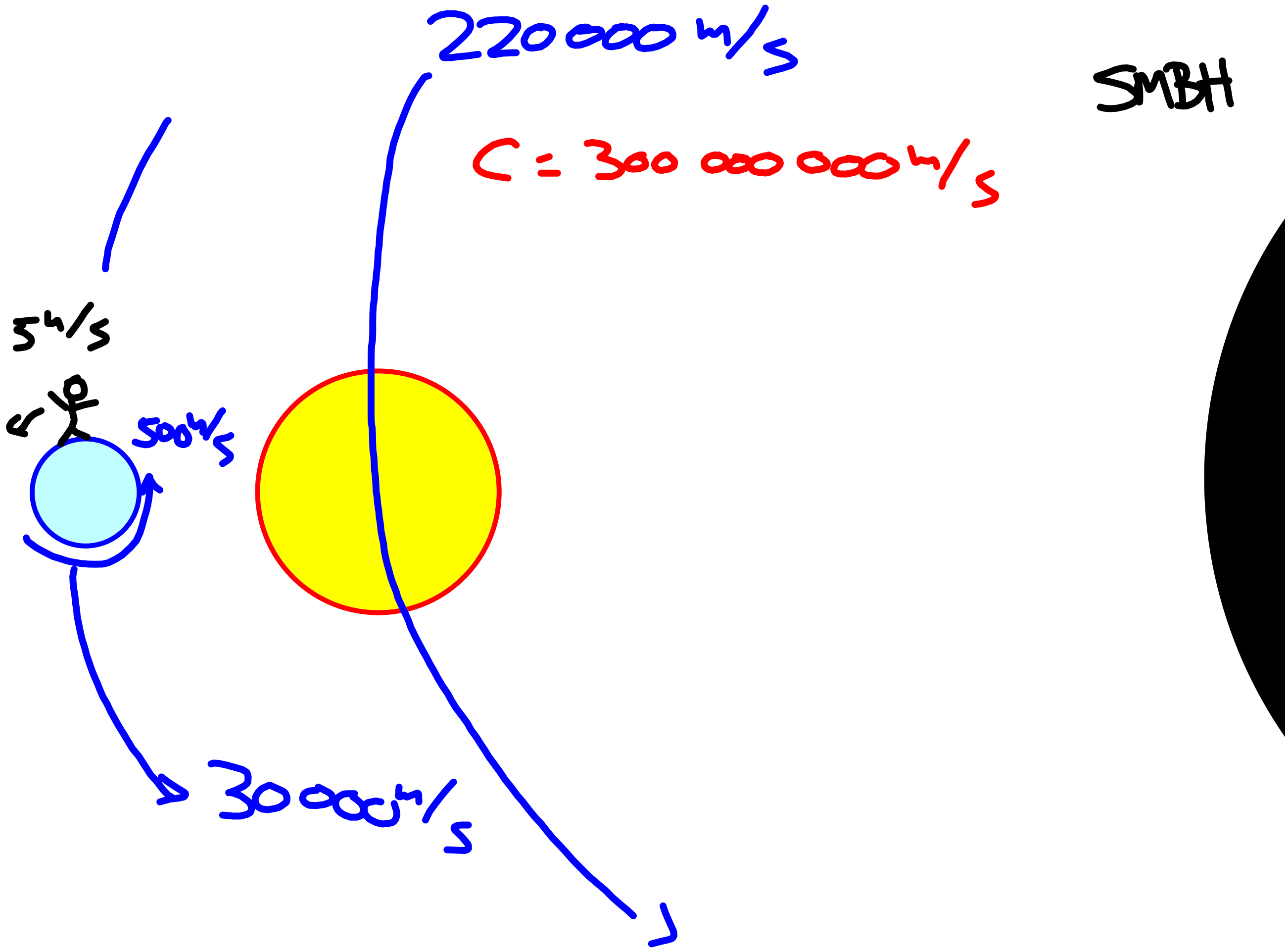
# *Selected Notes on Special Relativity*

# Relative Motion



## *Kinematics : Uniform Motion*

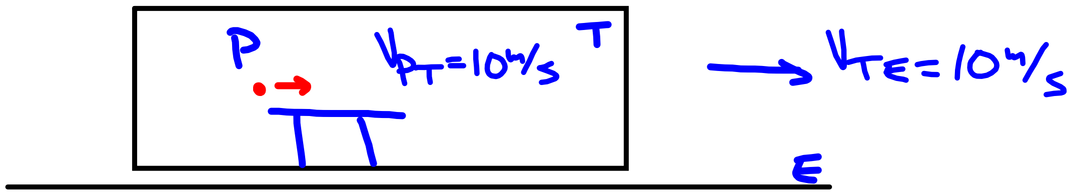
Mythbusters - Soccer Ball Shot from Truck.mp4

Uniform motion is defined as motion in a straight line at a constant speed.

$V = \frac{d}{t}$

T - train
E - earth
P - ping pong ball

### *Relative Motion*




$$\vec{V}_{PE} = \vec{V}_{PT} + \vec{V}_{TE} = 20\,m/s$$

generic relative vector addition

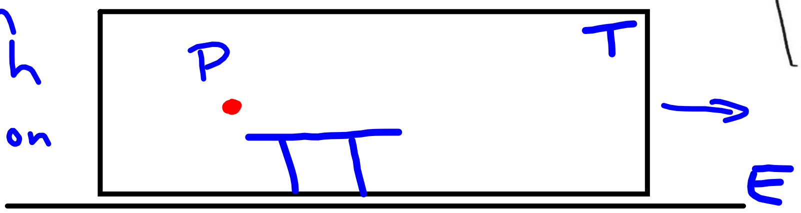
$$\vec{V}_{AD} = \vec{V}_{AB} + \vec{V}_{BC} + \vec{V}_{CD}$$

"Galilean Transformation"

## What about light?


$V_{PT} = 300\,000\,000\ m/s$

T - train
E - earth
P - photon




$V_{TE} = 10\ m/s$

$$\vec{V}_{PE} = \vec{V}_{PT} + \vec{V}_{TE}$$

$$= \cancel{300\,000\,010\ m/s}$$

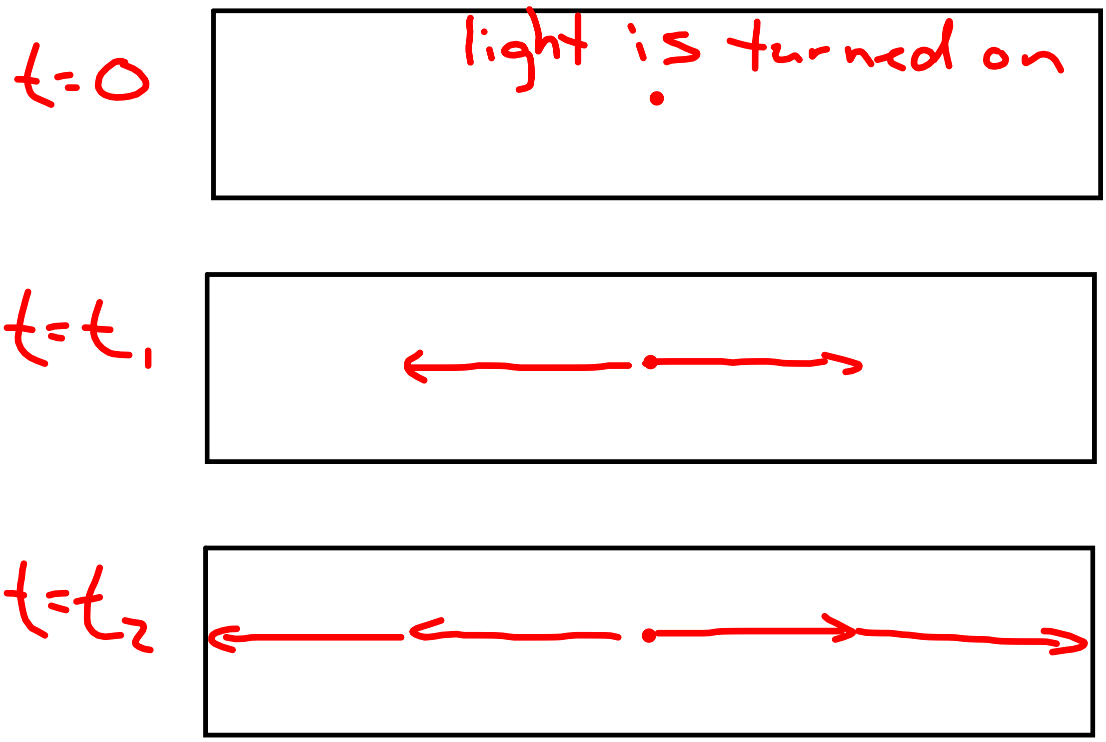
$$= 300\,000\,000\ m/s$$

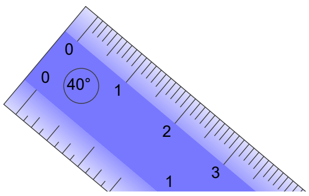
nothing can go faster than the speed of light relative to anything else.

## ***Relativity of Simultaneity***

Two events that occur simultaneously in one frame of reference are not simultaneous in another frame of reference.

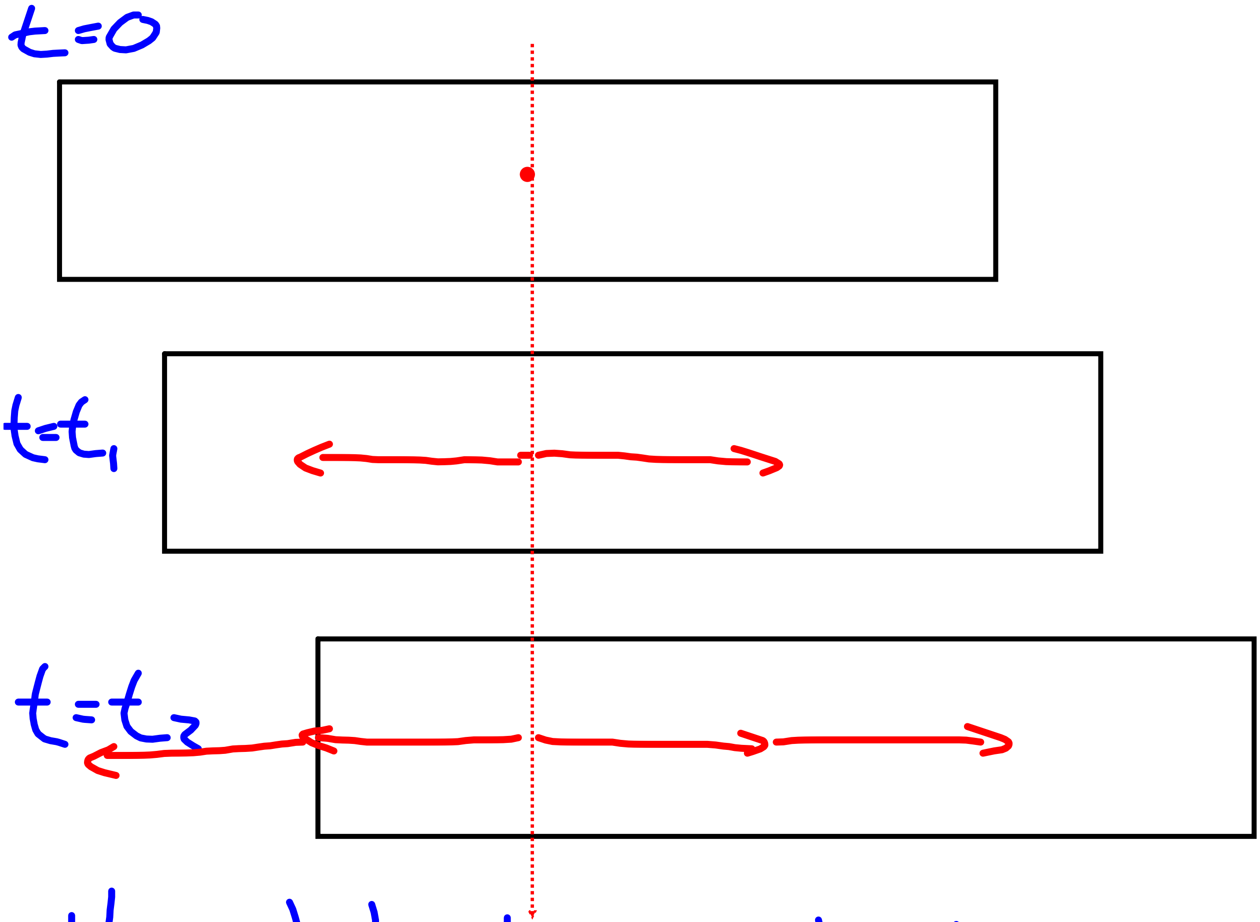
Frame of Reference on Train



Relativity of Simultaneity (cont'd)

Frame of Reference on Earth




the photon hit the back of the train before the front.

* the events are not simultaneous *

## Relativity of Simultaneity

In the frame of reference on the train (moving frame of reference) - the light hits the back and front of the train at the same time - the events are simultaneous.

In the frame of reference on the earth (stationary frame of reference) - the light hits the back of the train first and hits the front of the train at a later time - the events are not simultaneous.

### Which frame of reference is correct?

They both are.

Simultaneity depends on your frame of reference.

## Postulates of Special Relativity
(fact taken to be true)

1. The principle of relativity - all the laws of physics have the same form in inertial frames of reference. (inertial - non accelerating)
2. Speed of light is the same for all inertial frames of reference.

## Effects of Postulates of Special Relativity:

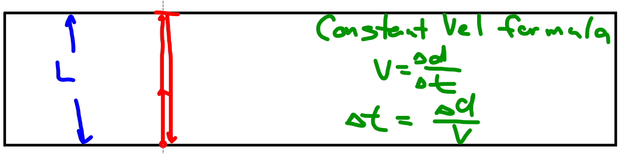
Time Dilation ✓
Length Contraction –
Velocity Additions - relative velocities do not simply add as expected ✓
Effective Mass Increases -
Mass / Energy equivalence – $E = mc^2$

## Time Dilation

Thought Experiment (light clock) - a beam of light is shone from the floor of a train car and reflected from a mirror on the top of the car. The time for the reflected beam is measured (calculated).

Case 1 : Frame of reference on train (moving frame of reference)

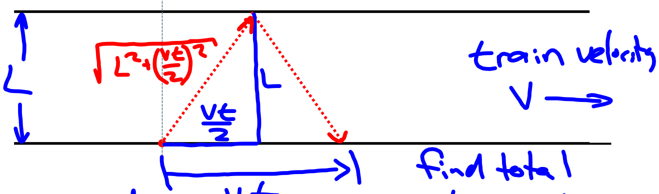
moving frame of reference → prime




Velocity of light = c

$$t' = \frac{2L}{c}$$

Case 2: Frame of reference on earth (stationary frame of reference)




find total time t

$$t = \frac{\Delta d}{v}$$

$$d = vt$$

$$t = \frac{2\sqrt{L^2+\left(\frac{vt}{2}\right)^2}}{c}$$

solve for t

$$\left(\frac{ct}{2}\right)^2 = L^2 + \left(\frac{vt}{2}\right)^2$$

$$\left(\frac{ct}{2}\right)^2 - \left(\frac{vt}{2}\right)^2 = L^2$$

$$t^2\left[\left(\frac{c}{2}\right)^2 - \left(\frac{v}{2}\right)^2\right] = L^2$$

$$t^2 = \frac{L^2}{\left(\frac{c}{2}\right)^2 - \left(\frac{v}{2}\right)^2}$$

$$t = \frac{L}{\sqrt{\left(\frac{c}{2}\right)^2 - \left(\frac{v}{2}\right)^2}}$$

$$t = \frac{L}{\sqrt{\frac{1}{4}(c^2 - v^2)}}$$

$$\frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{1}{2}} = 2$$

$$t = \frac{2L}{\sqrt{c^2 - v^2}}$$

moving frame of reference (on train)

$$t' = \frac{2L}{c}$$

stationary frame of reference (on earth)

$$t = \frac{2L}{\sqrt{c^2 - v^2}}$$

$$= \frac{2L}{\sqrt{c^2\left(1 - \frac{v^2}{c^2}\right)}}$$

$$= \frac{2L}{c\sqrt{1 - v^2/c^2}}$$

$$\boxed{t = t'\left(\frac{1}{\sqrt{1 - v^2/c^2}}\right)}$$

## Gamma Factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$t = t' \gamma$$

## Boundary Conditions

if $v = 0$, $\gamma = 1$, $t = t'$

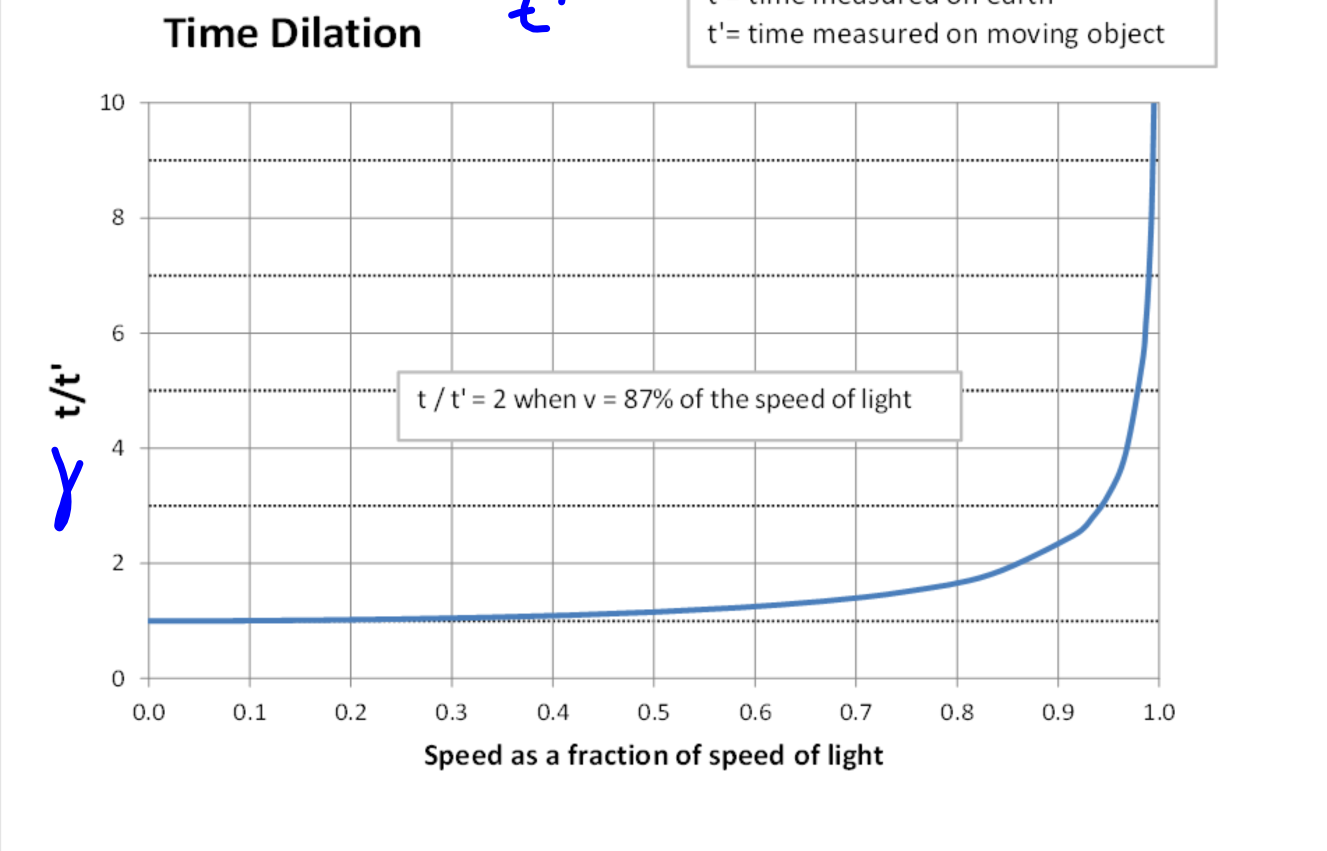
if $v = c$, $\gamma$ = undefined,

$\gamma$ is always $\geq 1$, $t$ is always $\geq t'$

Effects of relative motion on time differences $c = 3.0\times10^{8}$ m/s

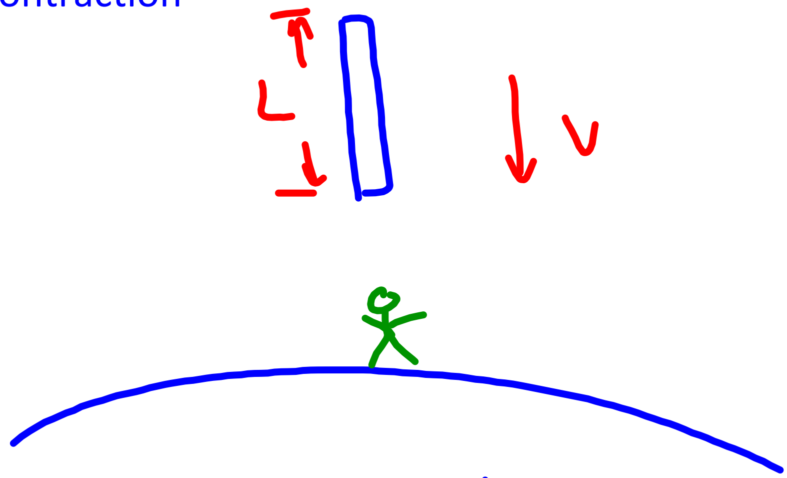
| V (m/s) | $\gamma$ |
|---|---|
| 0 m/s | 1 |
| 100 m/s | 1 |
| 10,000 m/s | 1.000000001 |
| 30,000,000 m/s (10% c) | 1.005 |
| 270,000,000 m/s (90% c) | 2.29 |

$t = t'\gamma \rightarrow \frac{t}{t'} = \gamma$




$\gamma = 2 \rightarrow v = 87\% c$

Length Contraction




$$L = \frac{L'}{\gamma}$$

## Word Problems Involving Time Dilation:

150 000 000 m/s

1. A spacecraft is travelling at 0.5c (i.e. 50% of the speed of light). If the passengers on the spacecraft measure their time away from earth to be 6 months, how much time has passed for the people on earth (based on their clocks)?

$V = \underline{0.5c}$ $(1.5 \times 10^8 \, m/s)$

$t' = 6$ months

$t = ?$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{(1.5 \times 10^8)^2}{(3.0 \times 10^8)^2}}}$$

$=$

$$= \frac{1}{\sqrt{1 - \frac{(0.5c)^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{(0.5)^2 \cancel{c^2}}{\cancel{c^2}}}}$$

$$= \frac{1}{\sqrt{1 - 0.25}}$$

$$= \frac{1}{\sqrt{0.75}}$$

$$= 1.15$$

$t = t' \gamma$

$= 6 \text{ months} \times 1.15$

$= 6.9$ months

## Word Problems (cont'd):

2. A spacecraft is travelling at 0.95c (i.e. 95% of the speed of light). If the passengers on the spacecraft measure their time away from earth to be 1 year, how much time has passed for the people on earth (based on their clocks)?

$\gamma = 3.20$

$t' = 1 \text{ year}$

$t = t'\gamma$

$= 3.20 \text{ years}$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$v = 285\,000\,000 \text{ m/s}$

$$\gamma = \frac{1}{\sqrt{1 - .95^2}}$$

$$\frac{(.95\cancel{c})^2}{\cancel{c^2}}$$

## Word Problems (cont'd):

3. How fast would a spacecraft have to be travelling (relative to the earth) if the occupants wanted to age one year, while the time passed on earth measured 50 years?

$t' = 1 \text{ year}$

$t = 50 \text{ years}$

$t = t'\gamma$

$50 = 1\gamma$

$\therefore \gamma = 50$

$$\boxed{\frac{1}{\sqrt{1 - v^2/c^2}} = 50}$$

$$\left(\frac{1}{50}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = \left[1 - \left(\frac{1}{50}\right)^2\right]$$

$$\sqrt{v^2} = \sqrt{\left[1 - \left(\frac{1}{50}\right)^2\right] c^2}$$

$$v = \sqrt{\left(1 - \left(\frac{1}{50}\right)^2\right)}\, c$$

## Word Problems (cont'd):

4. A clock on a moving spacecraft runs 1 sec slower per day relative to an identical clock on earth. What is the speed of the spacecraft?

earth $t = 86400\ s$

spaceship $t' = 86399\ s.$

1 day on earth $= 86400\ s$

$t = t'\gamma \rightarrow \gamma = \frac{t}{t'}$

$= \frac{86400}{86399}$

find $v$.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$\vdots$

$$v = \sqrt{1 - \left(\frac{1}{\gamma}\right)^2}\,c$$

$$v = \sqrt{1 - \left(\frac{86399}{86400}\right)^2}\,c$$

$= 0.004811\,c$

$= 1\,440\,000\ m/s$

## Word Problems (cont'd):

5. The average lifetime of a pi meson (sometimes called a pion) in its own frame of reference is 2.6x10$^{-8}$s. (This is the proper lifetime). If the meson moves with a speed 0.95c (relative to the earth), what is:

a. its mean lifetime as measured by an observer on earth.

b. the average distance is travels before decaying, as measured by an observer on earth?

a. $t' = 2.6\times10^{-8}$ s

$v = 0.95c$

find $\gamma$

$\underline{t} = t'\gamma \rightarrow 8.3\times10^{-8}$ s

b. $\Delta d = v \times t$

$= 0.95c \cdot \underline{t} \rightarrow 23.7$ m

## Word Problems (cont'd):

6. A precise bar measuring exactly 1m in length is moving along its' length at a speed 0.75c. What is the length of the bar as measured by a stationary observer?


$v = 0.75c \longrightarrow$

$L' = 1m$

$L = L'/\gamma$

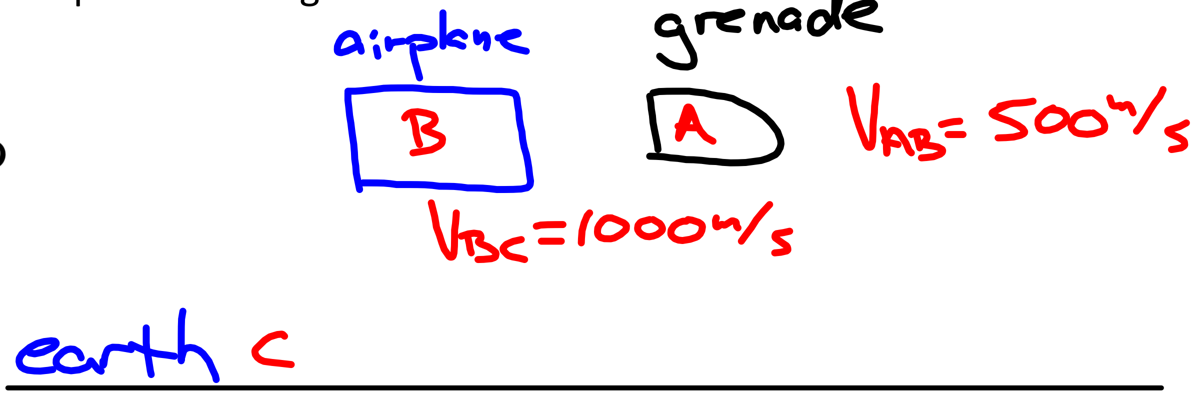
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$=$

## Part 2 : Relativistic Velocities:

***First let's review a non-relativistic example.*** $\longrightarrow +$

Airplane with a grenade launcher.




$$V_{AC} = V_{AB} + V_{BC}$$
$$= 1500\,m/s$$

Galilean Transformation

## Relativistic Velocities (cont'd)

***How do velocities add in a relativistic world?***

B
Part A : Rocket ship with a alpha ray gun example....

B A

$V_{AB} = 0.8c$

$V_{BC} = 0.7c$

$V_{AC} = V_{AB} + V_{BC} = 1.5c$ X

c

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## Lorentz Transformation

$$V_{AC} = \frac{V_{AB} + V_{BC}}{1 + \frac{V_{AB} V_{BC}}{c^2}}$$

## Solve our problem

$V_{AB} = 0.8c \quad V_{BC} = 0.7c$

$$V_{AC} = \frac{0.8c + 0.7c}{1 + \frac{(0.8\cancel{c})(0.7\cancel{c})}{\cancel{c^2}}}$$

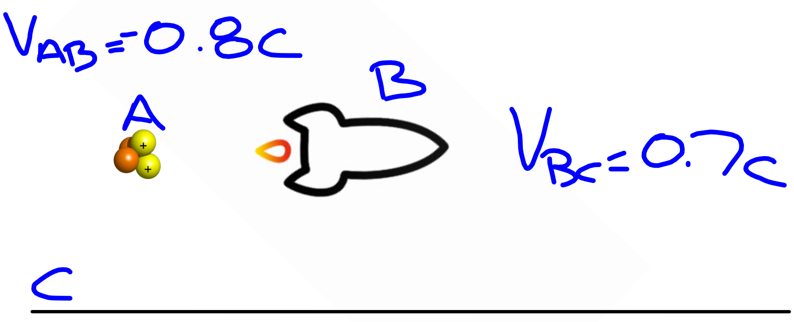
$$= \frac{1.5c}{1 + 0.56}$$

$$= \frac{1.5}{1.56} c$$

$$= 0.962c$$

**Relativistic Velocities (cont'd)**

Part B : Rocket ship with a alpha ray gun example....




$$V_{AC} = \frac{V_{AB} + V_{BC}}{1 + \frac{V_{AB}V_{BC}}{c^2}}$$

$$= \frac{-0.8c + 0.7c}{1 + \frac{(-0.8c)(0.7c)}{c^2}}$$

$$= \frac{-0.1c}{1 - 0.56}$$

$$= \frac{-0.1c}{0.44}$$

$$= -0.227c$$

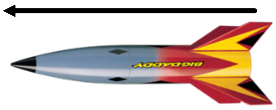
Challenge Problem : if 2 rockets are both flying at 50% the speed of the light relative to the earth and approaching each other.  What is the relative velocity of the two rockets with respect to each other

A

C

V=50%c with respect to the earth

V=50%c (with respect to earth)



$V_{AB} = 0.5c$

$V_{CB} = -0.5c$

$V_{BC} = +0.5c$

B

$$V_{AC} = \frac{V_{AB} + V_{BC}}{1 + \frac{V_{AB} V_{BC}}{c^2}}$$

240,000,000 m/s

*When do we have to take into account relativistic formulas?*

Set $\gamma = 1.0001$ $\quad \frac{t}{t'} = \frac{1.0001}{1.0000}$

find $v$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \left(\frac{1}{\gamma}\right)^2 \qquad -\frac{v^2}{c^2} = \left(\frac{1}{\gamma}\right)^2 - 1$$

$$\frac{v^2}{c^2} = 1-\left(\frac{1}{\gamma}\right)^2$$

$$v = \sqrt{1-\left(\frac{1}{\gamma}\right)^2}\cdot c$$

$$= 0.014\cdot c$$

$$= 4.2\times10^{6}\ m/s$$

4,200,000 m/s

10,000 m/s - fastest human

Velocity must be at least 50% the speed of light to worry about relativistic effects.

# Relativity Summary

1. $t = t'\gamma$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$t$ = time on stationary object

$t'$ = time on moving object

2. $$V_{AC} = \frac{V_{AB} + V_{BC}}{1 + \frac{V_{AB}V_{BC}}{c^2}}$$