RECALL:

Last class we looked at a) how to optimize (maximize) the area of a rectangle with a given perimeter & b) how to optimize (minimize) the perimeter of a rectangle given its area.

Success Criteria:

Aer last class, you should...

- be able to determine the dimensions and maximum area of a rectangle given a fixed perimeter when all four sides are enclosed.
- be able to determine the dimensions and maximum area of a rectangle given a fixed perimeter when only three sides are enclosed.
- be able to determine the dimensions and minimum perimeter of a rectangle given a fixed area.
Have you achieved last day's success criteria?

ENTRANCE CARD:

Test your understanding by answering the following quesons...

1. What is the smallest **perimeter** possible for a rectangle with an area of 36 m$^2$?
   
   \[
   A = 36 \quad A = l \cdot w \quad \sqrt{36} = l \quad P = 4l
   \]
   
   \[
   36 = l^2 \quad 6 = l \quad P = 2(6 + 6) = 24 \text{ m}
   \]

2. What is the largest **area** possible for a rectangle with a perimeter of 28 m?
   
   \[
   l = \frac{P}{4} = \frac{28}{4} = 7 \quad \therefore A = l \cdot w = 49 \text{ m}^2
   \]

3. Brandon wants to create an enclosure for his puppy in his backyard. He wants to use the side of the house as one side of the enclosure, so he only needs to fence 3 sides of the enclosure. He has 12 m of fencing. What are the **dimensions** and the **area** of the rectangle with maximum area?

   \[
   l = 2w
   \]

   \[
   \therefore l = \frac{P}{2} \quad \text{and} \quad w = \frac{P}{4}
   \]

   \[
   = \frac{12}{2} \quad = \frac{12}{4}
   \]

   \[
   = 6 \text{ m} \quad = 3 \text{ m}
   \]

   \[
   \therefore A = l \cdot w = 18 \text{ m}^2
   \]
TODAY:
Success Criteria:
By the end of today's lesson can you...

• determine the minimum surface area of a square-based prism given a fixed volume.

• determine the maximum volume of a square-based prism given a fixed surface area.
Investigation A:

How can you compare the surface areas of square-based prisms with the same volume?

Let's investigate...

1. Use 16 interlocking cubes to build as many different square-based prisms as possible with a volume of 16 cubic units.

2. Calculate the surface area of each prism and record your results in the table.

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Height</th>
<th>Volume</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>16</td>
<td>16</td>
<td>66</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1</td>
<td>16</td>
<td>48</td>
</tr>
</tbody>
</table>

3. What are the dimensions of the square-based prism that has the minimum, or optimal, surface area? 2 by 2 by 4

4. Describe the shape of this prism compared to the other prisms. closest to a cube
5. Predict the dimensions of the square-based prism with minimum surface area if you use:

   - a) 27 cubes
     \[ l = \sqrt[3]{27} \]
     \[ l \approx 3 \]
     \[ \therefore \text{3 by 3 by 3} \]

   - b) 64 cubes.
     \[ l = \sqrt[3]{64} \]
     \[ l = 4 \]
     \[ \therefore \text{4 by 4 by 4} \]

   - c) 125 cubes.
     \[ l = \sqrt[3]{125} \]
     \[ l = 5 \]
     \[ \therefore \text{5 by 5 by 5} \]

**REFLECT:** Summarize your findings.

a) Do any relationships exist between the length, width, and height of a square-based prism with minimum surface area for a given volume?

   \[ l = w = h \]

b) What is the ideal shape for minimizing the surface area of a square-based prism when given a fixed volume? **A cube.**

c) How can you predict the dimensions of a square-based prism with minimum surface area if you know the volume?

   \[ l = \sqrt[3]{V} \]
EX. 1. Cardboard Box Dimensions

a) The Pop-a-Lot popcorn company ships kernels of popcorn to movie theatres in large cardboard boxes with a volume of 500,000 cm³. Determine the dimensions of the square-based prism box, to the nearest tenth of a centimetre, that will require the least amount of cardboard.

\[ l = \sqrt[3]{V} \]
\[ l = \sqrt[3]{500,000} \]
\[ l = 79.4 \text{ cm} \]

\[ \therefore \text{the dimensions are} \]
79.4 cm by 79.4 cm by 79.4 cm

b) Find the amount of cardboard required to make this box, to the nearest tenth of a square metre. Describe any assumptions you have made.

\[ \text{SA} = 6(l \times l) \]
\[ = 6(79.4 \times 79.4) \]
\[ = 37,826.2 \text{ cm}^2 \]
\[ \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^2 = \left( \frac{1 \text{ m}}{10,000 \text{ cm}} \right)^2 \]
\[ = 3.8 \text{ m}^2 \]

\[ \therefore 3.8 \text{ m}^2 \text{ of cardboard are needed.} \]

Assumption: No overlap of cardboard.
Investigation B:

How can you compare the volumes of square-based prisms with the same surface area?

Let's investigate...

1. Each of the square-based prisms below has a surface area of 24 cm². Calculate the area of the base and the volume of each prism. Record your data in the table.

<table>
<thead>
<tr>
<th>Prism Number</th>
<th>Side length of Base (cm)</th>
<th>Area of Base (cm²)</th>
<th>Surface Area (cm²)</th>
<th>Height (cm)</th>
<th>Volume (cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>24</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>24</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>24</td>
<td>0.5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

2. What are the dimensions of the square-based prism that has the maximum, or optimal, volume?

   2 cm by 2 cm by 2 cm

3. Describe the shape of this prism compared to the other prisms.

   A cube
4. Predict the dimensions of the square-based prism with maximum volume if the surface area is 54 cm$^2$.

\[
5A = 6(l \times l)
\]
\[
54 = 6l^2
\]
\[
\frac{54}{6} = \frac{6l^2}{6}
\]
\[
9 = l^2
\]
\[
\sqrt{9} = l
\]
\[
3 = l
\]

\[\therefore 3\text{cm by 3cm by 3cm}\]

**REFLECT:** Summarize your findings.

a) Do any relationships exist between the length, width, and height of a square-based prism with maximum volume for a given surface area?

\[l = w = h\]

b) What is the ideal shape for maximizing the volume of a square-based prism when given a fixed surface area?

A cube

c) How can you predict the dimensions of a square-based prism with maximum volume if you know the surface area?

\[l = \sqrt{\frac{5A}{6}}\]
EX. 2. Maximize the Volume of a Square-Based Prism

a) Determine the dimensions of the square-based prism with maximum volume that can be formed using 5400 cm$^2$ of cardboard.

\[ SA = 6l^2 \]
\[ 5400 = 6l^2 \]
\[ \frac{5400}{6} = l^2 \]
\[ 900 = l^2 \]
\[ \sqrt{900} = l \]

\[ \therefore 30 = l \]

\[ \therefore \text{the dimensions are } 30 \text{ cm by } 30 \text{ cm by } 30 \text{ cm}. \]

b) What is the volume of the prism?

\[ V = l^3 \]
\[ = 30^3 \]
\[ = 27000 \]
\[ \therefore \text{the volume is } 27000 \text{ cm}^3. \]
Practice: p. 495 #2, 3, 5a, 7 & p. 501 #2, 3, 6, 7