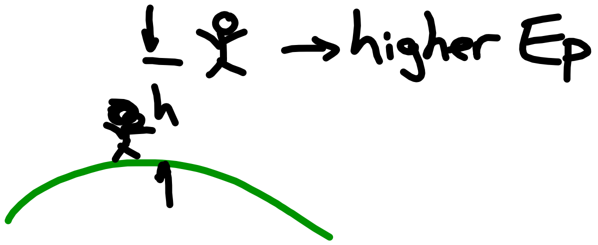
Mechanical Energy

Definition - Energy of position or motion

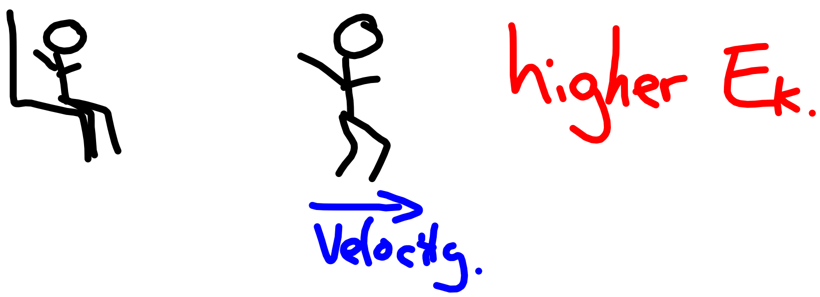
Gravitational Potential Energy - E_p

- energy possessed by an object due to it's height in a gravitational field.



Kinetic Energy - E_k

- energy possessed by an object due to its relative motion compared to a second object.



Examples

A. Energy of motion : ***What is the work required (i.e. the change in energy) to accelerate a 10kg object from 0 to 30m/s in 20s?***

$W = F \cdot d$
$= 15N \cdot 300m$
$= 4500J$

$F = ma$
$= m\left(\frac{V_2 - V_1}{\Delta t}\right)$ 2
$= 10\left(\frac{30-0}{20}\right)$
$= 15N$

$d = \left(\frac{V_1 + V_2}{2}\right)\Delta t$ 1
$= \left(\frac{0+30}{2}\right)20$
$= 300m$

Predict - What if we wanted to accelerate the same object as in Part A from 0 to 30m/s, but this time in 5s? Would the energy requirement be higher, lower or the same?

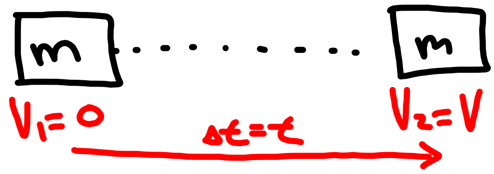
$W = F \cdot d$
$= 60N \cdot 75m$
$= 4500J$

$F = ma$
$= m\left(\frac{V_2 - V_1}{\Delta t}\right) = 60N$

$d = \left(\frac{V_1 + V_2}{2}\right)\Delta t = 75m$

Standard Formula for Kinetic Energy

Calculate the work required to accelerate an object of mass, m, from rest to velocity V (m/s) in t seconds.



$$W = F \cdot d$$

$$F = ma = m\left(\frac{V_2 - V_1}{\Delta t}\right) = m\left(\frac{V - 0}{t}\right) = \frac{mV}{t}$$

$$d = \left(\frac{V_1 + V_2}{2}\right)\Delta t = \left(\frac{0 + V}{2}\right)t = \frac{Vt}{2}$$

$$W = \left(\frac{mV}{\cancel{t}}\right)\left(\frac{V\cancel{t}}{2}\right) = \frac{mV^2}{2}$$

$$* \boxed{E_k = \frac{1}{2}mv^2}$$

called kinetic energy

- energy a moving object has relative to not moving.

Examples:

B. *Calculate the kinetic energy of an electron ($m=9.11x10^{-31}kg$) moving at 80% of the speed of light ($c=3.0x10^{8}m/s$)*

$E_k = \frac{1}{2}mv^2$

$V = 0.80 \times 3.0 \times 10^{8}\ m/s$
$= 2.4 \times 10^{8}\ m/s$

$= \frac{1}{2}(9.11 \times 10^{-31}kg)(2.4 \times 10^{8}\ m/s)^2$
$= 2.6 \times 10^{-14}\ J$

$kg\ m^2/s^2$
$= 1 J$

C. *Calculate the kinetic energy of 1500kg car moving at 50 km/hr.*

$E_k = \frac{1}{2}mv^2$

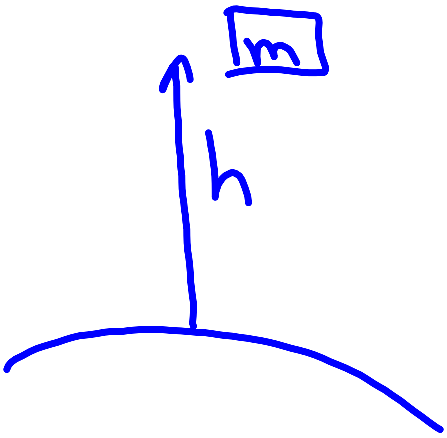
$= \frac{1}{2}(1500kg)(13.89\ m/s)^2$
$= 1.4 \times 10^{5}\ J$ ✓
~~144676 J~~
145000 J ✓
140000 J ✓

$m/s \xrightarrow{\times 3.6} km/hr$
$km/hr \xrightarrow{\div 3.6} m/s$

$50\ km/hr = 50 \div 3.6\ m/s$
$= 13.89\ m/s.$

Gravitational Potential Energy

energy an object has due to its height above the ground.



$W = \Delta E = F \times d$

$F_g = mg$ h

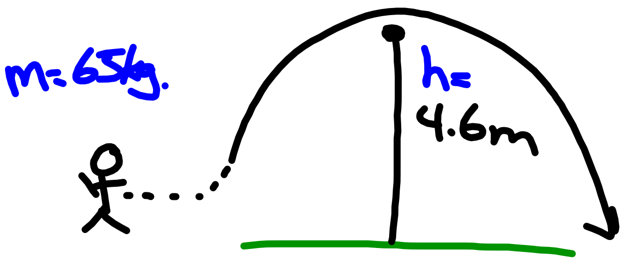
$\Delta E = mgh$

$$\boxed{E_p = mgh} \ *$$

$E_{p} =$

Example

A. Pole Vaulting : ***A 65 kg pole vaulter can clear a bar that is 4.6m above the track. What is the potential energy of the pole vaulter at the top of the jump?***



$E_p = mgh$

$= 65kg \times 9.8 m/s^2 \times 4.6m$

$= 2900 J$

Conservation of Energy

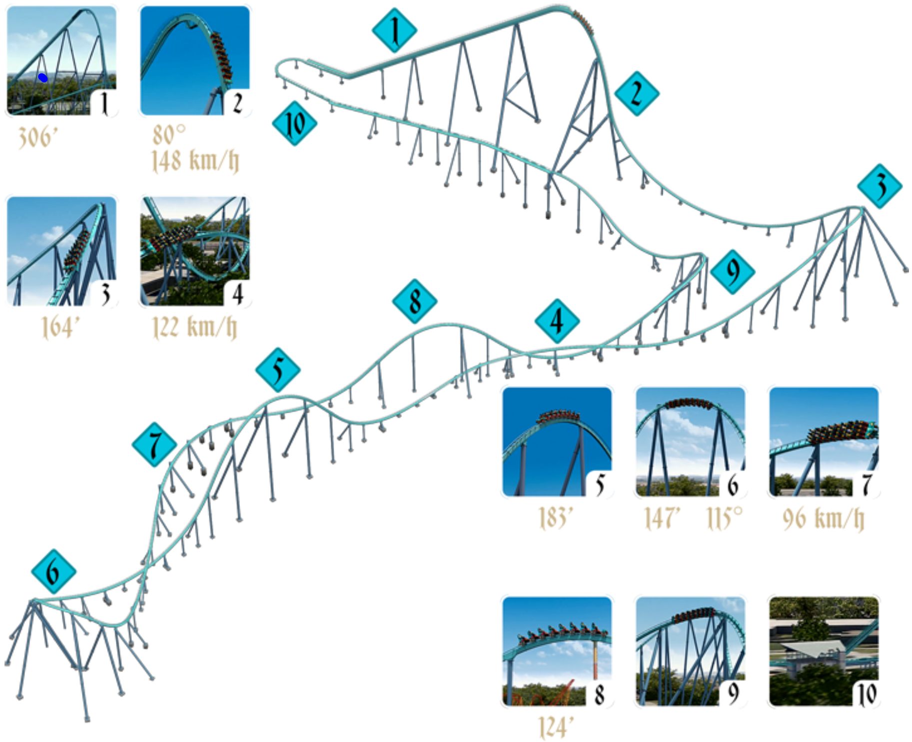
Energy can neither be created nor destroyed.

It can only change forms or be transferred from one object to another.

$$E_{Ti} = E_{Tf}$$

total initial energy = total final energy

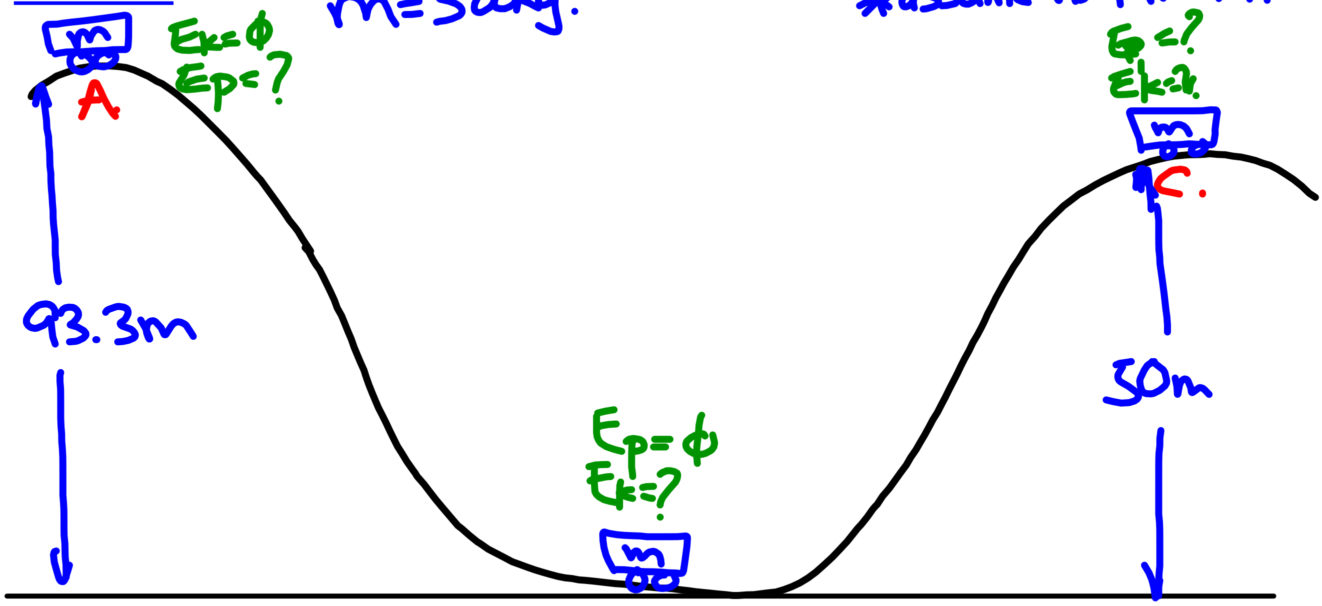
Leviathan



Leviathan

$m = 500kg.$

*assume no friction.



A.

$E_p = 457000$

$E_k = \Phi$

$E_T = 457000$

$V = \Phi\ m/s.$

B.

$E_p = 0$

$E_k = 457000$

$E_T = 457000$

$$V = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2 \times 457000}{500}}$$

$$= 42.8\ m/s.$$

$\sim 154\ km/hr.$

C

$500 \times 9.8 \times 50$

$E_p = 245\ 000$

$E_k = 212\ 000$

$E_T = 457000$

$$V = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2 \times 212000}{500}}$$

$$= 29.1\ m/s.$$

Warm Up / Practice Problems

1. Find the work required to slow down a 1,500kg automobile from 100 km/hr to 50 km/hr in a distance of 100m.

2. Find the work done by gravity when a 75 kg parachutist falls 750 metres.

1.

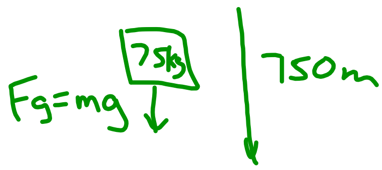
100 km/hr → $27.78\,m/s$ 50 km/hr → $13.89\,m/s$

$W = \Delta E = F \times d$

$W = \Delta E$
$= E_{k_2} - E_{k_1}$
$= \frac{1}{2}(1500)(13.89)^2 - \frac{1}{2}(1500)(27.78)^2$
$= -434000\,J$

$F = ma$
$\Delta d = 100,\ V_1 = 27.78\,m/s$
$V_2 = 13.89\,m/s$
$a = \frac{V_2^2 - V_1^2}{2\Delta d}$
$= -2.89\,m/s^2$
$F = ma$
$= 1500\,kg \times (-2.89\,m/s^2)$
$= -4340\,N$
$W = F \times d$
$= -4340\,N \cdot 100\,m$
$= -434000\,J$

#2



$W = F \times d$
$= m \cdot g \cdot \Delta d$
$= 75\,kg \times 9.8\,m/s^2 \times 750\,m$
$= 551250\,J$
$= 5.5 \times 10^5\,J$
$(550000\,J)$

What if all of the potential is not converted into kinetic energy?

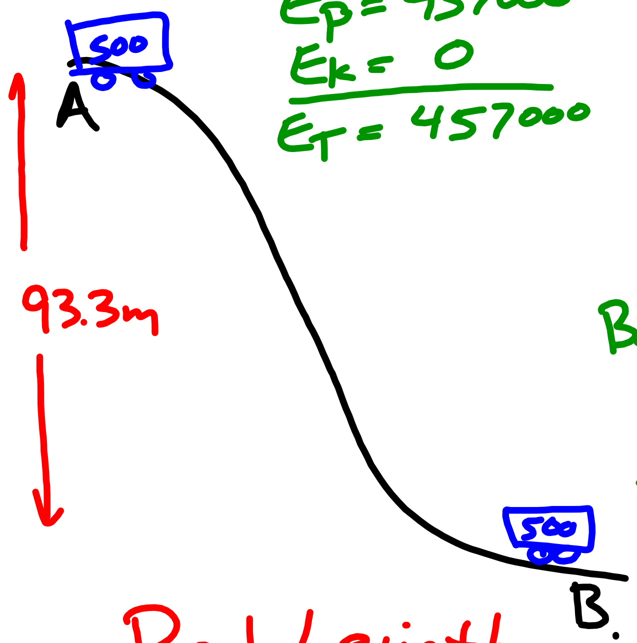
Where did it go?

Typically energy is "lost" to heat (friction), sound, light (radiant energy)

$$\text{efficiency} = \frac{\text{useful energy out}}{\text{total input energy}} \times 100\%$$

$$\boxed{\text{efficiency} = \frac{E_{out}}{E_{in}} \times 100\%}$$

Example #1 : Calculate the efficiency of the Leviathan (a gravity machine that convert potential energy into kinetic energy)



$E_p = 457000$
$E_k = 0$

$E_T = 457000$

Bottom (no losses)
$E_p = 0$
$E_k = 457000 \rightarrow V = \sqrt{\frac{2E_k}{m}}$
$= 42.8\ m/s$

$E_T = 457000$

Real Leviathan

Velocity $= 148\ km/hr$
$= 41.1\ m/s \Rightarrow E_k = \frac{1}{2}mv^2$
$= 423000\ J$

energy
$E_p = 0$
$E_k = 423000$
$E_{lost} = 34000$ *

$E_T = 457000$

$$\text{efficiency} = \frac{E_{out}}{E_{in}} \times 100\%$$
$$= \frac{423000}{457000} \times 100\%$$
$$= 92.6\%$$

Example #2 - Efficieny of a light bulb

An incandescent lamp requires 1.0×10^4 J of electrical energy to create 400J of useful (i.e. visible) light energy. What is the effeciency of the incandescent bulb?
Where did the lost energy go

conversion from
electrical energy to light energy.

$$\text{efficiency} = \frac{E_{out}}{E_{in}} \times 100\% = \frac{400\,J}{1.0 \times 10^4\,J} \times 100\%$$
$$= 4\%$$

lost to heat,
lost to other forms of radiation.

Power

Power is the rate of energy transformation or doing work.

$$\text{Power} = \frac{\text{change of energy}}{\text{time}}$$

$$= \frac{\text{work}}{\text{time}}$$

$$\boxed{P = \frac{\Delta E}{t}}$$

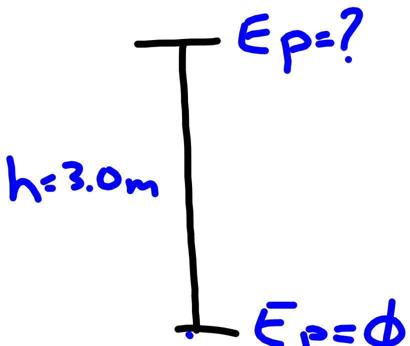
power can be energy consumed or generated.

$$\text{units} = \frac{\text{Joules}}{\text{sec}}$$

$$= \text{Watt}$$

Examples:

1. A 75 kg man climbs up a 3.0 m ladder in 4.7s. What power must he generate to make this climb?



$P = \frac{\Delta E}{t}$

ΔE = gain in potential

$P = \frac{\text{gain in potential}}{t}$

$= \frac{mgh}{t}$

$= \frac{(75\,kg)(9.8\,m/s^2)(3.0\,m)}{4.7\,s}$

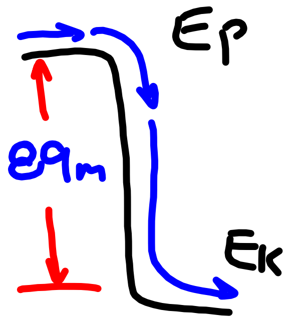
$= 470\ \frac{kg\,m^2/s^2}{s}$

$= 470\ kg\,m^2/s^3$

$\boxed{P = 470\ \text{Watts}}$

2a. The Sir Adam Beck II generating station in Niagara Falls has a capacity of 1500MW.
The water driving the turbines drops 89 metres.
How many kg of water are needed per second, to generate 1500MW of power?

k → 1,000
M → 1,000,000



1500MW
$= 1\,500\,000\,000\text{ W}$
$= 1.5\times10^9\text{ W}$

generates 1.5×10^9 Joules of energy every second.

→ converting Ep of water to Ek

∴ need 1.5×10^9 J of Ep in the water.

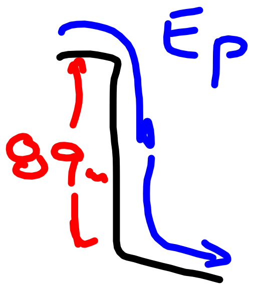
$E_p = mgh$

$$m = \frac{E_p}{gh}$$
$$= \frac{1.5\times10^9\text{ J}}{(9.8\text{ m/s}^2)(89\text{ m})}$$
$$= 1.7\times10^6\text{ kg}$$
$$= 1.7\times10^6\text{ litres}$$

2b. The actual volume of water available to the plant is 2.2 x10^6 kg per second. What is the efficiency of the power plant in converting the potential energy of the water into electrical energy

Power output = 1.5×10^9 W

Available Power = ?



$E_p = mgh$

$= (2.2 \times 10^6 \text{ kg})(9.8)(89\text{ m})$

$= 1.92 \times 10^9$ J

every second

or $= 1.92 \times 10^9$ W

$E_{ff} = \frac{P_{out}}{P_{in}} \times 100 \quad \left(\frac{E_{out}}{E_{in}} \times 100 \right)$

$= \frac{1.5 \times 10^9 \text{ W}}{1.92 \times 10^9 \text{ W}} \times 100$

$= 78\%$

Home use of Power

Energy consumption in the house is measured in kWh's.

how many Joules in a kWh?

$1\,kWh = 1000\,W \cdot h$

$= 1000\,J/s \cdot h$

$= 1000\,J/\cancel{s} \cdot (3600\,\cancel{s})$

$= 3{,}600{,}000\,J$

Example

Calculate the cost to operate a 60 Watt light bulb for eight hours, if the cost per kWh is 7.2 cents.

1. calculate total energy used in Joules.
2. multiply by rate to get cost.

$P = \frac{\Delta E}{t}$

1. $E = P \times t = 60\,J/s \times (8 \times 60 \times 60)$

$= 1728000\,J$

2. Cost = energy used × rate

$= (1{,}728{,}000\,J)(7.2¢/3{,}600{,}000\,J)$

$= 3.5¢$

Text Book Practice

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plus handouts - see website