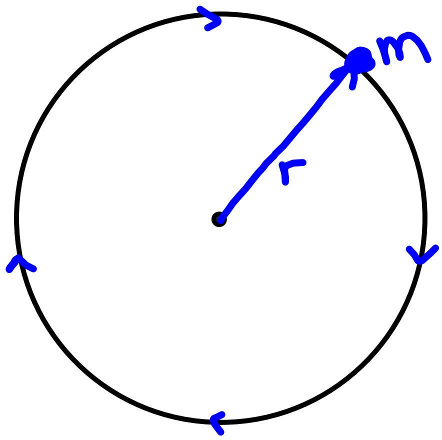
Nov, 2019

Centripetal Motion and Gravity
Some Summary Notes

Uniform Circular Motion - summary



moving at a constant speed in a circular motion.

$a_c = \frac{v^2}{r}$ "centripetal" acceleration → "centre seeking"

$F_c = \frac{mv^2}{r}$

what if we don't know velocity (speed)?

given (measure) → radius (m)
→ time (s)
→ mass (kg)

distance travelled in 1 rev = circumference = $2\pi r$
time to travel 1 rev = T (period)

$v = \frac{dist}{time} = \frac{2\pi r}{T}$ $F_c = \frac{mv^2}{r}$

$F_c = \frac{m(2\pi r)^2}{rT^2}$

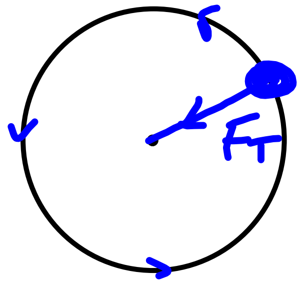
$F_c = \frac{4\pi^2 r m}{T^2}$

$f = \frac{1}{T}$ or $T = \frac{1}{f}$

$F_c = 4\pi^2 r m f^2$

f → cycles/sec
T → sec/cycle

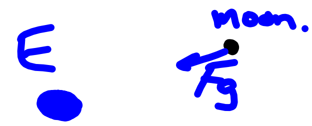
Causes of Centripetal Forces



$F_{net} = F_T$

Tension
→ puck on string

Gravity → orbit of a planet or moon



Friction → between tires & car
to help turn corners

Normal Force → supporting a car in a loop-de-loop

Electromagnetic Force → electrons in "orbit"
around nucleus of atom

PLANETARY DATA SHEET

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO
Mass (10^{24}kg)	0.33	4.87	5.97	0.642	1899	568	86.8	102	0.0125
Diameter (km)	4879	12,104	12,756	6792	142,984	120,536	51,118	49,528	2390
Radius (km)	2,440	6,052	6,378	3,396	71,492	60,268	25,559	24,764	1,195
Density (kg/m^3)	5427	5243	5515	3933	1326	687	1270	1638	1750
Gravity (m/s^2)	3.7	8.9	9.8	3.7	23.1	9	8.7	11	0.6
Escape Velocity (km/s)	4.3	10.4	11.2	5	59.5	35.5	21.3	23.5	1.1
Rotation Period (hours)	1407.6	-5832.5	23.9	24.6	9.9	10.7	-17.2	16.1	-153.3
Length of Day (hours)	4222.6	2802	24	24.7	9.9	10.7	17.2	16.1	153.3
Distance from Sun (10^6 km)	57.9	108.2	149.6	227.9	778.6	1433.5	2872.5	4495.1	5870
Perihelion (10^6 km)	46	107.5	147.1	206.6	740.5	1352.6	2741.3	4444.5	4435
Aphelion (10^6 km)	69.8	108.9	152.1	249.2	816.6	1514.5	3003.6	4545.7	7304.3
Orbital Period (days)	88	224.7	365.2	687	4331	10,747	30,589	59,800	90,588
Orbital Velocity (km/s)	47.9	35	29.8	24.1	13.1	9.7	6.8	5.4	4.7
Orbital Inclination (degrees)	7	3.4	0	1.9	1.3	2.5	0.8	1.8	17.2
Orbital Eccentricity	0.205	0.007	0.017	0.094	0.049	0.057	0.046	0.011	0.244
Axial Tilt (degrees)	0.01	177.4	23.4	25.2	3.1	26.7	97.8	28.3	122.5
Mean Temperature (C)	167	464	15	-65	-110	-140	-195	-200	-225
Number of Moons	0	0	1	2	63	62	27	13	3
Ring System?	No	No	No	No	Yes	Yes	Yes	Yes	No
Global Magnetic Field?	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Unknown

	SUN
Mass (10^{24}kg)	1,980,000
Diameter (km)	1,390,000
Radius (km)	695,000
Density (kg/m^3)	1,400
Gravity (m/s^2)	274
Escape Velocity (km/s)	618
Rotation Period (hours)	594
Length of Day (hours)	na
Distance from Sun (10^6 km)	na
Perihelion (10^6 km)	na
Aphelion (10^6 km)	na
Orbital Period (days)	na
Orbital Velocity (km/s)	na
Orbital Inclination (degrees)	na
Orbital Eccentricity	na
Axial Tilt (degrees)	na
Mean Temperature (C)	6000
Number of Moons	na
Ring System?	na
Global Magnetic Field?	Yes

	MOON
Mass (10^{24}kg)	0.073
Diameter (km)	3475
Radius (km)	1,738
Density (kg/m^3)	3340
Gravity (m/s^2)	1.6
Escape Velocity (km/s)	2.4
Rotation Period (hours)	655.7
Length of Day (hours)	708.7
Distance from Earth (10^6 km	0.384
Perihelion (10^6 km)	0.363
Aphelion (10^6 km)	0.406
Orbital Period (days)	27.3
Orbital Velocity (km/s)	1
Orbital Inclination (degrees)	5.1
Orbital Eccentricity	0.055
Axial Tilt (degrees)	6.7
Mean Temperature (C)	-20
Number of Moons	0
Ring System?	No
Global Magnetic Field?	No

source : http://nssdc.gsfc.nasa.gov/planetary/factsheet/index.html

Force of Gravity Review

Using the universal gravity formula calculate the force of gravity between the earth and the sun.

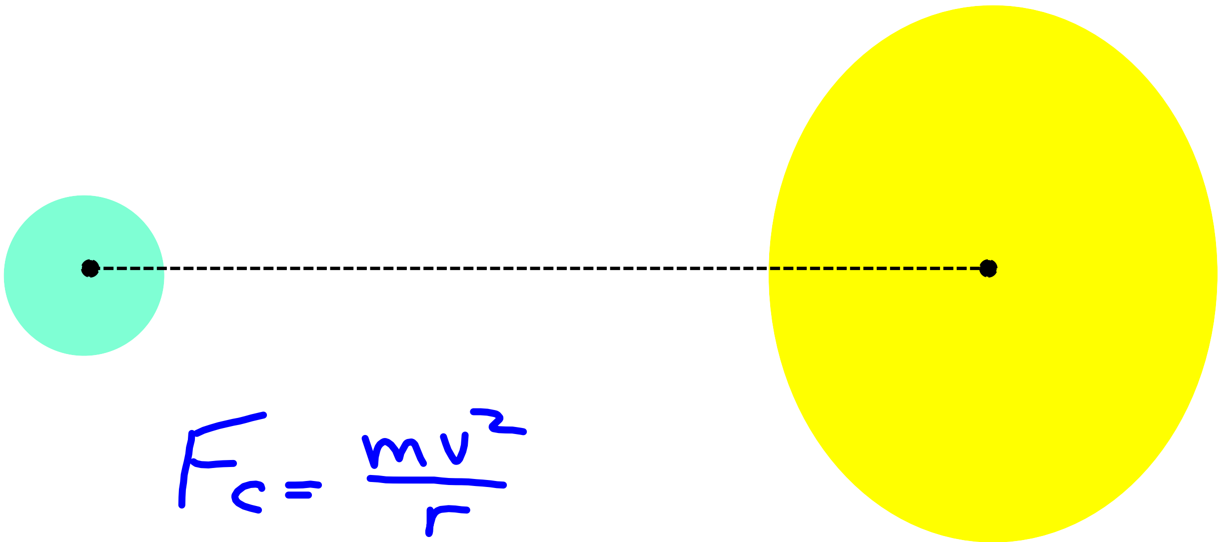
$$F_g = \frac{G m_1 m_2}{r^2}$$

$$F_g = \frac{(6.67\times10^{-11})(5.97\times10^{24})(1.98\times10^{30})}{(149.6\times10^{9})^2}$$

$$= 3.5\times10^{22}\,N$$

Centripetal Force

Using centripetal acceleration calculate the force of gravity between the earth and the sun.



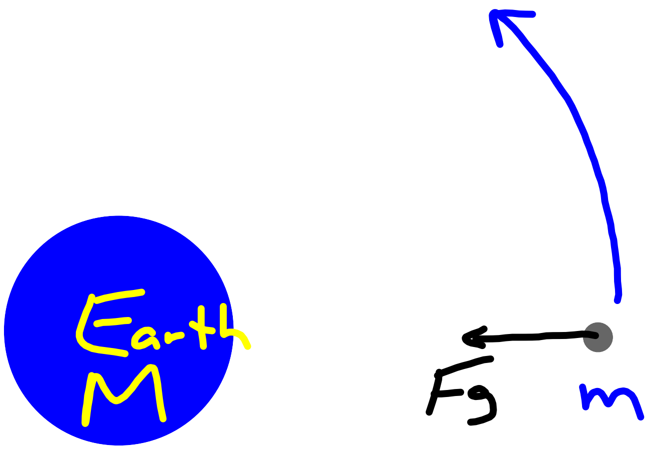
$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{4\pi^2 m r}{T^2}$$

$$F_c = \frac{4\pi^2 (5.97\times10^{24})(149.6\times10^{9})}{(365.2\times24\times3600)^2}$$

$$= 3.5\times10^{22}\,N$$

Gravity and Centripetal Forces (a deeper dive)



if only forces affecting motion are gravity.

then $F_{net} = F_g$

$$\frac{\cancel{m}v^2}{r} = \frac{G\cancel{m}M}{r^2}$$

$$V = \sqrt{\frac{GM}{r}}$$

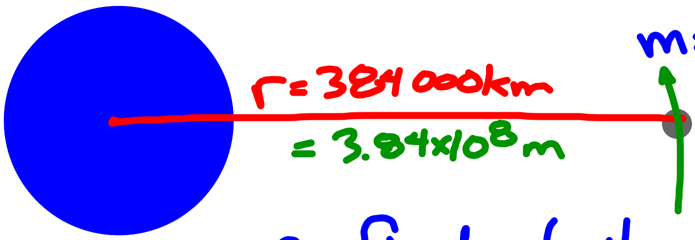
$$M = \frac{v^2 r}{G}$$

Example : Earth - Moon System

$G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$

$M = 5.98 \times 10^{24}\ kg$

$m = 7.3 \times 10^{22}\ kg$.



a. find velocity of moon's orbit in m/s.

b. find period (T) of moon in days.

a. $V = \sqrt{\frac{GM}{r}} = 1020\ m/s$ $(\sim 3700\ km/hr)$

b. constant speed $V = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{V}$$

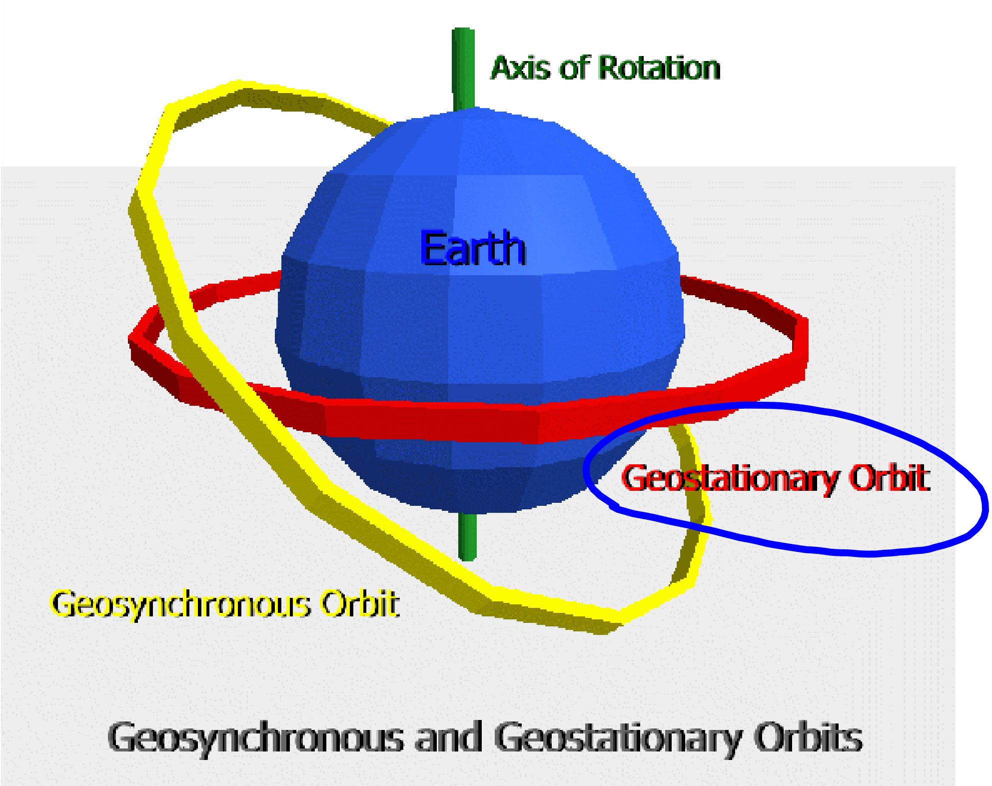
$$= \frac{2\pi \times 3.84 \times 10^8\ m}{1020\ m/s}$$

$$= 2.37 \times 10^6\ s$$

$\div 60 \div 60 \div 24$

$$= 27.3\ days$$

Geostationary Orbit



Geostationary Orbit

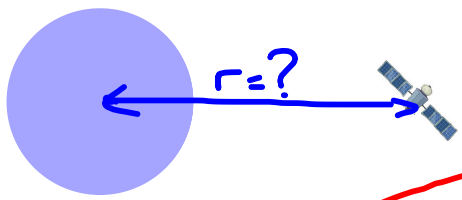
Two Requirements:

1. Path of the orbit must be above the equator
2. Period of the orbit must be 24 hours.

Using data from your planetary data sheet, calculate the orbital radius of a satellite in geosynchronous orbit.

$T = 24\text{hrs}$

$M_E = 5.97\times10^{24}\text{kg}$



$$M = \frac{v^2 r}{G} \qquad v = \frac{2\pi r}{T}$$

$$M = \frac{4\pi^2 r^3}{G T^2}$$

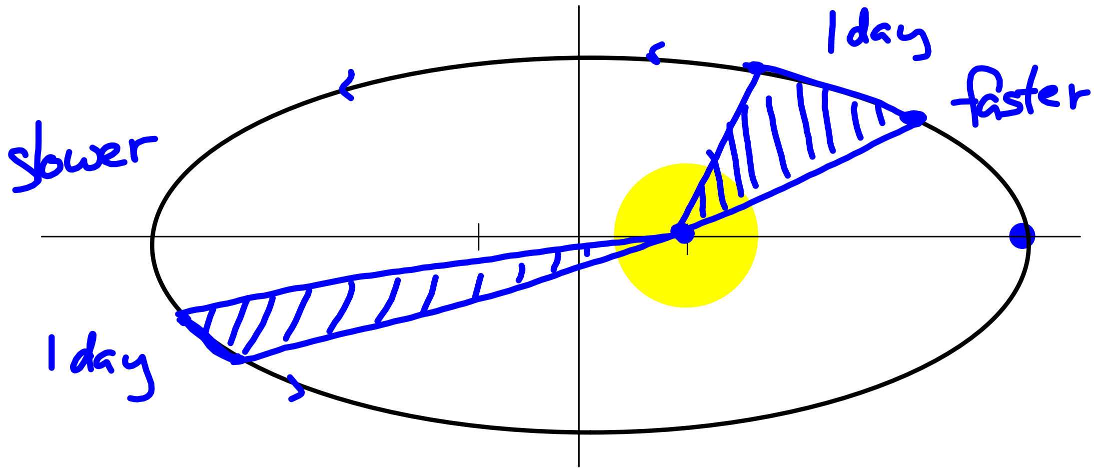
$$r = \sqrt[3]{\frac{G T^2 M}{4\pi^2}}$$

$= 42,000,000\text{m}$

$= 42,000\text{km}$

altitude $\sim 36,000\text{km}$

Kepler's Laws



1. All planets (satellites) orbit in elliptical patterns.

2. in equal time periods the planet will sweep out equal areas.

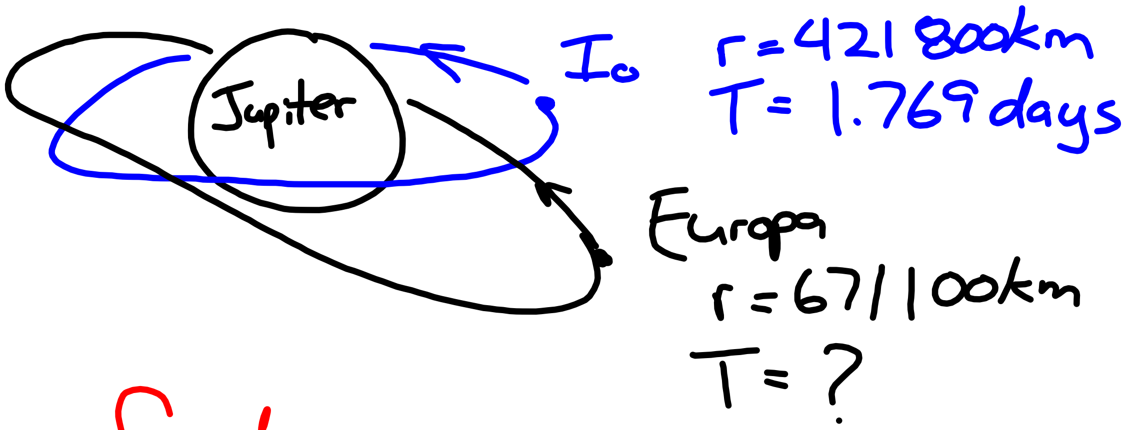
3. ratio of radius cubed to period (T) squared is a constant for orbiting planets.

$$M = \frac{4\pi^2 r^3}{G T^2}$$

$$\frac{r^3}{T^2} = \text{constant}$$

Kepler's Laws - example

Jupiter's moon Io has an orbital period of 1.77 days and orbits at a radius of 422000km. A second moon Europa orbits at 671000km, find the period of Europa's orbit.



find T_E

$$\frac{T_E^2}{r_E^3} = \frac{T_I^2}{r_I^3}$$

$$T_E = \sqrt{\frac{r_E^3 \times T_I^2}{r_I^3}}$$

$$= \sqrt{\frac{(671100)^3 \times (1.769)^2}{(421800)^3}}$$

$= 3.55$ days.

Uniform Circular Motion

* $a_c = \frac{v^2}{r}$

$F = \frac{mv^2}{r}$

$F = \frac{4\pi^2 m r}{T^2}$

$F = 4\pi^2 m r f^2$

$\frac{GmM}{r^2} = \frac{mv^2}{r}$

$M = \frac{v^2 r}{G}$

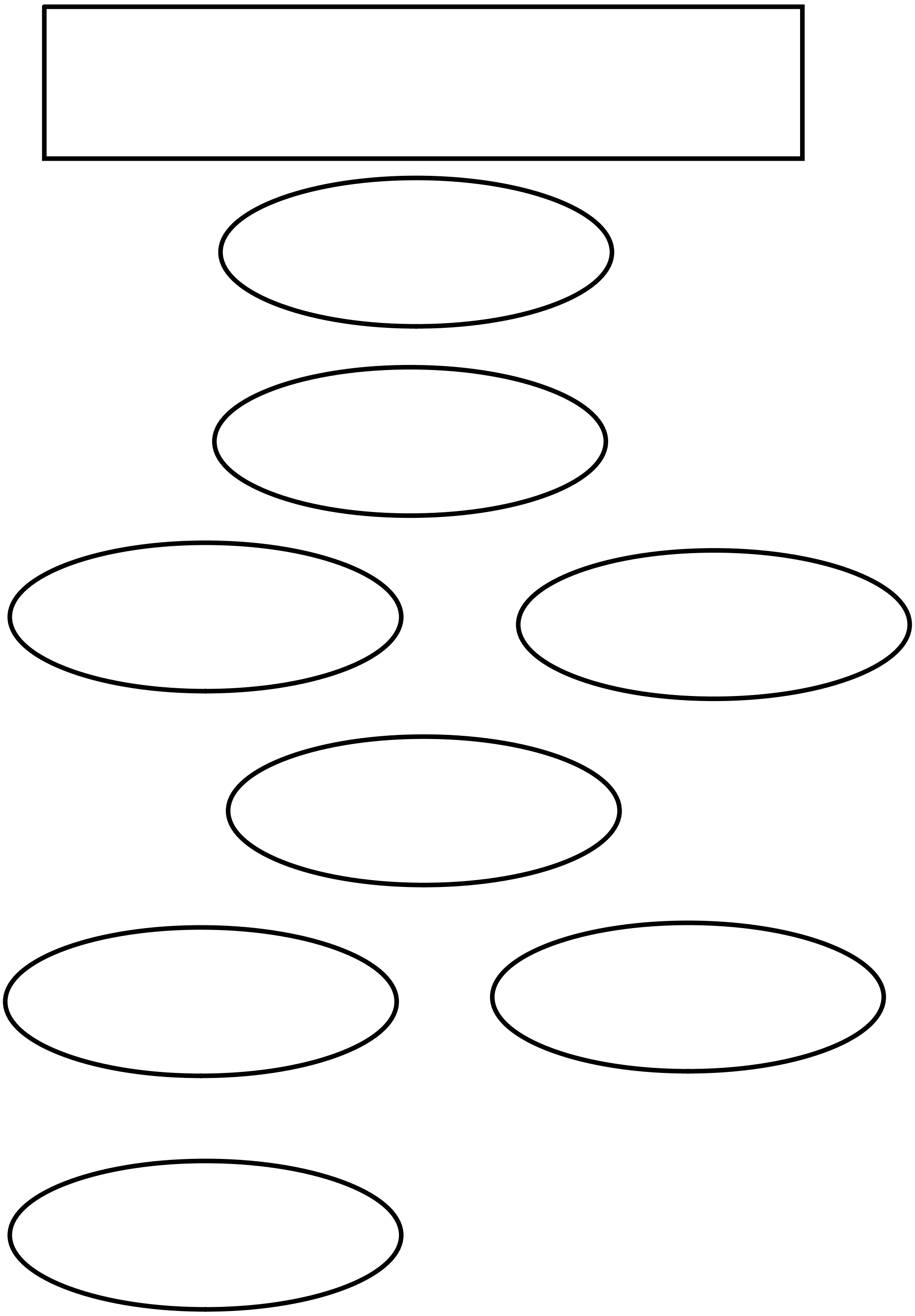
$v = \sqrt{\frac{GM}{r}}$

$M = \frac{4\pi^2 r^3}{GT^2}$

$v = \frac{dist}{time} = \frac{2\pi r}{T}$

$f = \frac{1}{T}$

$\frac{r^3}{T^2}$ constant



Homework

Handout - Centripetal Force meets Gravitational Force (answers on line)

Coming Up

Centripetal and Sport

Vertical Loops