + 25(10) = 150 + 250 = 400$$

The equation describing this problem has a couple of restrictions that are not necessarily part of all equations. Since the values of the variables are representing physical coins, then the values can not be negative numbers, nor can they be fractions. The possible values for d are between 0 and 40. The possible values for q are between 0 and 16. The possible values for variables can be described as the *domain* and *range*. Since all values for these variables must be whole numbers, then this equation is describing a *discrete function*.

All of these concepts: independent variable, dependent variable, domain, range, and discrete functions are investigated in mathematics courses students may study in the future.

Problem of the Week

Problem A

Granny's Quilt

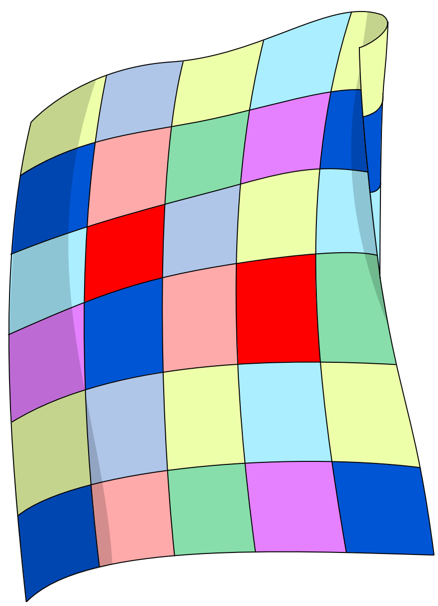
Granny is making a quilt for her new grandchild. She has 5 different rolls of fabric: green, blue, red, orange, yellow. Each square of her quilt must have 3 different colours.

Granny wants each square to be a unique combination of colours.

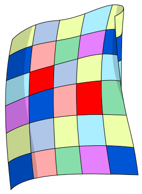
A) What is the maximum number of squares she can have in her quilt?

B) How many times does red appear in a square?

C) Would the answer above change for a different colour? Why?



Strand Patterning and Algebra



Problem of the Week

Problem A and Solution

Granny's Quilt

Problem

Granny is making a quilt for her new grandchild. She has 5 different rolls of fabric: green, blue, red, orange, yellow. Each square of her quilt must have 3 different colours.

Granny wants each square to be a unique combination of colours.

A) What is the maximum number of squares she can have in her quilt?

B) How many times does red appear in a square?

C) Would the answer above change for a different colour? Why?

Solution

A) There are 10 different combinations of three colours, chosen from five colours, assuming that the order of the colours does not matter.

- blue, green, red
- blue, green, orange
- blue, green, yellow
- blue, orange, red
- blue, orange, yellow
- blue, red, yellow
- green, orange, red
- green, orange, yellow
- green, red, yellow
- orange, red, yellow

B) Red appears in patterns six times each.

C) Since each colour has the same chance of appearing the square, it does not matter which colour you pick. Each colour appears six times in the patterns.

Teacher's Notes

It is relatively easy to calculate how many different combinations you can make by selecting three colours out of five choices. If you have to select three different items then you have all 5 options for the first selection, 4 choices left for the second selection, and 3 choices left for the last selection. This means you have $5 \times 4 \times 3 = 60$ possible selections.

However, if the order of the colours you select does not matter, within those 60 selections there are many duplicates. For example, selecting blue, then red, then orange, results in the same three colours as selecting orange, then red, then blue. So you need to eliminate these duplicates. When arranging three different items into specific positions, you have 3 choices for the position of the first item, then you have 2 choices for the position of the second item, and only 1 position left for the last item. This means you have $3 \times 2 \times 1 = 6$ different arrangements of three items. So for each set of three colours in your selection, there is a total of 6 different arrangements of those colours. Eliminating duplicates, there are $60 \div 6 = 10$ combinations of three different colours out of five choices.

Some people may find actually listing all of the selections in a way that ensures that you do not miss or duplicate one of the combinations more difficult. Here is one way. Make a table with five columns, where the header of the columns are the colour choices. In each row, you can use the digit 1 to indicate you are selecting that colour and the digit 0 to indicate you are not selecting that colour. Each row needs to have three 1s and two 0s that form a five digit number representing a three colour selection. Place the five digit numbers in the table in ascending order like this:

B	G	O	R	Y
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0

These numbers represent the 10 different combinations of colours for the quilt.

Problem of the Week
Problem A
Colin's Coloured Blocks

Colin arranges blocks in a 12 × 12 grid. He starts in position A1 then follows a snake pattern to fill the grid. When he reaches the end of the first row, he continues filling up the second row from right to left. When he reaches the third row, he fills from left to right and so on. All the odd numbered rows are filled from left to right and the even numbered rows are filled from right to left.

Colin uses yellow, blue, red and green blocks to fill the grid. He starts with 1 yellow block, then 3 blue blocks, then 5 red blocks, then 7 green blocks. He continues the pattern starting with yellow blocks again. Each change of colour has 2 more blocks than the previous colour. The beginning of the pattern is shown here:

	A	B	C	D	E	F	G	H	I	J	K	L
1	Y	B	B	B	R	R	R	R	R	G	G	G
2	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
3	Y											
4												
5												
6												
7												
8												
9												
10												
11												
12												

A) What is the colour of the block at position F8 on the grid?

B) What is the colour of the block at position J9 on the grid?

C) What is the colour of the block at position B11 on the grid?

D) If Colin starts with 50 blocks of each colour, how many yellow, blue, red, and green blocks does he have left over after filling the grid?

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Colin's Coloured Blocks

Problem

Colin arranges blocks in a 12×12 grid. He starts in position A1 then follows a snake pattern to fill the grid. When he reaches the end of the first row, he continues filling up the second row from right to left. When he reaches the third row, he fills from left to right and so on. All the odd numbered rows are filled from left to right and the even numbered rows are filled from right to left.

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A) What is the colour of the block at position F8 on the grid?

B) What is the colour of the block at position J9 on the grid?

C) What is the colour of the block at position B11 on the grid?

D) If Colin starts with 50 blocks of each colour, how many yellow, blue, red, and green blocks does he have left over after filling the grid?

Solution

After filling up the grid it looks like this:

	A	B	C	D	E	F	G	H	I	J	K	L
1	Y	B	B	B	R	R	R	R	R	G	G	G
2	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
3	Y	B	B	B	B	B	B	B	B	B	B	B
4	R	R	R	R	R	R	R	R	R	R	R	R
5	R	G	G	G	G	G	G	G	G	G	G	G
6	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
7	Y	Y	Y	Y	Y	Y	Y	Y	Y	B	B	B
8	B	B	B	B	B	B	B	B	B	B	B	B
9	B	B	B	B	R	R	R	R	R	R	R	R
10	R	R	R	R	R	R	R	R	R	R	R	R
11	R	G	G	G	G	G	G	G	G	G	G	G
12	G	G	G	G	G	G	G	G	G	G	G	G

A) F8 is blue.

B) J9 is red.

C) B11 is green.

D) The total number of yellow blocks used is $1 + 9 + 17 = 27$.
The total number of blue blocks used is $3 + 11 + 19 = 33$.
The total number of red blocks used is $5 + 13 + 21 = 39$.
The total number of green blocks used is $7 + 15 + 23 = 45$.

If Colin starts with 50 blocks of each colour, he will have:

$50 - 27 = 23$ yellow blocks left over,
$50 - 33 = 17$ blue blocks left over,
$50 - 39 = 11$ red blocks left over, and
$50 - 45 = 5$ green blocks left over.

Teacher's Notes

Calculating the number of blocks for each colour is an example of determining the value of an *arithmetic series*. An arithmetic series is a sum, where the terms in the sum have a common difference. For example, in this arithmetic series $1 + 9 + 17 = 27$, the difference between the first and second terms is 8, and the difference between the second and third terms is 8. In fact, all of the series used to determine the total number of blocks of each colour have a common difference of 8. This common difference can be explained by the fact that we repeat the colours in a regular pattern. In between repeating the colours, we increase the number of blocks in the pattern by 2. We switch colours 4 times before returning to the same colour in the pattern. Since $2 \times 4 = 8$, the difference in the number of blocks we will use the next time with the same colour will be 8.

There is a formula for calculating the result of an arithmetic sequence.

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

where n is the number of terms in the sequence, a_1 is the value of the first term, and d is the common difference. Using this formula we could calculate the total number of red blocks for example. In this case, we would have to figure out that Colin uses 5 red blocks the first time (a_1), and that red blocks are used 3 times (n). We know that the common difference is 8 (d). Then we calculate the sum as:

$$\begin{aligned} S_n &= \frac{(3)}{2}[2(5) + ((3)-1)(8)] \\ &= \frac{3}{2}[10 + (2)(8)] \\ &= \frac{3}{2}[26] \\ &= (3)(13) \\ &= 39 \end{aligned}$$

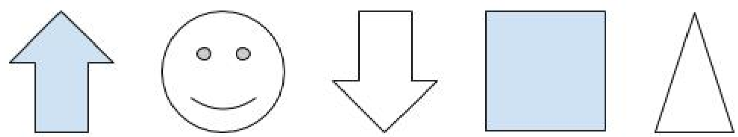
Adding up three terms is probably easier than using the formula to calculate the number of blocks in this problem. However, if we wanted to know how many block are used if the pattern continues for another 50 cycles, the formula would be useful.

Problem of the Week

Problem A

Passive Patterns

The core of a pattern is the shortest string of objects that repeat in a given order. Below we have the core of a pattern that contains five objects.



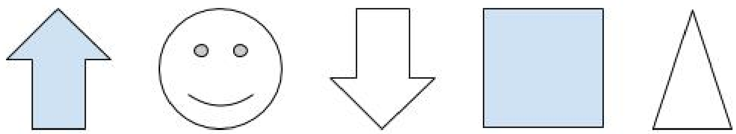
What would the 15^{th} object be in this pattern? the 28^{th}?, the 101^{st}?

Strand Patterning and Algebra

Problem of the Week
Problem A and Solution
Passive Patterns

Problem

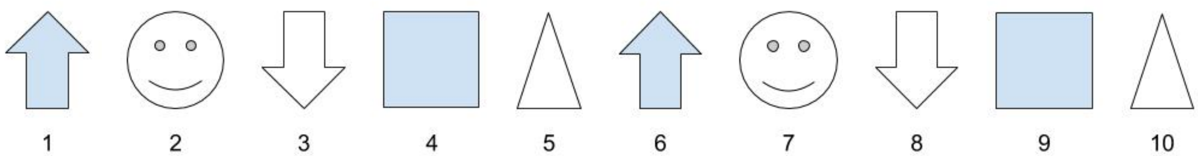
The core of a pattern is the shortest string of objects that repeat in a given order. Below we have the core of a pattern that contains five objects.



What would the 15^{th} object be in this pattern? the 28^{th}?, the 101^{st}?

Solution

You may draw out the shapes and begin to realize the repetition with the pattern as follows:



If you continue numbering, then every shape whose position is a multiple of 5 will be a triangle. So the 15^{th} shape will be a triangle.

Shapes 1, 6, 11, 16, 21, etc. will all be up arrows. In other words, any numbers that end with a 1 or a 6 in the pattern sequence will be an up arrow.
Shapes 2, 7, 12, 17, 22, etc. will all be smiley faces. In other words, any numbers that end with a 2 or a 7 in the pattern sequence will be a smiley face.
Shapes 3, 8, 13, 18, 23, etc. will all be a down arrow. In other words, any numbers that end with a 3 or an 8 in the pattern sequence will be a down arrow.
Shapes 4, 9, 14, 19, 24, etc. will all be a square. In other words, any numbers that end with a 4 or a 9 in the pattern sequence will be a square.

So the 18^{th} shape will be a down arrow and the 101^{st} shape will be an up arrow.

Teacher's Notes

Determining which shape appears at the various positions can be accomplished using the *mod* operation. For positive integers, calculating *mod* is the same as finding the remainder in division. For example, 17 mod 5 = 2, since when we divide 17 by 5 we get a remainder of 2. This is helpful for this problem, since the positions that have the same value *mod* 5 will have the same shape. In other words, since from the core of the pattern:

1 mod 5 = 1, and the shape at position 1 is an up arrow,
2 mod 5 = 2, and the shape at position 2 is a smiley face,
3 mod 5 = 3, and the shape at position 3 is a down arrow,
4 mod 5 = 4, and the shape at position 4 is a square, and
5 mod 5 = 0, and the shape at position 5 is a triangle,

to determine the shape at some random position k, we calculate k mod 5.

Whatever answer we get must be a number between 0 and 4, since the remainder of a division must be smaller than the divisor. We match the result of this calculation with the core results. Whatever number we get determines the shape at that position.

Since 15 mod 5 = 0, the shape at the 15^{th} position is a triangle.
Since 18 mod 5 = 3, the shape at the 18^{th} position is a down arrow.
Since 101 mod 5 = 1, the shape at the 101^{st} position is an up arrow.
If we wanted to know what shape is at position 32836749, we calculate 32836749 mod 5 = 4, which tells us there is a square at that position.

If the length of the core changed, that would change the number that follows *mod* in our calculation. For example, if we had a core that had 8 different shapes repeated, then our calculations would be k mod 8. The *mod* operation is used in many applications in mathematics and computer science.

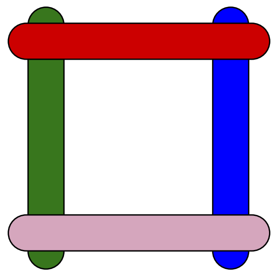
Problem of the Week

Problem A

Crafty Construction

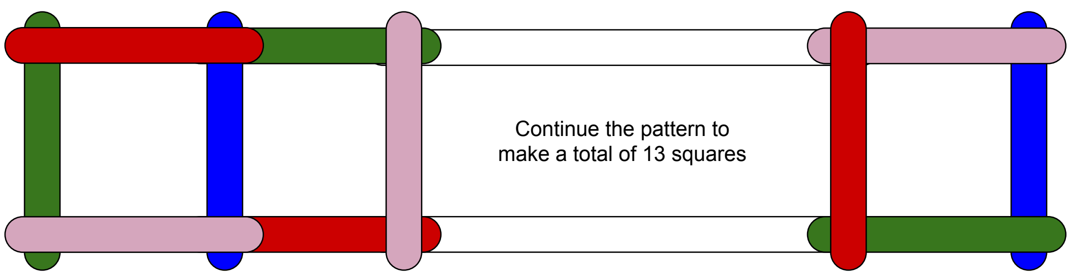
Sam is making square picture frames for his friends and family. He has one box of popsicle sticks and there are 50 sticks in a box. He needs to make 13 frames.

A) If Sam builds individual frames like this:



will he have enough material to build 13 individual frames? Explain your answer.

B) Instead of individual frames, he decides to connect the frames in a line so that any two frames share at most one popsicle stick.



If he built the frames this way, would he have enough sticks in a box of 50 to make connected frames for 13 pictures?

C) Can you draw another layout of the 13 frames where any two frames share at most one popsicle stick?

D) What is the fewest number of popsicle sticks you would need to make connected frames for 13 pictures?

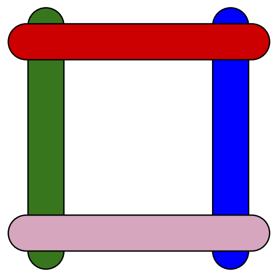
Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Crafty Construction

Problem

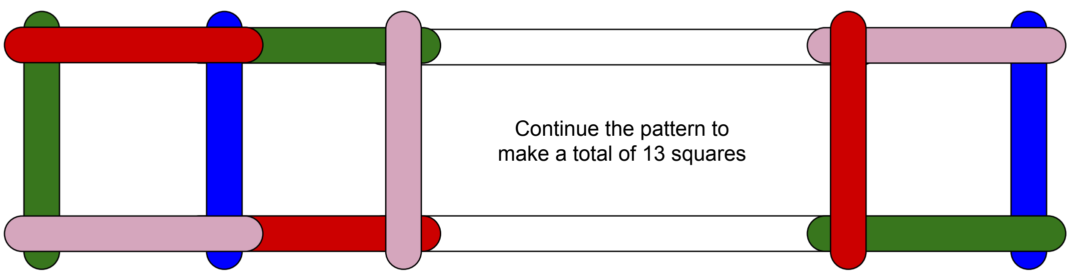
Sam is making square picture frames for his friends and family. He has one box of popsicle sticks and there are 50 sticks in a box. He needs to make 13 frames.

A) If Sam builds individual frames like this:



will he have enough material to build 13 individual frames? Explain your answer.

B) Instead of individual frames, he decides to connect the frames in a line so that any two frames share at most one popsicle stick.



If he built the frames this way, would he have enough sticks in a box of 50 to make connected frames for 13 pictures?

C) Can you draw another layout of the 13 frames where any two frames share at most one popsicle stick?

D) Can you find a way of making 13 connected frames that uses less than 35 popsicle sticks?

Solution

A) Since each frame requires 4 sticks, we can calculate the total number of sticks required this way: $13 \times 4 = 52$

We could also make a table showing the number of frames and the number of sticks required and see the following pattern:

# of Frames	**# of Sticks**
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36
10	40
11	44
12	48
13	52

Either way, since it will take 52 popsicle sticks to build 13 individual frames, Sam does not have enough popsicle sticks in the box.

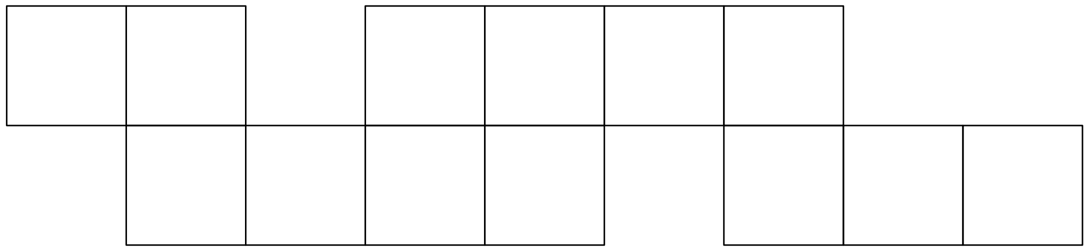
B) Since the first frame takes 4 sticks and the other 12 frames take 3 sticks each, we can calculate the total number of sticks required by adding 4 to 12×3. This gives a total of $4 + 36 = 40$.

Another way to look at this is to make a table to see the following pattern:

# of Frames	**# of Sticks**
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28
10	31
11	34
12	37
13	40

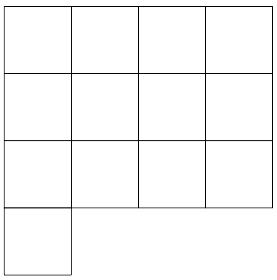
Either way, since it will take 40 popsicle sticks to build 13 connected frames, Sam does have enough popsicle sticks in the box.

C) There are many designs you could make to hold 13 pictures. Here is one:



This one takes only 39 popsicle sticks.

D) The most efficient use of popsicle sticks is to share as many sides as possible with other frames. If you connect the frames to form a shape as close as possible to a square, you can make 13 frames using only 34 popsicle sticks. For example:



Teacher's Notes

Optimization problems appear everywhere in real life, and mathematicians have many techniques to help maximize or minimize some process. Businesses try to minimize costs, farmers try to maximize yield, people try to minimize the time they spend doing things they do not like and maximize the time they spend doing things they do like. Areas of advanced mathematical study such as calculus and linear programming can be used to solve some optimization problems. The key to being able to use these techniques is to translate the real world problem into a mathematical form. This process is called abstraction, and is an essential element of mathematics.

At the end of this particular problem students are asked to minimize the number of popsicle sticks necessary to frame 13 pictures. Another similar optimization problem is to find the maximum area of a shape given a fixed perimeter. For example, what would the maximum area of a quadrilateral be if its perimeter is 100 cm? It can be proven (although not easily) that the solution in this case will be a square. You could try drawing different rectangles that have a perimeter of 100 cm to see that none of them have an area greater than the square that has sides with length 25 cm each. In the general case, to maximize the area of a shape that has n sides and a fixed perimeter you must form a *regular polygon*. A shape where the lengths of all of the sides are equal is known as a regular polygon.