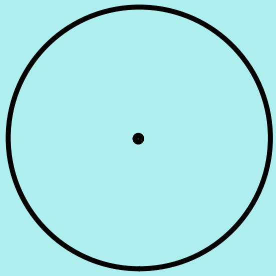
Causes of Centripetal Forces



Tension ✓

Gravity ✓

Normal Force

Friction

~~Electromagnetic Force~~

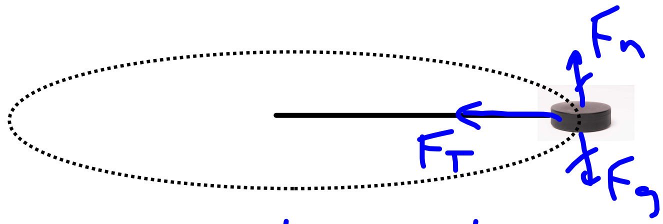
Horizontal vs Vertical Loops

Let's revisit the hockey puck on a string problem from earlier in the unit.

Horizontal Loops (on an ice surface)

m = 0.160 kg
r = 0.600 m
T = 0.500 s

$F_f = \phi$



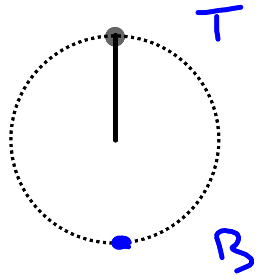
circular motion $\therefore F_{net} = F_c$

$F_{net} = F_T \quad \therefore F_T = F_c$

$= \frac{4\pi^2 mr}{T^2}$

$= 15.2\text{N}$

Vertical Loops



m = 0.160 kg
r = 0.600 m
T = 0.500 s

$V = \frac{2\pi r}{T}$

Find F_T

$F_g = mg = 1.6\,N$

$F_{net} = F_c = 15.2\,N$

Top

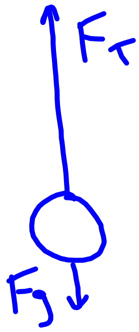
↓ $F_{net} = F_g + F_T$

$F_T = F_{net} - F_g$

$= 15.2 - 1.6$

$= 13.6\,N$

Bottom



↑ $F_{net} = 15.2\,N$

↑ $F_{net} = F_T - F_g$

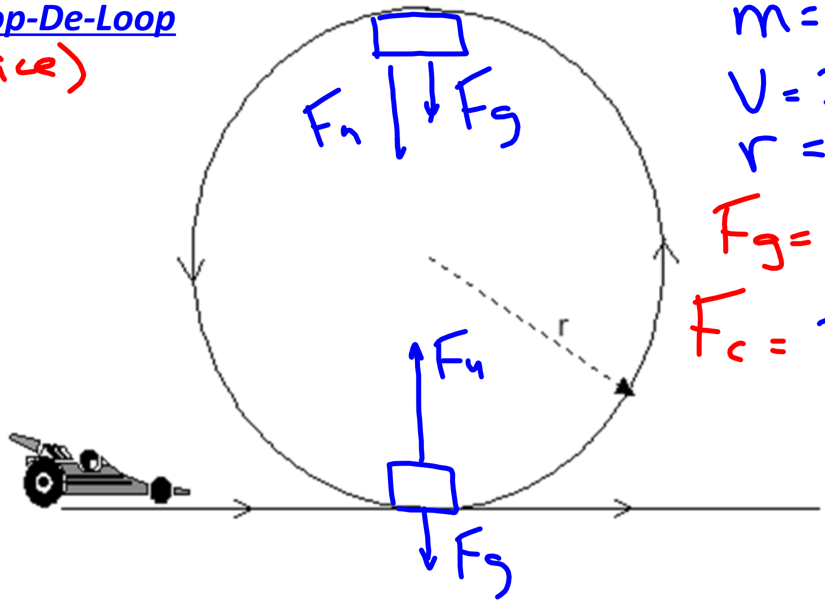
$F_T = F_{net} + F_g$

$= 15.2 + 1.6$

$= 16.8\,N$

Tension at bottom of a loop will always be 2 Fg's bigger than top.

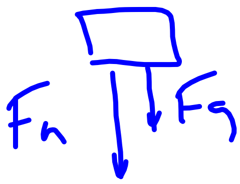
The Loop-De-Loop
(practice)



$m = 200\,kg$.
$v = 20\ m/s$
$r = 25\ m$
$F_g = 1960\,N$
$F_c = 3200\,N$

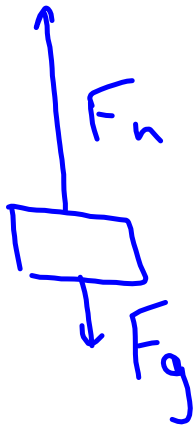
Find normal force at bottom & top.
(assume constant v)

Top



↓ $F_{net} = F_n + F_g$
$F_n = F_{net} - F_g$
$= 1240\,N$

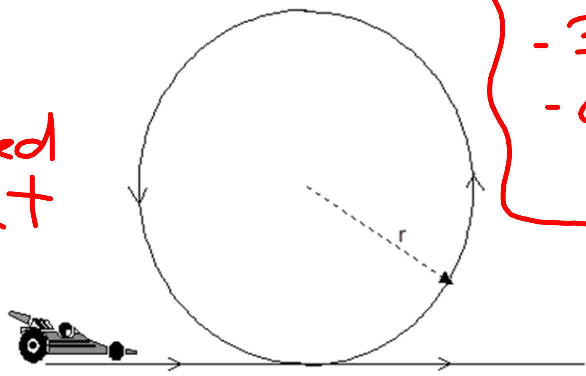
Bottom



↑ $F_{net} = F_n - F_g$
$F_n = F_{net} + F_g$
$= 5160\,N$

5^{th} Gear Video

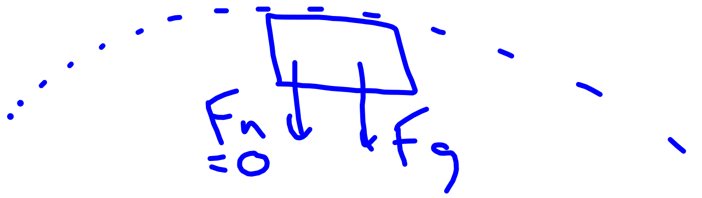
Minimum speed to not fall out of loop.



movie
- 36 mph (16 m/s)
- dia 40 ft

$r \sim 6.1\,m$

assume at top



at point of falling $F_n = 0$

$$F_{net} = \cancel{F_n} + F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$\boxed{V = \sqrt{rg}}$$

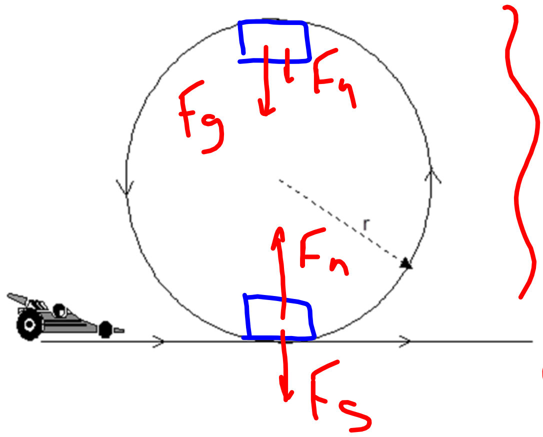
critical velocity (minimum speed to not fall out)

in our example $r = 6.1\ m$

$V = 7.7\ m/s$ at top of loop.

Energy Analysis of the 5th Gear Video,

what is the minimum speed at the bottom top of the loop?



dia - 40ft

V_{min} - 36mph

$r = 6.1m$

$V_{min} = 16\ m/s$ (from movie)

bottom — top

$$E_{TOT\,bot} = E_{TOT\,top}$$

$h = 2 \times r$

$$E_{kb} = E_{kT} + E_{pT}$$

$$\frac{1}{2}mV_b^2 = \frac{1}{2}mV_T^2 + mg(2r)$$

min speed top $V_T = \sqrt{rg}$

$$\frac{1}{2}\cancel{m}V_b^2 = \frac{1}{2}\cancel{m}\left(\sqrt{rg}\right)^2 + \cancel{m}g2r$$

$$\frac{1}{2}V_b^2 = \frac{1}{2}rg + 2rg$$

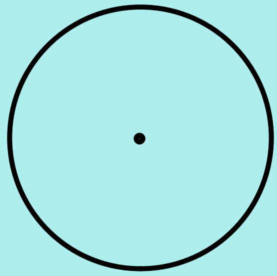
$$\frac{1}{2}V_b^2 = \frac{1}{2}(rg + 4rg)$$

$$V_b^2 = 5rg$$

$$V_b = \sqrt{5rg}$$

$$V_b = 17.3\ m/s$$

'g' Forces

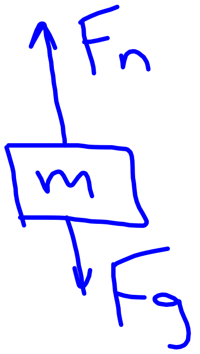


"g-force"

A measurement of the relative magnitude of a force or acceleration that causes a perception of weight.

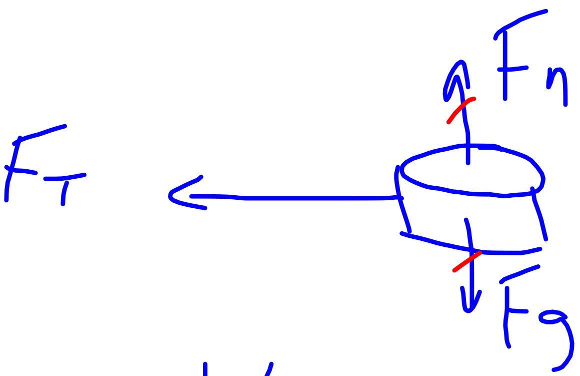
"g-forces" have a direction and are normally stated as a ratio of the force or acceleration compared to gravity (unitless)

Vertical g's

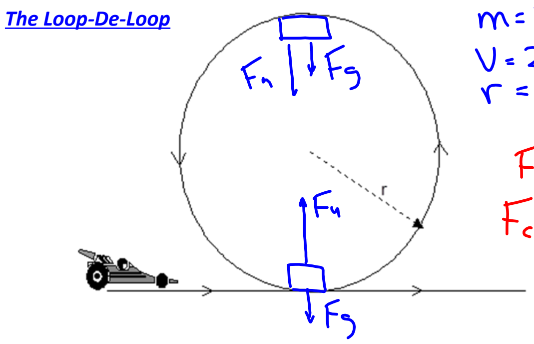


$$g\text{'s} = \frac{\text{support force}}{\text{force gravity}} = \frac{F_n}{F_g}$$

horizontal g's



$$\text{horizontal } g\text{'s} = \frac{F_{net}}{F_g} = \frac{F_T}{F_g}$$

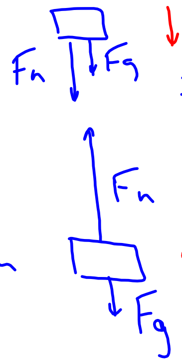


$m = 200 kg$.
$V = 20 m/s$
$r = 25 m$

$F_g = 1960 N$
$F_c = 3200 N$
$= F_{net}$

Find normal force at bottom & top.
(assume constant v)

Top



$\downarrow F_{net} = F_n + F_g$
$F_n = F_{net} - F_g$
$= 1240 N$

Bottom

$\uparrow F_{net} = F_n - F_g$
$F_n = F_{net} + F_g$
$= 5160 N$

3. g's at bottom

$$= \frac{F_n}{F_g} = \frac{5160 N}{1960 N} = 2.6 g's$$

4. g's at top

$$= \frac{F_{n\,top}}{F_g} = \frac{1240}{1960} = 0.6 g's$$

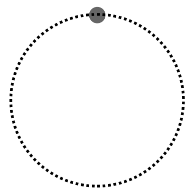
5. min speed set
$F_n = \phi$.

$F_{net} = F_g$

$$\frac{mv^2}{r} = mg$$

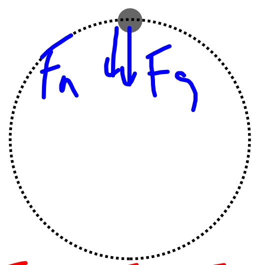
$$V = \sqrt{rg} = \sqrt{25m \times 9.8 m/s^2}$$
$$= 15.6 m/s.$$

G Forces in Vertical Loops (without mass)



"g" force $= \frac{\text{support force}}{\text{force of gravity}}$

"g" forces at top



$F_{net} = F_n + F_g$

$\frac{mv^2}{r} = F_n + mg$

$F_n = \frac{mv^2}{r} - mg$

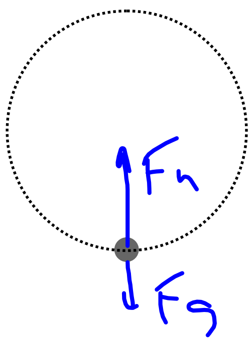
$$g's = \frac{F_n}{F_g}$$

$$= \frac{\frac{mv^2}{r} - mg}{mg}$$

$$= \frac{\frac{v^2}{r} - g}{g}$$

$$= \frac{v^2}{rg} - 1$$

Bottom g's



$F_{net} = F_n - F_g$

$\frac{mv^2}{r} = F_n - mg$

$F_n = \frac{mv^2}{r} + mg$

$$g's = \frac{F_n}{F_g}$$

$$= \frac{\frac{mv^2}{r} + mg}{mg}$$

$$= \frac{v^2}{rg} + 1$$

Homework

Handout - problems 1-6 on loop-de-loop problem sheet

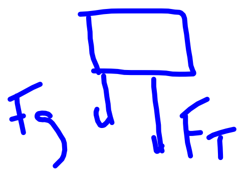
1. A mass of 0.500 kg is being spun on a string in a vertical circle with a radius of 0.750m. The mass is rotating at a frequency of 0.900 c/s.
   a. Determine the force of gravity on the mass.
   b. Find the centripetal force needed to keep it spinning at a constant speed in the vertical circle.
   c. Calculate the tension in the string at the very top and the very bottom of the circle.

a. $F_g = mg = 4.9N$

b. $F_c = 4\pi^2 mrf^2 = 12.0N = F_{net}$

c. Top

F_{net} ↓

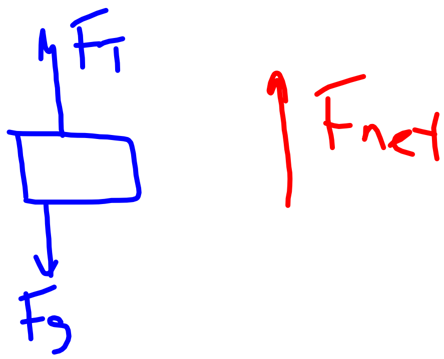


$F_{net} = F_g + F_T$

$\therefore F_T = F_{net} - F_g$

$= 7.1N$

Bottom



$F_{net} = F_T - F_g$

$\therefore F_T = F_{net} + F_g$

$= 16.9N$

$+2F_g$'s

2. An object of mass 3.0 kg is whirled around in a vertical circle or radius 1.3m with a constant velocity of 6.0 m/s. Calculate the maximum and minimum tension in the string.

max tension - bottom

$$F_T = F_{net} + F_g$$
$$= F_c + F_g$$
$$= 112 N$$

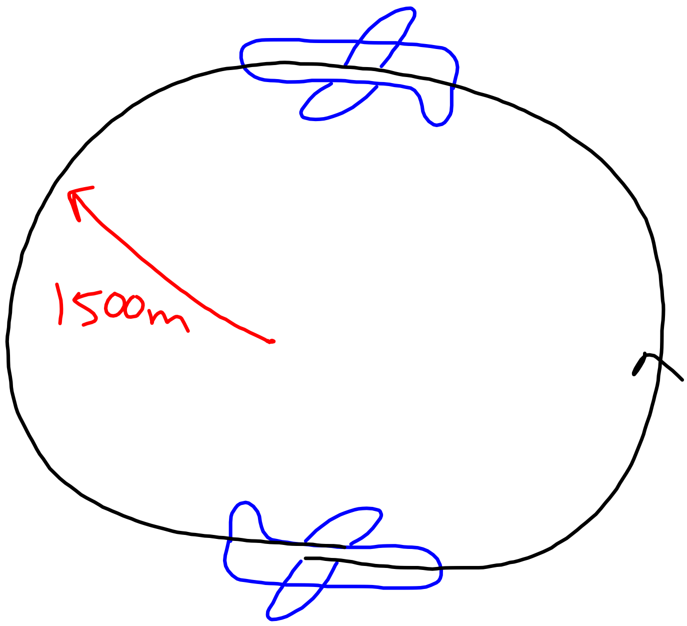
$$F_c = \frac{mv^2}{r}$$

min tension - top

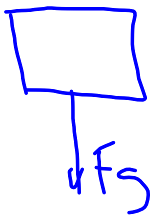
$$F_T = F_{net} - F_g$$
$$= F_c - F_g$$
$$= 54 N$$

3. A plane is flying in a vertical loop of 1500m radius. At what speed is the plane flying at the top of the loop if the vertical force exerted by the air on the plane is zero at this point?

$v = ?$



FBD (top) $(F_{air} = 0)$



$$F_{net} = F_g$$

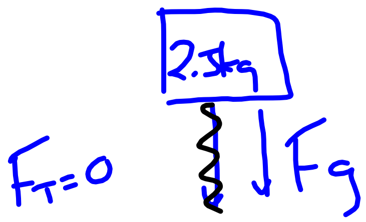
$$\frac{mv^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= \sqrt{1500\text{m} \cdot 9.8\text{m/s}^2}$$

$$v = 121\text{ m/s}$$

4. When you whirl a ball on a cord in a vertical circle, you find a critical speed at the top for which the tension in the cord is zero. This is because the force of gravity on the object itself supplies the necessary centripetal force. How slowly can you swing a 2.5 kg ball like this so that it will just follow a circle with radius 1.5m?



$$F_{net} = F_T + F_g \quad (F_T = 0)$$

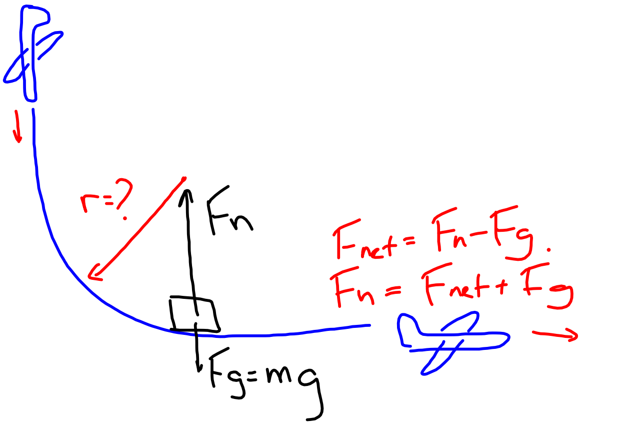
$$\frac{mv^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= 3.8 \text{ m/s}$$

5. The pilot of an airplane, which has been diving at a speed of 540km/h, pulls out of the dive at a constant speed.
a. What is the minimum radius of the plane's circular path in order that the acceleration of the pilot at the lowest point will not exceed 7g?
b. What force is applied on an 80kg pilot by the plane seat at the lowest point of the pull out?

150 m/s



$$g's = \frac{F_n}{F_g}$$

$$g's = \frac{F_{net} + F_g}{F_g} \qquad F_{net} = F_c = \frac{mv^2}{r}$$

$$g's = \frac{\frac{mv^2}{r} + mg}{mg}$$

$$7 = \frac{\frac{v^2}{r} + g}{g}$$

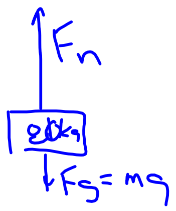
$$7g = \frac{v^2}{r} + g$$

$$6g = \frac{v^2}{r}$$

$$r = \frac{v^2}{6g} = \frac{(150\,m/s)^2}{6 \times 9.8\,m/s^2}$$

$$= 383\,m$$

b.



$$F_n = F_{net} + F_g$$

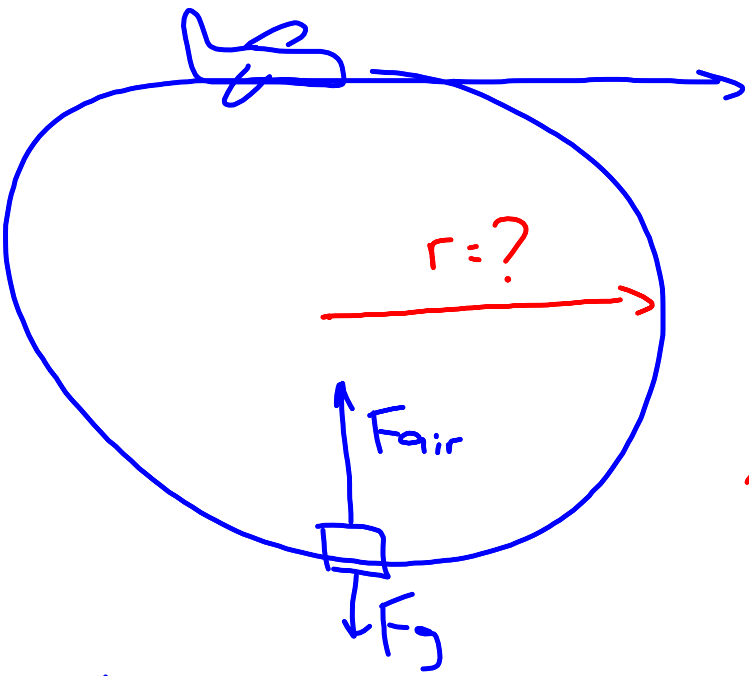
$$= \frac{mv^2}{r} + mg$$

$$= 5490\,N \qquad (7\,mg)$$

6. Snoopy is flying his vintage war plane in a "loop-de-loop" path chasing the Red Baron. His instruments tell him that the plane is level (i.e. at the bottom of the loop) and travelling at 180km/h. He is sitting on a set of bathroom scales and notes that they read four times the normal force of gravity on him. What is the radius of the loop?

$V = 50\ m/s$

"g"'s = 4



$F_{net} = \frac{mv^2}{r}$

bottom

$$g's = \frac{F_{net} + F_g}{F_g}$$

$$g's = \frac{v^2}{rg} + 1$$

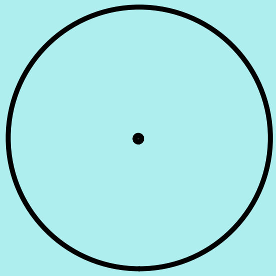
$$4 = \frac{v^2}{rg} + 1$$

$$3 = \frac{v^2}{rg}$$

$$r = \frac{v^2}{3g} = \frac{(50)^2}{3 \cdot 9.8}$$

$$= 85\ m$$

Causes of Centripetal Forces



Tension

Gravity

Normal Force

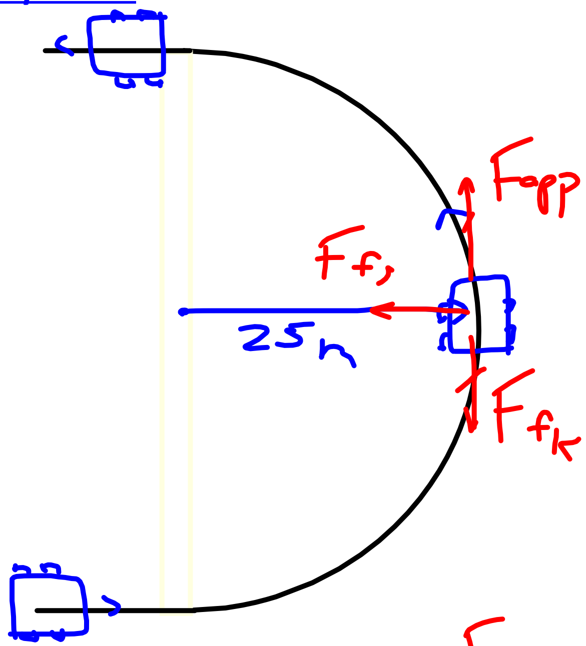
Friction

~~Electromagnetic Force~~

Friction as a Source of Centripetal Force

Part A : Calculate the frictional force that "keeps" the car from skidding out of the corner.

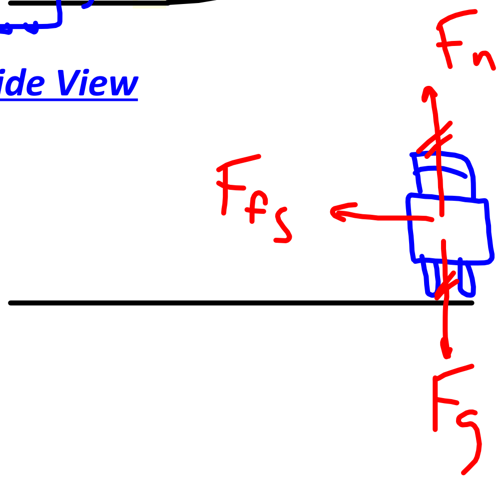
Top View



$m = 2000$ kg
$r = 25$ m
$\mu_s = 0.85$
$\mu_k = 0.65$
$V = 12$ m/s

$$F_{fs} \neq \mu_s F_n$$

Side View



$$F_{net} = F_{fs}$$

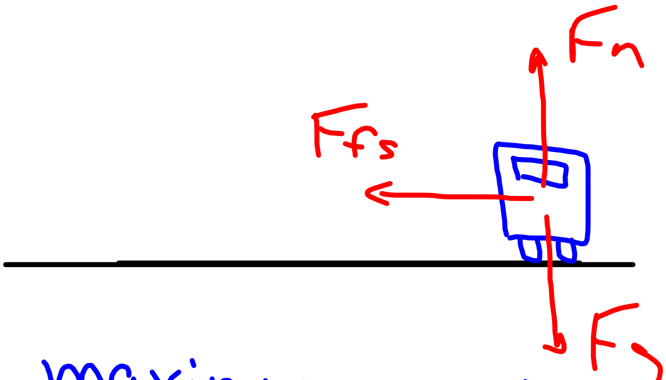
$$\frac{mv^2}{r} = F_{fs}$$

$$F_{fs} = \frac{(2000\,kg)(12\,m/s)^2}{25\,m}$$

$$= 11{,}500\,N$$

Part B : Calculate the maximum speed the car could take into the corner without skidding out.

Side View



$m = 2000$ kg

$r = 25$ m

$\mu_s = 0.85$

$\mu_k = 0.65$

$V_{max} = ???$

maximum speed is when $F_{net} = F_{fs}$

$$\frac{mv^2}{r} = \mu_s F_n$$

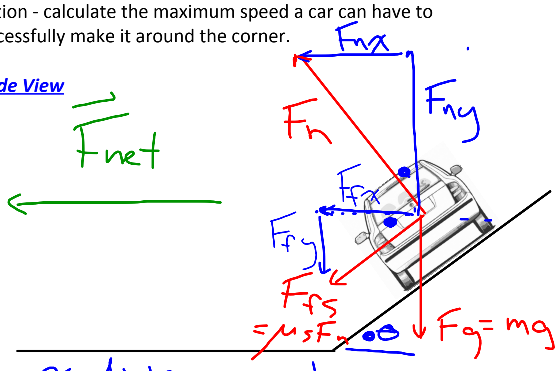
$$\frac{mv^2}{r} = \mu_s mg$$

$$V_{max} = \sqrt{\mu_s r g} = 14.4 \text{ m/s}$$

Part C : Maximum Speed on a Banked Corner

Given a ramp angle, radius of curvature and coefficient of static friction - calculate the maximum speed a car can have to successfully make it around the corner.

Side View



x dir'n

$$F_{fx} + F_{nx} = F_{net}$$

$$\frac{mv^2}{r} = F_{fx} + F_{nx} \quad ①$$

y dir'n

$$F_{ny} = F_g + F_{fy}$$

$$F_{ny} = mg + F_{fy} \quad ②$$

① $$\frac{mv^2}{r} = \mu_s F_n \cos\theta + F_n \sin\theta$$

$$= F_n(\mu_s \cos\theta + \sin\theta)$$

② $$F_n \cos\theta = mg + \mu F_n \sin\theta$$

$$F_n(\cos\theta - \mu\sin\theta) = mg$$

$$F_n = \frac{mg}{\cos\theta - \mu\sin\theta}$$

Sub'n ① for F_n

$$\frac{mv^2}{r} = \frac{mg}{\cos\theta - \mu\sin\theta}(\mu\cos\theta + \sin\theta)$$

$$v = \sqrt{\frac{rg(\mu\cos\theta + \sin\theta)}{\cos\theta - \mu\sin\theta}}$$

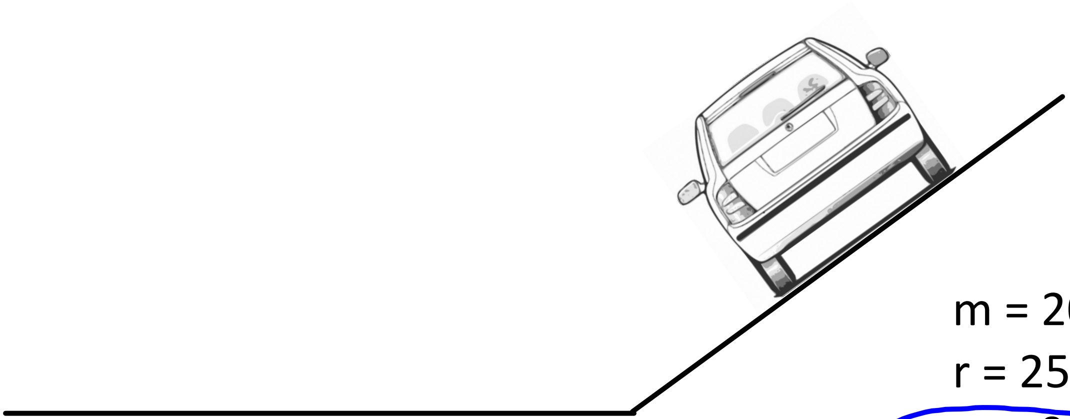
boundary condition

$\theta = 0$, $v = \sqrt{\mu r g}$

$\mu = 0$ (no friction)

$v = \sqrt{rg\tan\theta}$

Part C : Calculate the maximum speed the car could take into the ramped corner without skidding out.



m = 2000 kg
r = 25 m
μ_s = 0.85
μ_k = 0.65
θ = 24
V_{max} = ???

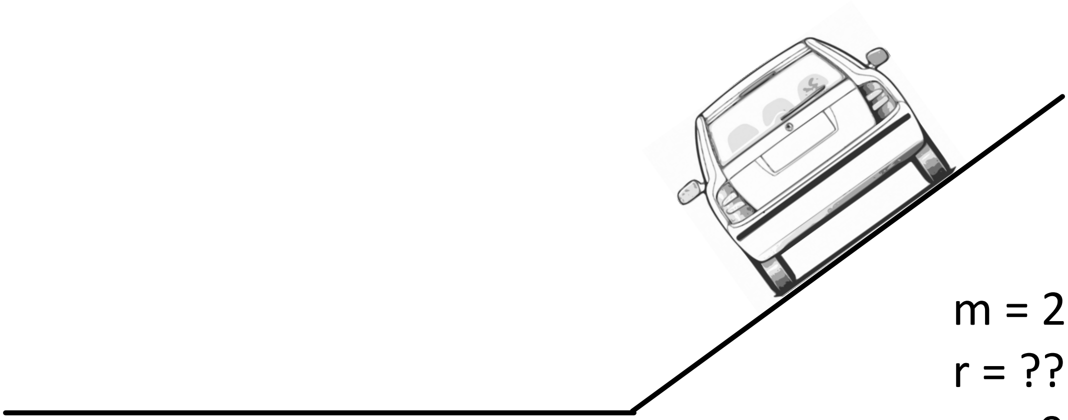
$$V_{max} = \sqrt{\frac{rg(\mu\cos\theta + \sin\theta)}{(\cos\theta - \mu\sin\theta)}}$$

$= 22.6\ m/s$

(22.5952)

recall when
$\theta = 0$ (flat)
$V_{max} = 14.4\ m/s$

Part D : Calculate the minimum radius of the curve for a given speed.

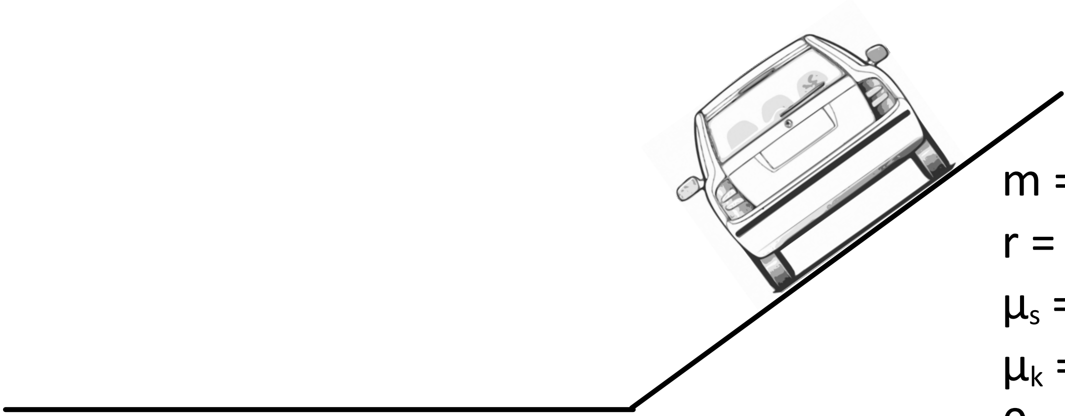


$m = 2000$ kg
$r = ??$
$\mu_s = 0.85$
$\mu_k = 0.65$
$\theta = 24$
$V_{max} = 30$m/s

$$r = \frac{v^2(\cos\theta - \mu\sin\theta)}{g(\mu\cos\theta + \sin\theta)}$$

$= 44$ m

Part E : Find the formula for the minimum angle for a given velocity. & radius.



m = 2000 kg
r = 25 m
μ_s = 0.85
μ_k = 0.65
θ = ??
V_{max} = 30m/s

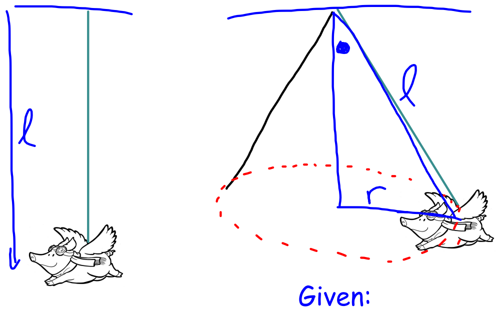
$$\theta = \tan^{-1}\left(\frac{V^2 - \mu rg}{\mu V^2 + rg}\right)$$

$$\theta = 34^\circ$$

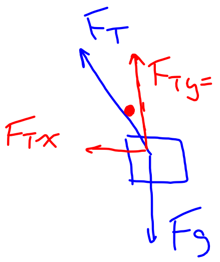
Conical Pendulums
The Flying Pig

Calculate the tension in the string and predict the radius of curvature

$r = \ell \sin\theta$



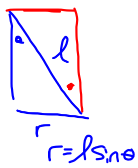
Given:
length = 2.12 m
mass = 0.15 25 kg
period = 2.84



$F_{Ty} = F_g = F_T \cos\theta$

$F_{Tx} = F_{net}$

$F_{Tx} = F_T \sin\theta$



$\frac{mv^2}{r} = F_T \sin\theta$

$\frac{4\pi^2 m r}{T^2} = F_T \sin\theta$

$\frac{4\pi^2 m \ell \sin\theta}{T^2} = F_T \sin\theta$

① $F_T = \frac{4\pi^2 m \ell}{T^2}$

$= 1.58 N$

② $r = ?$

$F_{Ty} = mg$

$F_T \cos\theta = mg$

$\theta = \cos^{-1}\left(\frac{mg}{F_T}\right)$

$= 20°$

$r = \ell \sin\theta$

$= 0.73 m$