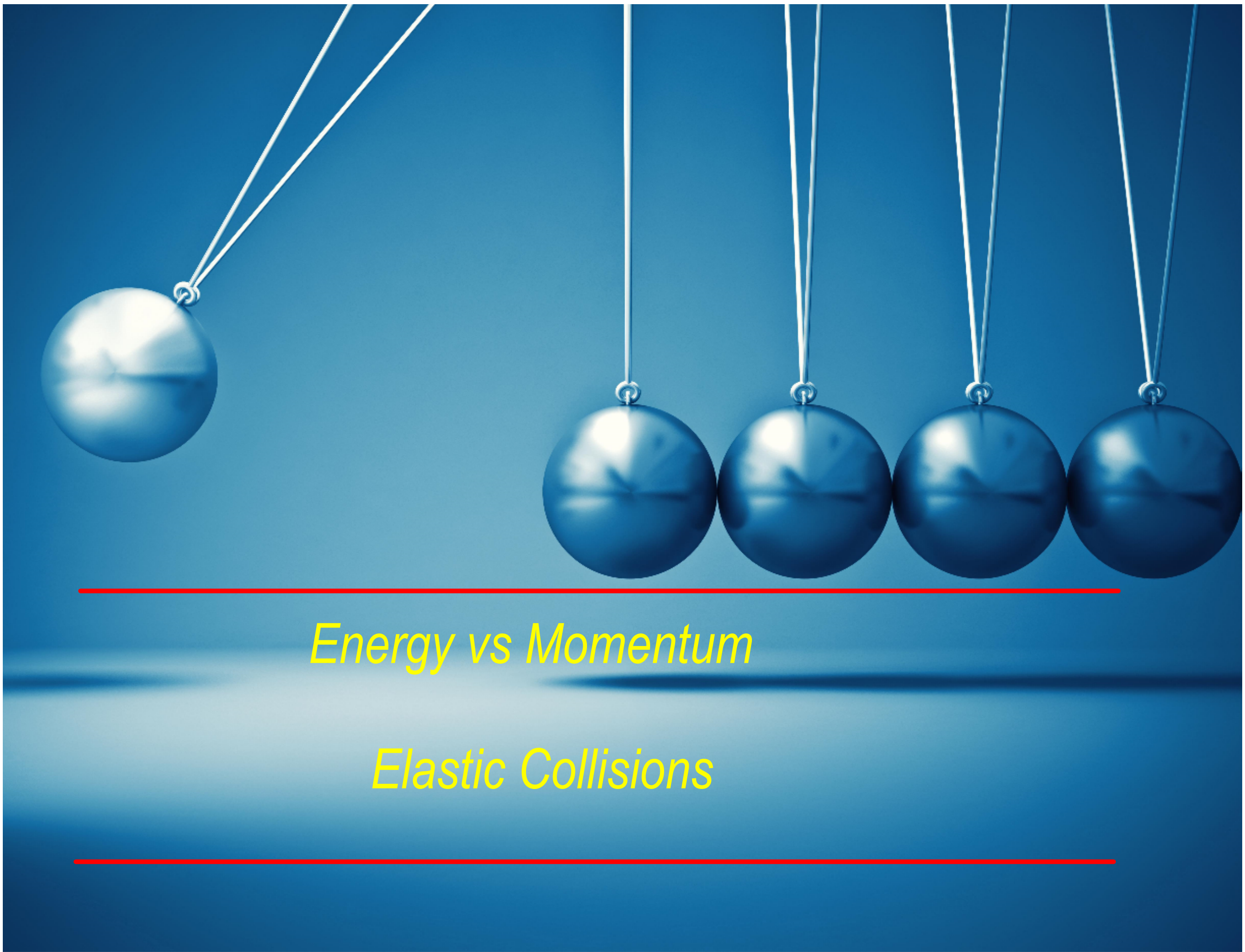

Energy vs Momentum
Elastic Collisions

Energy

recall from Grade 11

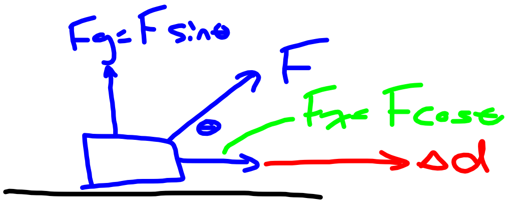
Energy is the capacity to do work

Work : the energy transferred to an object by an applied force over a measured distance.

$$W = \Delta E = \vec{F} \cdot \Delta \vec{d}$$

Scalars

Vectors



force applied on an angle

$$W = \Delta E = F\cos\theta\, \Delta d$$

only the component of force in the direction of motion affects the ΔE or W.

Units on W or energy

$$1\ \text{Joule} = 1\ \text{N}\cdot\text{m} \qquad 1\ \text{N} = 1\ \text{kg}\,\text{m/s}^2$$
$$= 1\ \text{kg}\,\text{m}^2/\text{s}^2$$

Mechanical Energy

energy of motion

$$E_k = \tfrac{1}{2}mv^2$$

energy of position

$$E_g = mgh$$

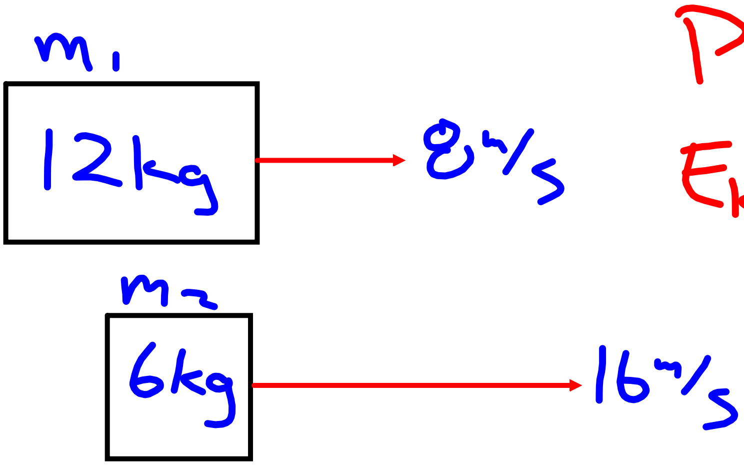
Conservation of Energy

$$E_{Ti} = E_{Tf}$$

total initial energy = total final energy

Relating Momentum to Kinetic Energy

Objects with the same momentum may have different kinetic energy values.



$\vec{P}_1 = 96\,kgm/s$

$E_{k1} = 384\,J$

$\vec{P}_2 = 96\,kgm/s$

$E_{k2} = 768\,J$

Example : Determine the kinetic energy of a 160g hockey puck that has a momentum of 4.8 kg·m/s

$V = \vec{P}/m = 30\,m/s$

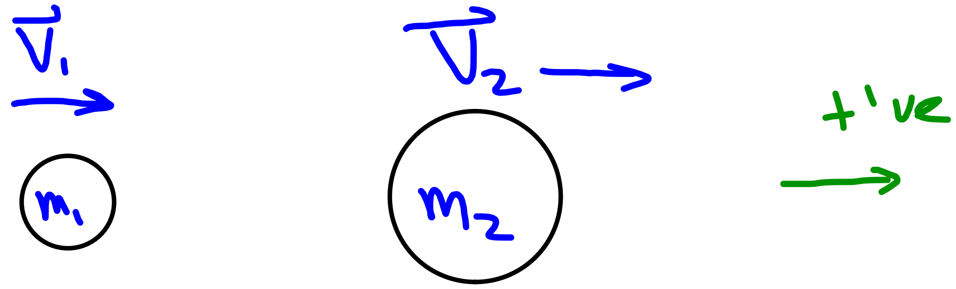
$E_k = \frac{1}{2}mv^2 = 72\,J$

$E_k = \frac{1}{2}m\left(\frac{P}{m}\right)^2$

$$\boxed{E_k = \frac{P^2}{2m}}$$

Elastic vs Inelastic Collisions

given



unknowns $\vec{v}_1{}'$ $\vec{v}_2{}'$

Momentum is always conserved

① $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$

Total Energy is always conserved

if kinetic energy is conserved then the collision is called elastic

if kinetic energy is lost (or gained) the collision is called inelastic.

elastic collision

$$E_{kTOT} = E_{kTOT}'$$

$$\cancel{\tfrac{1}{2}}m_1v_1^2 + \cancel{\tfrac{1}{2}}m_2v_2^2 = \cancel{\tfrac{1}{2}}m_1v_1'^2 + \cancel{\tfrac{1}{2}}m_2v_2'^2$$

② $m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$

Solving Elastic Collision Problems

① $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

② $m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$

① $m_1(v_1 - v_1') = m_2(v_2' - v_2)$

② $m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$

let $v_2 = 0$. (assume m_2 stationary)

① $m_1(v_1 - v_1') = m_2v_2'$

② $m_1(v_1^2 - v_1'^2) = m_2v_2'^2$

② $m_1(v_1 + v_1')(v_1 - v_1') = m_2v_2'^2$

② ÷ ①

$$\frac{\cancel{m_1}(v_1 + v_1')\cancel{(v_1 - v_1')}}{\cancel{m_1}\cancel{(v_1 - v_1')}} = \frac{\cancel{m_2}v_2'^{\cancel{2}}}{\cancel{m_2}\cancel{v_2'}}$$

③ $v_1 + v_1' = v_2'$

Sub ③ into ① & solve for v_1'

$m_1(v_1 - v_1') = m_2(v_1 + v_1')$

$m_1v_1 - m_1v_1' = m_2v_1 + m_2v_1'$

$m_1v_1 - m_2v_1 = m_1v_1' + m_2v_1'$

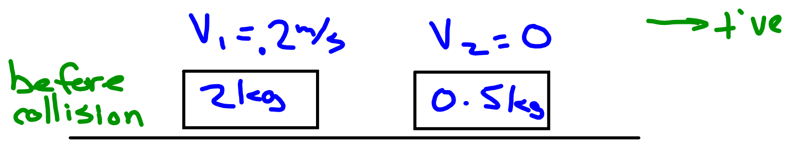
$v_1(m_1 - m_2) = v_1'(m_1 + m_2)$

$$\boxed{v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1} \ *$$

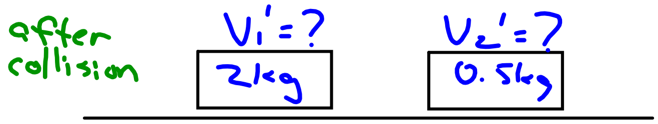
$$\boxed{v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1} \ *$$

Collision Cart Examples:

Example : 2 Objects collide in an elastic collision (1 object moving, 1 object stationary)



find V_1' & V_2'



$P_T = P_T'$

① $0.4 = 2V_1' + 0.5V_2'$ $\vec{p}$.

① $V_2' = 0.8 - 4V_1'$ sub into ② & solve for V_1'

$E_{k_T} = E_{k_T}'$

$\cancel{\tfrac{1}{2}}(2)(.2)^2 + \cancel{\tfrac{1}{2}}(.5)(0)^2 = \cancel{\tfrac{1}{2}}(2)V_1'^2 + \cancel{\tfrac{1}{2}}(0.5)V_2'^2$

② $0.08 = 2V_1'^2 + 0.5V_2'^2$

$0.08 = 2V_1'^2 + 0.5(0.8 - 4V_1')^2$

$0.08 = 2V_1'^2 + 0.5(0.64 - 6.4V_1' + 16V_1'^2)$

$0.08 = 2V_1'^2 + 0.32 - 3.2V_1' + 8V_1'^2$

$10V_1'^2 - 3.2V_1' + 0.24 = 0$

$V_1' = 0.2$ or 0.12

initial vel

final vel

$V_2' = 0$

$0.32\ m/s$

Collision Cart Examples:

Example : 2 Objects collide in an elastic collision (1 object moving, 1 object stationary)

→ +ve

before collision

$V_1 = .2\ m/s$ $\quad$ $V_2 = 0$

| 2kg | 0.5kg |

find V_1' & V_2'

after collision

$V_1' = ?$ $\quad$ $V_2' = ?$

| 2kg | 0.5kg |

$$\left.\begin{array}{l}\text{elastic}\\ \text{linear}\\ V_2 = 0\end{array}\right\} \checkmark$$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1$$

$$= \left(\frac{2 - .5}{2 + .5}\right) .2\ m/s$$

$$= \left(\frac{1.5}{2.5}\right) .2\ m/s$$

$$= 0.12\ m/s$$

$$V_2' = \left(\frac{2\,m_1}{m_1 + m_2}\right) V_1$$

$$= \left(\frac{2 \times 2}{2 + .5}\right) .2\ m/s$$

$$= \left(\frac{4}{2.5}\right) .2\ m/s$$

$$= 0.32\ m/s$$

Collisions

Momentum is conserved

$\vec{p}_i=\vec{p}_f$

Energy is Conserved

$E_i=E_f$

Elastic Collision

Kinetic Energy is Conserved

Inelastic Collision

Kinetic Energy is NOT Conserved

Special Case : Head On Collision

- no angular components
- one object at rest at beginning

$V_2=0$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1$$

$$V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1$$

Three Scenarios for V_1'

1. $m_1>m_2$ – (a big mass hits a smaller mass), then m_1-m_2 is positive and V_1' is positive, meaning m_1 keeps going in the same direction but at a lesser speed
2. $m_1<m_2$ – (a smaller mass hits a bigger mass), then m_1-m_2 is negative and V_1' is negative, meaning m_1 bounces back in the opposite direction to what it was going before the collision
3. $m_1=m_2$, (equal masses) then m_1-m_2 is zero and V_1' is zero.

Scenario for V_2'

The second equation shows that m_2 rebounds in the same direction that m_1 came into the collision (i.e. V_2' and V_1 are in the same direction).

If m1=m2, then $V_2'=V_1$ (m_2 rebounds at the same speed that m_1 started with).