

2.3 Applying Newton's Laws of Motion

When riders in a roller coaster are pulled toward the top of the first and highest hill, they are putting faith in the calculations used to determine the strength of the cable pulling on the coaster. The design engineers must analyze the relationship between the tension in the cable and the forces of gravity and friction acting on the coaster cars and all the passengers. This is just one example of a situation in which forces are analyzed. In this section you will expand your problem-solving skills as you apply Newton's three laws of motion to a variety of situations.

Solving Problems in a Systematic Way

Newton's laws of motion can be used to solve a variety of problems. One approach to solving most problems involves following a series of steps.

- Step 1. Read the problem carefully and check the definitions of any unfamiliar words.
- Step 2. Draw a system diagram. Label all relevant information, including any numerical quantities given. (For simple situations, you can omit this step.)
- Step 3. Draw an FBD of the object (or group of objects) and label all the forces. Choose the $+x$ and $+y$ directions. (Try to choose one of these directions as the direction of the acceleration.)
- Step 4. Calculate and label the x - and y -components of all the forces on the FBD.
- Step 5. Write the second-law equation(s), $\Sigma F_x = ma_x$ and/or $\Sigma F_y = ma_y$, and substitute for the variables on both sides of the equation(s).
- Step 6. Repeat steps 3 to 5 for any other objects as required.
- Step 7. Solve the resulting equation(s) algebraically.
- Step 8. Check to see if your answers have appropriate units, a reasonable magnitude, a logical direction (if required), and the correct number of significant digits.

SAMPLE problem 1

A contractor is pushing a stove across a kitchen floor with a constant velocity of 18 cm/s [fwd]. The contractor is exerting a constant horizontal force of 85 N [fwd]. The force of gravity on the stove is 447 N [down].

- Determine the normal force (\vec{F}_N) and the force of friction (\vec{F}_f) acting on the stove.
- Determine the total force applied by the floor (\vec{F}_{floor}) on the stove.

Solution

$$\vec{F}_{\text{app}} = 85 \text{ N [fwd]}$$

$$\vec{F}_g = 447 \text{ N [down]}$$

$$\vec{v} = 18 \text{ cm/s [fwd]}$$

(a) $\vec{F}_N = ?$
 $\vec{F}_f = ?$

We can omit the system diagram (step 2) because this problem involves the motion of a single object in a simple, one-dimensional situation. We begin by drawing an FBD of the stove, as in **Figure 1(a)**.

Since the stove is moving at a constant velocity, $a_x = 0$ and $\sum F_x = 0$. Since the stove has no motion in the vertical direction, $a_y = 0$ and $\sum F_y = 0$. To solve for the normal force, we substitute using the vertical components of the forces:

$$\begin{aligned}\sum F_y &= ma_y = 0 \\ F_N + (-F_g) &= 0 \\ \therefore F_N &= F_g = 447 \text{ N}\end{aligned}$$

Since the direction of the normal force is upward, the final value is 447 N [up].

To determine the force of friction, we substitute using the horizontal components of the forces.

$$\begin{aligned}\sum F_x &= ma_x = 0 \\ F_{\text{app}} + (-F_f) &= 0 \\ \therefore F_{\text{app}} &= F_f = 85 \text{ N}\end{aligned}$$

The friction is 85 N [backward].

(b) $\vec{F}_{\text{floor}} = ?$

As shown in **Figure 1(b)**, the force applied by the floor on the stove has two components. We determine the magnitude of the required force as follows:

$$\begin{aligned}|\vec{F}_{\text{floor}}| &= \sqrt{(F_N)^2 + (F_f)^2} \\ &= \sqrt{(447 \text{ N})^2 + (85 \text{ N})^2} \\ |\vec{F}_{\text{floor}}| &= 455 \text{ N}\end{aligned}$$

We determine the direction of the force from trigonometry:

$$\begin{aligned}\phi &= \tan^{-1} \frac{F_N}{F_f} \\ &= \tan^{-1} \frac{447 \text{ N}}{85 \text{ N}} \\ \phi &= 79^\circ\end{aligned}$$

The force of the floor on the stove is $4.6 \times 10^2 \text{ N}$ [79° above the horizontal].

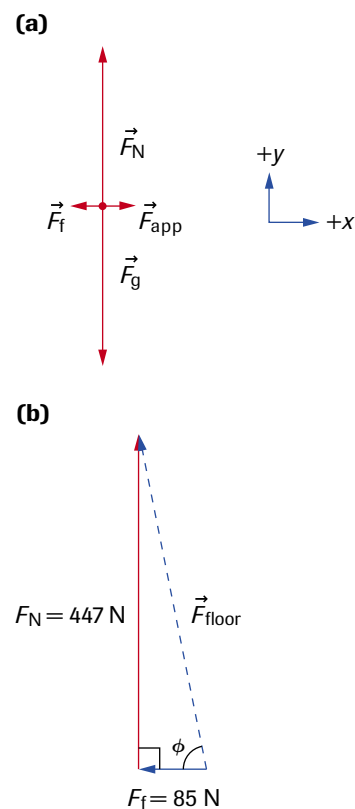


Figure 1

(a) FBD of the stove

(b) Force exerted by the floor on the stove

LEARNING TIP

Using Components

Notice that when using components, vector notation is omitted. Remember that the symbol F_g represents the *magnitude* of the force \vec{F}_g and is positive. If the $+y$ direction is chosen to be upward, then the y -component of the force of gravity is negative and is written $-F_g$.

▶ SAMPLE problem 2

Sleds A and B are connected by a horizontal rope, with A in front of B. Sled A is pulled forward by means of a horizontal rope with a tension of magnitude 29.0 N. The masses of A and B are 6.7 kg and 5.6 kg, respectively. The magnitudes of friction on A and B are 9.0 N and 8.0 N, respectively. Calculate the magnitude of

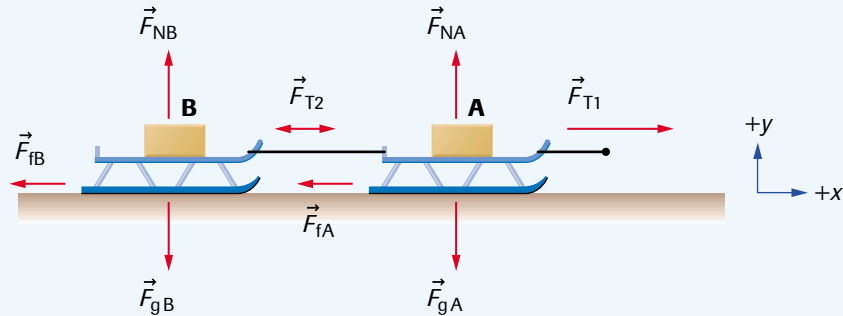
- the acceleration of the two-sled system
- the tension in the rope connecting the sleds

Figure 2
System diagram of sleds A and B for Sample Problem 2

Solution

We begin by drawing a system diagram and labelling the pertinent information, as in **Figure 2**.

$$\begin{aligned} \vec{F}_{T1} &= 29.0 \text{ N} & F_{fA} &= 9.0 \text{ N} \\ m_A &= 6.7 \text{ kg} & F_{fB} &= 8.0 \text{ N} \\ m_B &= 5.6 \text{ kg} \end{aligned}$$



- (a) To determine the magnitude of the acceleration of the two-sled system, we note that the tension in the front rope, \vec{F}_{T1} , determines the acceleration of the entire system. **Figure 3** is the FBD of the system.

Horizontally for the system:

Applying Newton's second law:

$$\begin{aligned} \sum F_{\text{system},x} &= m_{\text{system}} a_{\text{system},x} \\ a_x &= \frac{\sum F_x}{m} \\ &= \frac{F_{T1} + (-F_{f,\text{total}})}{m_A + m_B} \\ &= \frac{29.0 \text{ N} - (9.0 \text{ N} + 8.0 \text{ N})}{(6.7 \text{ kg} + 5.6 \text{ kg})} \\ a_x &= 0.98 \text{ m/s}^2 \end{aligned}$$

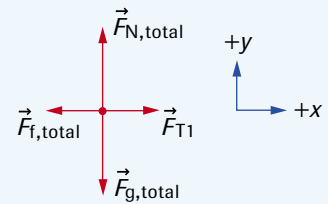


Figure 3
FBD of the two-sled system

The magnitude of the acceleration is 0.98 m/s^2 .

- (b) To determine the magnitude of the tension in the second rope, \vec{F}_{T2} , we analyze the forces that are acting only on sled B. **Figure 4** is the FBD of sled B. Knowing that the magnitude of the acceleration of sled B is the same as the acceleration of the system (i.e., 0.98 m/s^2), we can apply the second-law equation to the horizontal components of the motion:

$$\begin{aligned} \sum F_{Bx} &= m_B a_{Bx} \\ F_{T2} - F_{fB} &= m_B a_{Bx} \\ F_{T2} &= F_{fB} + m_B a_{Bx} \\ &= 8.0 \text{ N} + (5.6 \text{ kg})(0.98 \text{ m/s}^2) \\ F_{T2} &= 13 \text{ N} \end{aligned}$$

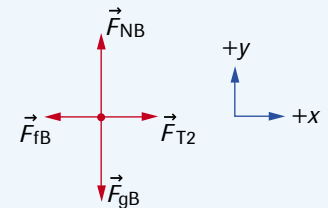


Figure 4
FBD of sled B

The magnitude of the tension in the connecting ropes is 13 N. We would have obtained the same value by drawing an FBD for sled A instead, since the tension force applied by the connecting rope to A is equal in magnitude, although opposite in direction, to the tension force applied by the connecting rope to B. The value of 13 N seems reasonable, since the tension must be large enough not only to overcome the friction of 8.0 N, but also to cause acceleration.

LEARNING TIP

Multiple Ropes

In Sample Problem 2, there are two sleds and two ropes. However, if there were three or more objects and three or more ropes, you would need to calculate the tensions in those ropes. You would first find the acceleration of the system, and then go to the last object and work your way step by step to the first object. These types of problems can also be solved by setting up and solving simultaneous equations. For example, Sample Problem 2 can be solved by setting up a second-law equation in the horizontal plane for each object. These two equations have two unknowns (the acceleration and the tension in the connecting rope). The simultaneous equations can then be solved for the unknowns.

SAMPLE problem 3

You attach a loonie ($m_L = 6.99 \text{ g}$) and a dime ($m_D = 2.09 \text{ g}$) to the ends of a thread. You put the thread over a smooth horizontal bar and pull the thread taut. Finally, you release your hands, letting the loonie drop and the dime rise. Friction between the thread and the bar is negligible, and the magnitude of \vec{g} is 9.80 m/s^2 . Determine the magnitude of

- (a) the acceleration of the coins
(b) the tension in the thread

Solution

We begin by drawing the system diagram (**Figure 5(a)**), including as much information as possible. Since the loonie has more mass than the dime, the system of coins and thread will accelerate counterclockwise over the bar. We choose the direction of the acceleration as the positive direction for each coin: downward for the loonie and upward for the dime. Because there is only one thread, there is only one tension, \vec{F}_T .

- (a) To determine the magnitude of the acceleration of the coins, we analyze the FBDs of the two coins separately (**Figures 5(b)** and **(c)**). The second-law equation with y -components in the coordinate system for the loonie yields the following:

$$\begin{aligned}\sum F_y &= ma_y \\ F_{gL} - F_T &= m_L a_y\end{aligned}$$

Both F_T and a_y are unknown. Next, we apply the second-law equation with y -components in the coordinate system for the dime:

$$F_T - F_{gD} = m_D a_y$$

If we add the two equations, we find that one of the unknowns, F_T , disappears.

$$\begin{aligned}F_{gL} - F_{gD} &= m_L a_y + m_D a_y \\ F_{gL} - F_{gD} &= a_y (m_L + m_D) \\ a_y &= \frac{F_{gL} - F_{gD}}{m_L + m_D} \\ &= \frac{m_L g - m_D g}{m_L + m_D} \\ &= \frac{(m_L - m_D)g}{m_L + m_D} \\ &= \frac{(6.99 \times 10^{-3} \text{ kg} - 2.09 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{(6.99 \times 10^{-3} \text{ kg}) + (2.09 \times 10^{-3} \text{ kg})} \\ a_y &= 5.29 \text{ m/s}^2\end{aligned}$$

The magnitude of the acceleration is 5.29 m/s^2 .

- (b) To determine the magnitude of the tension, we can substitute the acceleration into either of the two second-law equations. Using the second-law equation for the dime:

$$\begin{aligned}F_T &= F_{gD} + m_D a_y \\ &= m_D g + m_D a_y \\ &= m_D (g + a_y) \\ &= (2.09 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2 + 5.29 \text{ m/s}^2) \\ F_T &= 3.15 \times 10^{-2} \text{ N}\end{aligned}$$

The magnitude of the tension in the thread is $3.15 \times 10^{-2} \text{ N}$.

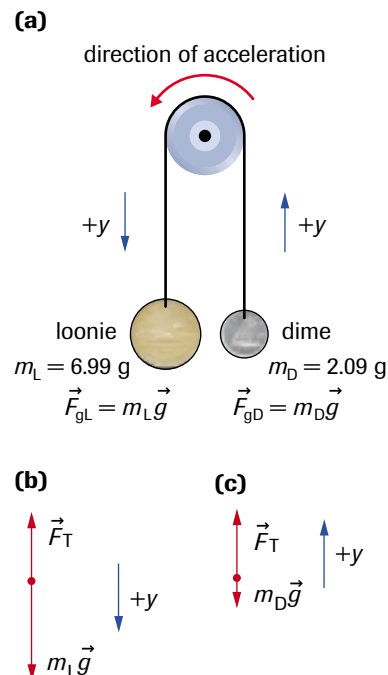


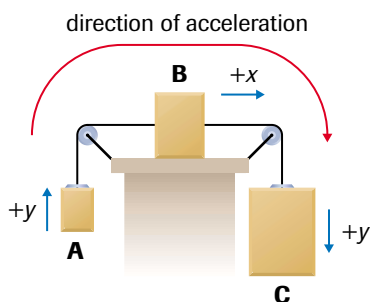
Figure 5

- (a) The system diagram for the coins in Sample Problem 3
(b) The FBD of the loonie
(c) The FBD of the dime

LEARNING TIP

Directions in Pulley Problems

When you solve a problem that involves at least one pulley, choose a general positive direction for the entire system of objects. You should then assign a $+x$ or $+y$ direction for each object so that it is in the general positive direction. For example, the system of objects below will tend to accelerate in a clockwise direction. The positive direction will be upward for mass A, to the right for mass B, and downward for mass C.



Answers

- 35 g
- (a) $2.74 \times 10^3 \text{ N}$ [up]
(b) 35 kg
- (a) 1.55 m/s^2
(b) 295 N
- 1.2 kg
- (a) 4.8 m/s^2
(b) 19 N
- (a) 194 N
(b) 2.4 N
(c) 0.79 m/s
(d) 2.9 N
- (a) $6.2 \times 10^2 \text{ N}$
(b) 2.0 m/s^2
- (b) $mg - F_A \sin \theta$
(c) $F_A \cos \theta$

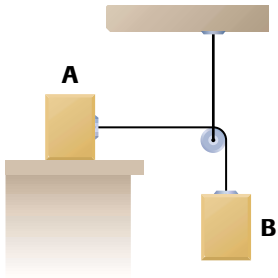


Figure 6
For question 5

Practice

Understanding Concepts

- You apply a force of 0.35 N [up] to lift a fork. The resulting acceleration of the fork is 0.15 m/s^2 [up]. Determine the mass of the fork in grams.
- A hot-air balloon experiences an acceleration of 1.10 m/s^2 [down]. The total mass of the balloon, the basket, and the contents of the basket is 315 kg.
 - What is the upward (buoyant) force on the system?
 - The balloonist wishes to change the acceleration to zero. There is no fuel left to heat the air in the balloon. Determine the mass of the ballast that must be discarded overboard. (Neglect air resistance.)
- A tree house has a vertical “fire pole” of smooth metal, designed for quick exits. A child of mass 35.7 kg slides down the pole with constant acceleration, starting from rest. The pole is 3.10 m high. The journey to the ground takes 2.00 s.
 - What is the magnitude of the downward acceleration of the child?
 - What is the magnitude of the upward force of friction exerted by the pole on the child?
- When an external net force is applied to a particular mass m , an acceleration of magnitude “ a ” results. When the mass is increased by 2.0 kg and the same net force is applied, the acceleration is $0.37a$. Determine the mass, m .
- Blocks A and B are connected by a string passing over an essentially frictionless pulley, as in **Figure 6**. When the blocks are in motion, Block A experiences a force of kinetic friction of magnitude 5.7 N. If $m_A = 2.7 \text{ kg}$ and $m_B = 3.7 \text{ kg}$, calculate the magnitude of
 - the acceleration of the blocks
 - the tension in the string
- A boy pushes a lawn mower ($m = 17.9 \text{ kg}$) starting from rest across a horizontal lawn by applying a force of 32.9 N straight along the handle, which is inclined at an angle of 35.1° above the horizontal. The magnitude of the mower’s acceleration is 1.37 m/s^2 , which lasts for 0.58 s, after which the mower moves at a constant velocity. Determine the magnitude of
 - the normal force on the mower
 - the frictional force on the mower
 - the maximum velocity of the mower
 - the force applied by the boy needed to maintain the constant velocity
- A skier ($m = 65 \text{ kg}$) glides with negligible friction down a hill covered with hard-packed snow. If the hill is inclined at an angle of 12° above the horizontal, determine the magnitude of
 - the normal force on the skier
 - the skier’s acceleration (*Hint*: Remember to choose the $+x$ direction as the direction of the acceleration, which in this case is downward, parallel to the hillside.)

Applying Inquiry Skills

- Groups of physics students are each given a force scale (to measure an applied force of magnitude F_A), an electronic balance (to measure the mass, m), a rectangular wooden block with a hook at one end, a protractor, and a piece of string. Each group must determine the force of kinetic friction acting on the block as it is pulled with a constant velocity along a horizontal lab bench. However, the applied force must be at an angle θ above the horizontal.
 - Draw a system diagram and an FBD of the block for this investigation.
 - Derive an equation for the magnitude of the normal force on the block in terms of the given parameters F_A , g , m , and θ .
 - Derive an equation for the magnitude of friction on the block in terms of F_A and θ .

Making Connections

9. A physics student of mass 55.3 kg is standing on a weigh scale in an elevator. The scale shows the magnitude of the upward normal force (in newtons) on the student.
- Determine the reading on the scale when the elevator has an acceleration of 1.08 m/s^2 [up].
 - The force calculated in (a) can be called the “apparent weight.” How does the student’s apparent weight in this case compare with the true weight? What happens to the apparent weight when the elevator is undergoing downward acceleration? undergoing free fall?
 - To check your answers in (b), determine the student’s apparent weight when the elevator has first a downward acceleration of 1.08 m/s^2 , then a downward acceleration of 9.80 m/s^2 .
 - The term “weightless” is used to describe a person in free fall. Why is this term used? Is the term valid from the physics point of view? Explain your answer.
 - Repeat (a) when the elevator has a constant velocity of 1.08 m/s [up].

Answers

9. (a) 602 N
 (c) 482 N; 0 N
 (e) 542 N

Applying Newton’s Third Law of Motion

Newton’s third law of motion, the action–reaction law, always relates to two objects. When drawing a system diagram of action–reaction pairs of forces, both objects can be placed in the same diagram. However, when solving problems related to only one of objects, an FBD must be drawn for that object.

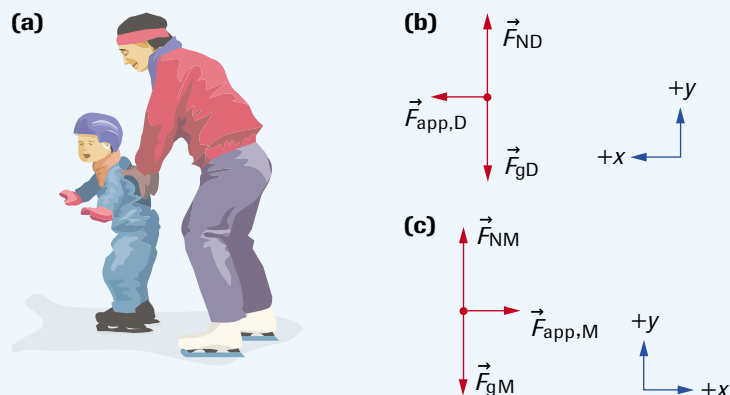
SAMPLE problem 4

An indoor skating rink is the venue for a family fun day. In one event, children on skates position themselves along a blue line facing one end of the rink. Behind each child, a parent, also on skates, prepares to push horizontally to see who can push their child the farthest. In this competition, a certain mother has a mass of 61 kg, and her daughter a mass of 19 kg. Both skaters experience negligible friction as long as their skates point straight ahead. At the starting bell, the mother pushes the child with a constant applied force of magnitude 56 N for 0.83 s. Determine the magnitude of

- the daughter’s acceleration
- the mother’s acceleration
- the maximum velocity of the daughter

Solution

Figure 7(a) is the system diagram. **Figures 7(b)** and **(c)** are the respective FBDs of the daughter and the mother. From Newton’s third law of motion, it is evident that if the force of the mother on the daughter is 56 N in one direction, the force of the daughter on the mother is 56 N in the opposite direction.

**Figure 7**

- The system diagram for the mother–daughter pair in Sample Problem 4
- The FBD of the daughter
- The FBD of the mother

- (a) Use the second-law equation for the daughter's horizontal motion:

$$\sum F_{Dx} = 56 \text{ N}$$

$$m_D = 19 \text{ kg}$$

$$a_{Dx} = ?$$

$$\begin{aligned} a_{Dx} &= \frac{\sum F_{Dx}}{m_D} \\ &= \frac{56 \text{ N}}{19 \text{ kg}} \\ a_{Dx} &= 2.9 \text{ m/s}^2 \end{aligned}$$

The magnitude of the daughter's acceleration is 2.9 m/s^2 .

- (b) Use the second-law equation for the mother's horizontal motion:

$$\sum F_{Mx} = 56 \text{ N}$$

$$m_M = 61 \text{ kg}$$

$$a_{Mx} = ?$$

$$\begin{aligned} a_{Mx} &= \frac{\sum F_{Mx}}{m_M} \\ &= \frac{56 \text{ N}}{61 \text{ kg}} \\ a_{Mx} &= 0.92 \text{ m/s}^2 \end{aligned}$$

The magnitude of the mother's acceleration is 0.92 m/s^2 .

- (c) $v_i = 0$

$$a = 2.9 \text{ m/s}^2$$

$$\Delta t = 0.83 \text{ s}$$

$$v_f = ?$$

$$\begin{aligned} v_f &= v_i + a\Delta t \\ &= 0 + (2.9 \text{ m/s}^2)(0.83 \text{ s}) \\ v_f &= 2.4 \text{ m/s} \end{aligned}$$

The magnitude of the maximum velocity of the daughter is 2.4 m/s .

Practice

Understanding Concepts

10. A train consisting of two cars pulled by a locomotive experiences an acceleration of 0.33 m/s^2 [fwd]. Friction is negligible. Each car has a mass of $3.1 \times 10^4 \text{ kg}$.
- Determine the force exerted by the first car on the second car.
 - Determine the force exerted by the locomotive on the first car.
11. Two books are resting side by side, in contact, on a desk. An applied horizontal force of 0.58 N causes the books to move together with an acceleration of 0.21 m/s^2 horizontally. The mass of the book to which the force is applied directly is 1.0 kg . Neglecting friction, determine
- the mass of the other book
 - the magnitude of the force exerted by one book on the other

Answers

10. (a) $1.0 \times 10^4 \text{ N}$ [fwd]
(b) $2.0 \times 10^4 \text{ N}$ [fwd]
11. (a) 1.8 kg
(b) 0.37 N

SUMMARY Applying Newton's Laws of Motion

- It is wise to develop a general strategy that helps solve the great variety of types of problems involving forces, no matter how different each problem may at first appear.
- The skill of drawing an FBD for each object in a given problem is vital.
- For motion in two dimensions, it is almost always convenient to analyze the perpendicular components of the forces separately and then bring the concepts together.

Section 2.3 Questions

Understanding Concepts

1. A basketball is thrown so that it experiences projectile motion as it travels toward the basket. Air resistance is negligible. Draw an FBD of the ball (a) as it is rising, (b) as it arrives at the top of its flight, and (c) as it moves downward.
2. A shark, of mass 95 kg, is swimming with a constant velocity of 7.2 m/s [32° above the horizontal]. What is the net force acting on the shark?
3. **Figure 8** shows three masses (5.00 kg, 2.00 kg, and 1.00 kg) hung by threads.
 - (a) Draw an FBD for the bottom mass. Determine the magnitude of the tension in the lowest thread.
 - (b) Repeat (a) for the middle mass and the tension in the middle thread.
 - (c) Repeat (a) for the top mass and the tension in the highest thread.

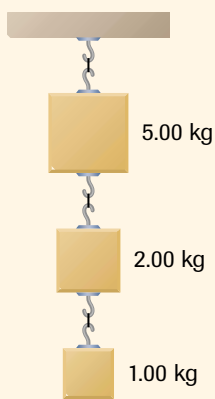


Figure 8

4. Just after a space shuttle is launched (**Figure 9**), its acceleration is about $0.50g$ [up]. The shuttle's mass, including fuel, is approximately 2.0×10^6 kg.
 - (a) Calculate the approximate magnitude of the upward force on the shuttle.
 - (b) What causes the upward force?



Figure 9

The space shuttle *Endeavour* is launched carrying astronauts to the International Space Station.

5. Two boxes, of masses $m_1 = 35$ kg and $m_2 = 45$ kg, are hung vertically from opposite ends of a rope passing over a rigid horizontal metal rod. The system starts moving from rest. Assuming that friction between the rod and the rope is negligible, determine the magnitude of
 - (a) the acceleration of the boxes
 - (b) the tension in the rope
 - (c) the magnitude of each box's displacement after 0.50 s

6. Two blocks are held in place by three ropes connected at point P, as shown in **Figure 10**. The magnitude of the force of static friction on block A is 1.8 N. The magnitude of the force of gravity on blocks A and B is 6.7 N and 2.5 N, respectively.
- Draw an FBD for block B. Determine the magnitude of the tension in the vertical rope.
 - Draw an FBD for block A. Determine the magnitudes of the tension in the horizontal rope and of the normal force acting on block A.
 - Draw an FBD of point P. Calculate the tension (the magnitude and the angle θ) in the third rope.

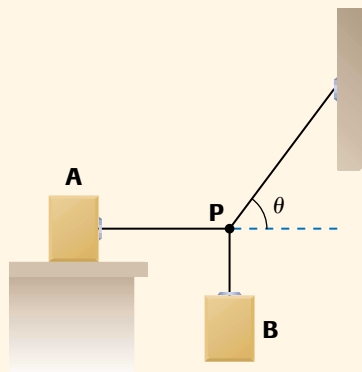


Figure 10

7. A store clerk pulls three carts connected with two horizontal cords (**Figure 11**) to move products from the storage room to the display shelves. The masses of the loaded carts are: $m_1 = 15.0$ kg; $m_2 = 13.2$ kg; and $m_3 = 16.1$ kg. Friction is negligible. A third cord, which pulls on cart 1 and is at an angle of 21.0° above the horizontal, has a tension of magnitude 35.3 N. Determine the magnitude of
- the acceleration of the carts
 - the tension in the last cord
 - the tension in the middle cord

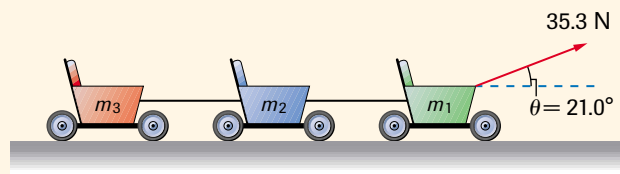


Figure 11

8. A hotel guest starts to pull an armchair across a horizontal floor by exerting a force of 91 N [15° above the horizontal]. The normal force exerted by the floor on the chair is 221 N [up]. The acceleration of the chair is 0.076 m/s² [fwd].
- Determine the mass of the chair.
 - Determine the magnitude of the friction force on the chair.

9. A child on a toboggan slides down a hill with an acceleration of magnitude 1.5 m/s². If friction is negligible, what is the angle between the hill and the horizontal?
10. Blocks X and Y, of masses $m_X = 5.12$ kg and $m_Y = 3.22$ kg, are connected by a fishing line passing over an essentially frictionless pulley (**Figure 12**).
- Show that block X slides up the incline with a positive acceleration. Determine the magnitude of that acceleration. Friction is negligible.
 - Determine the magnitude of the tension in the fishing line.

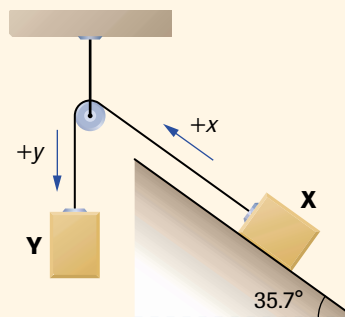


Figure 12

11. A figure skater of mass $m = 56$ kg pushes horizontally for 0.75 s with a constant force against the boards at the side of a skating rink. Having started from rest, the skater reaches a maximum speed of 75 cm/s. Neglecting friction, determine the magnitude of
- the (constant) acceleration
 - the force exerted by the skater on the boards
 - the force exerted by the boards on the skater
 - the displacement of the skater from the boards after 1.50 s

Applying Inquiry Skills

12. (a) A constant net force is applied to various masses. Draw a graph of the magnitude of the resulting acceleration as a function of mass.
- (b) A variable net force is applied to a constant mass. Draw a graph of the resulting acceleration as a function of the net force.

Making Connections

13. Discuss the physics principles illustrated by the following statement: "In front-end collisions, airbag deployment can pose extreme danger if the passenger is not wearing a seat belt or if the passenger is a small child."