Unit 0 - Necessary Skills

- Review

  - Scientific Notation & Significant Digits

- Analyzing Relationships
  - numerical method
  - graphing method
  - analyzing real life data - pendulum lab

Sig digs and Scientific Notation (quick review)

The speed of light is 299,792,458m/s, how long will it take a beam of light to travel 59.0cm? (express your answer with the correct number of significant digits). 3 sig figs

$V = \frac{\Delta d}{\Delta t}$

$\Delta d = 0.59\,m$

$V = 299792458\,m/s$

$\Delta t = \frac{\Delta d}{V} = \frac{0.59}{299792458}$

$= 0.000000001|...s$

$\cancel{= 1.00 \times 10^{-9}\,s}$

$= 1.968 \times 10^{-9}\,s$

$= 1.97 \times 10^{-9}\,s$

$= 1.97\,n\,secs$

Analyzing Experimental Data

Sketch a graph of the following relations.

$(B = \sqrt{A})$

$A=B$

$A=B^2$

$A=B^3$

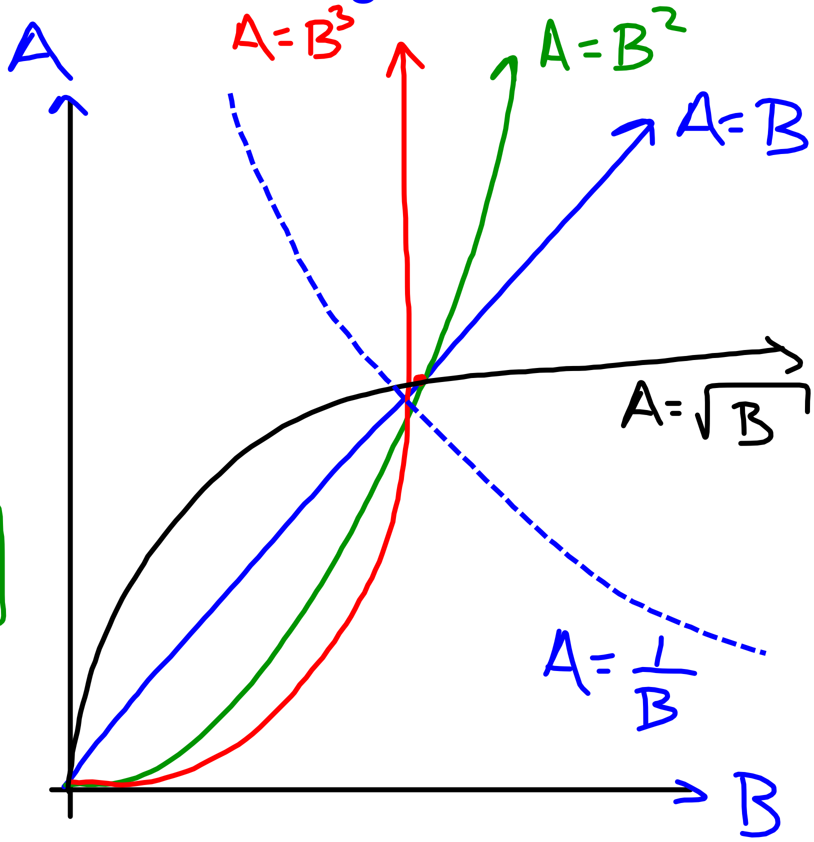
$B=\sqrt{A} \rightarrow A=B^2$

$A=\sqrt{B}$

$A=1/B$

$A=\frac{1}{B}$

$A=B^{-1}$

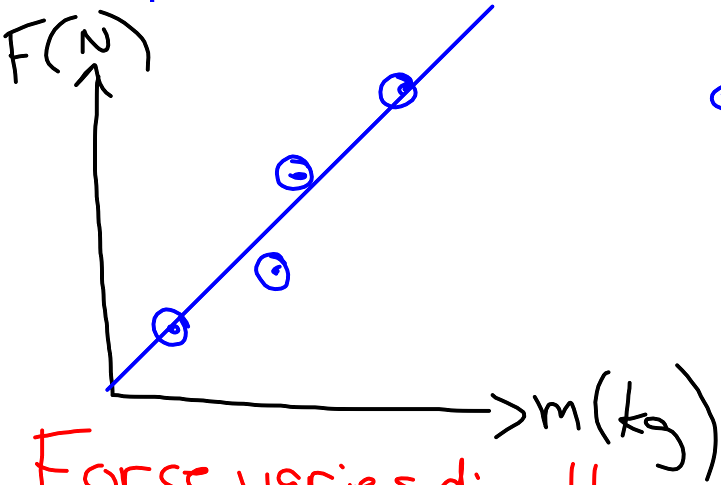


Analyzing Relations

How to identify and analyze relationships between variables (measurements).

in other words - how does changing one variable affect other variable(s)

Example 1 : Force and mass



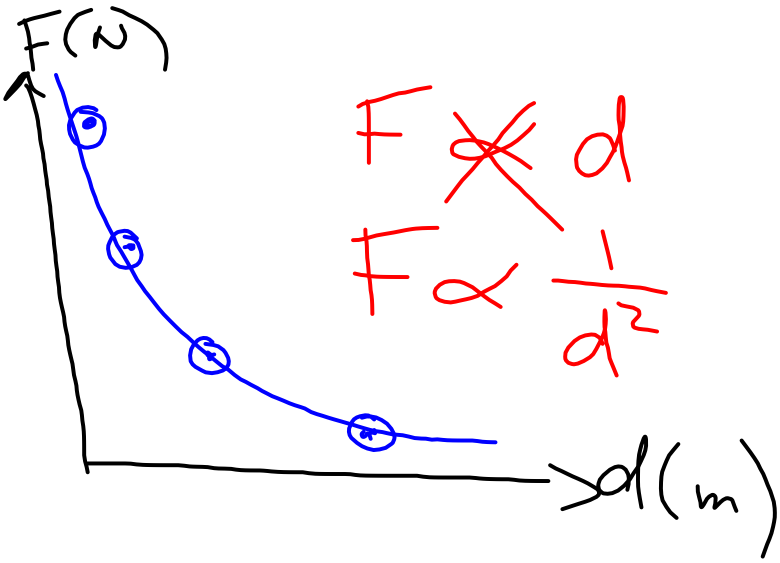
⊙ experimental data.

Force varies directly with mass

$$F \propto m$$

$\propto$ - varies directly with.

Example 2 : Force and distance

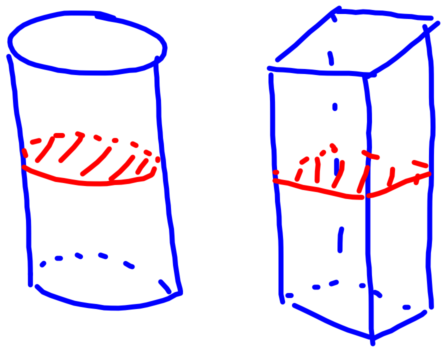


$$F \cancel{\propto} d$$

$$F \propto \frac{1}{d^2}$$

Real life example : How does scaling up an object affect it's physical properties?

example : rectangular prism / cylinder



Geometric Properties

volume

surface area

cross sectional area.

Physical Properties

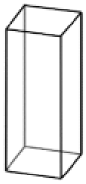
mass

ability to regulate/control heating/cooling.

strength of object

Rectangular Prism

To determine what happens as the linear dimensions of an object increase

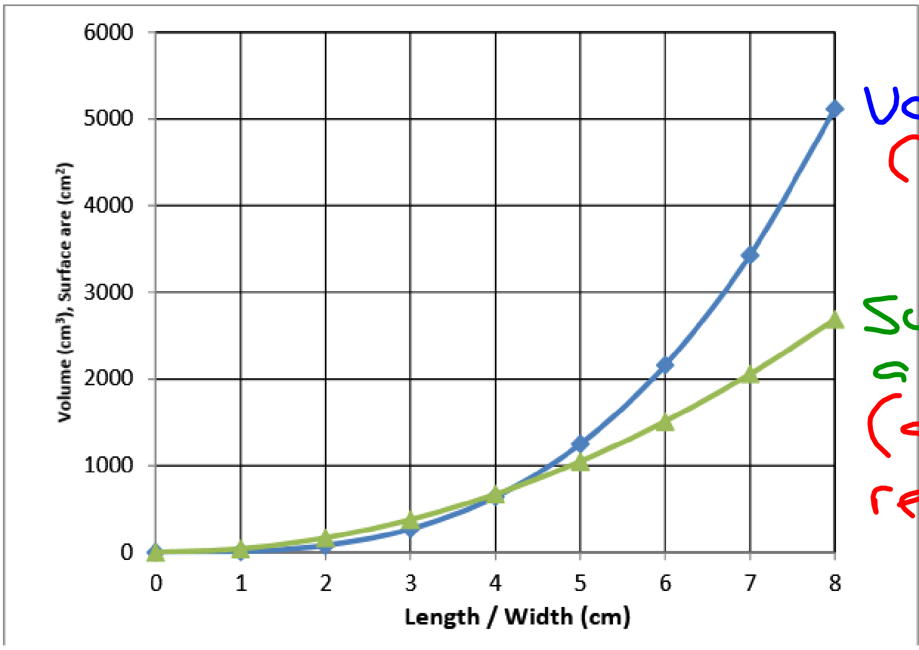


length (cm)	width (cm)	height (cm)	surface area (cm^2)	volume (cm^3)
0	0	0	0	0
1	1	10	42	10
2	2	20	168	80
3	3	30	378	270
4	4	40	672	640
5	5	50	1,050	1,250
6	6	60	1,512	2,160
7	7	70	2,058	3,430
8	8	80	2,688	5,120

x3 x2 x9 x4 x27 x8

1. For each new set of dimensions calculate the surface area and volume.

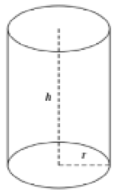
2. Plot both the surface area and volume on the grid below.



Volume (mass)

Surface area (ability to regulate heat)

Cylinder



radius	height	surface area (cm^2)	volume (cm^3)
0	0	0	0
1	10	69.1	31.4
2	20	276.5	251
3	30	622.0	848
4	40	1,105.8	2,011
5	50	1,727.9	3,927
6	60	2,488.1	6,786
7	70	3,386.6	10,776
8	80	4,423.4	16,085

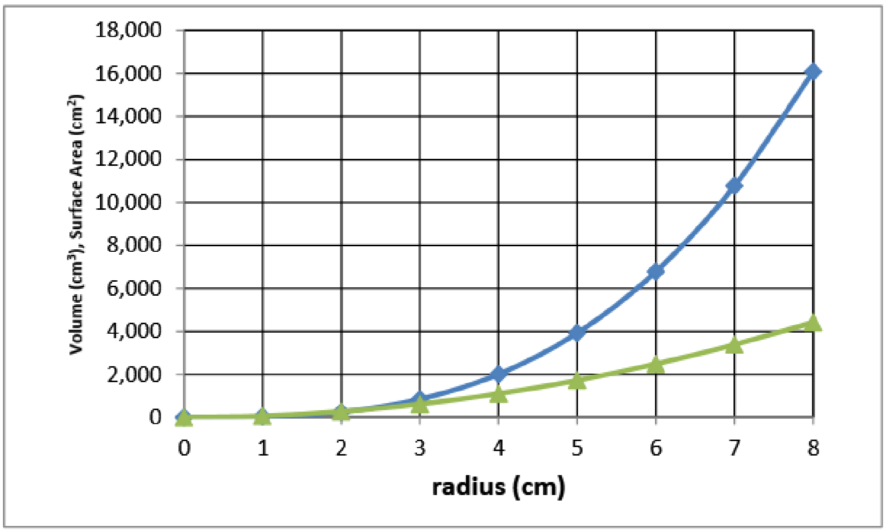
$\times 4$

$\times 4^2$ 16

$\times 4^3 = \times 64$

1. For each new set of dimensions calculate the surface area and volume.

2. Plot both the surface area and volume on the grid below.



Summary : Prisms and Cylinders (& any randomly shaped object)

As the linear dimensions ↑, the volume (mass) increases at a much higher rate than the surface area (ability to regulate heat) and the cross sectional area (strength).

Summary : How variables change with each other:

It's easy to think that if you double the size of something, all other variables (like strength and weight) also double.

That makes sense on a flat piece of paper, but it isn't necessarily true with real 3D physical objects.

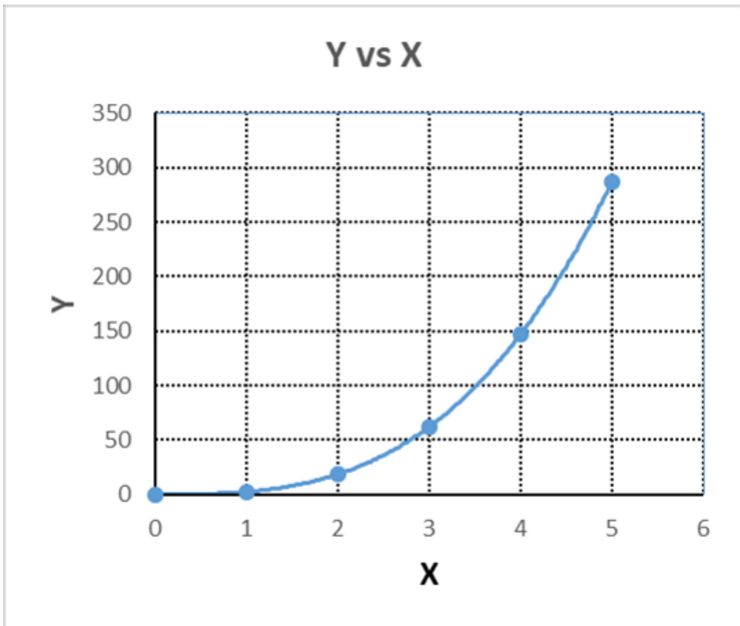
When our cylinders or prisms increased in size, the Strength of the object (xsa) didn't increase as fast as the weight (vol) and the object could collapse under it's own weight.

The way one variable makes another variable change is what we are studying ....

Analyzing Relations

Determing the Porportionality Statement & Equation

X	Y
1	2.3
2	18.4
3	62.1
4	147.2
5	287.5



$y = f(x)$

Analyzing Relations : Numerical Method

Goal : Given a set of data determine the proportionality statement and equation relating the two variables.

The multiplier method

example #1:

X (s)	Y (m)
1	5
2	10
3	15
4	20
5	25

×2 ×3 ×2 ×3

if multipliers are equal, then

$y \propto x$

to convert from proportionality statement to an equation, replace

$\propto \rightarrow = k$

where k is the constant of proportionality.

$y = kx$

solve for k by subbing in values for $(x, y) \rightarrow (1, 5)$

$5\,m = k \cdot 1\,s$

$k = 5\,m/s$

$\therefore\ y = (5\,m/s)x$

<u>*the multiplier method (cont'd)*</u>

example #2:

X (s)	Y (m)
1	6
2	24
3	54
4	96
5	150

×2, ×3, ×4 (x); ×4, ×9, ×16 (y)

$y \propto x^2$

$y = kx^2$

$(1, 6)$

(x, y)

$k = \frac{y}{x^2}$

$= \frac{6\,m}{(1\,s)^2}$

$= 6\ m/s^2$

$\therefore y = (6\ m/s^2)x^2$

check $(2, 24)$

L.S. R.S.

24 $\quad 6 \cdot (2)^2$

✓ $\quad = 24$

the multiplier method (cont'd)

example #3:

$E = ?$

V (m/s)	E (J)
2	12
4	48
6	108
8	192
10	300

×2 ×3 ×4 ×9

$E \propto v^2$

$E = kv^2$

$k = \frac{E}{v^2}$

$\frac{12\ J}{(2\ m/s)^2}$

$= 3 \frac{J}{\frac{m^2}{s^2}}$

$= 3 \frac{J\ s^2}{m^2}$

$= 3 \frac{J\ s^2}{m^2}$

$E = 3 \frac{J\ s^2}{m^2} v^2$

the multiplier method (cont'd)

example #4:

lm - lumens

$I = ?$

d (m)	I (lm)
10.0	10.0
20.0	2.50
30.0	1.11
40.0	0.625
50.0	0.400

×2 ×4

$\div 4 \quad \times \frac{1}{4}$

$\div 16 \quad \times \frac{1}{16}$

$$I \propto \frac{1}{d^2}$$

$$I = k\left(\frac{1}{d^2}\right)$$

$$I = \frac{k}{d^2}$$

$$k = I \cdot d^2$$

(10, 10)

$$k = 10\,lm \cdot (10\,m)^2$$

$$= 1000\,lm\,m^2$$

$$I = \frac{1000\,lm \cdot m^2}{d^2}$$

Handout

Numerical Analysis

Analyzing Realtionship Between y & a

a	3	6	9	12	15	18	21
y	2	4	6	8	10	12	14

$$y = \frac{2}{3}a$$

Analyzing Relationship Between y & b

b	120	60	40	30	24	20	17.143
y	2	4	6	8	10	12	14

$$y = \frac{240}{b}$$

Analyzing Relationship Between y & c

c	120	30	13.3	7.5	4.8	3.3	2.4
y	2	4	6	8	10	12	14

$$y = \sqrt{\frac{480}{c}}$$

$$= \frac{4\sqrt{30}}{\sqrt{c}}$$

Homework

Analyzing Relations - Practice Data Sets

Find the proportion statement and determine the equation that describes the relations between the data sets below

#1

time (s)	1	2	3	4	5	6	7
dist (m)	28	56	84	112	140	168	196

$d = 28t \qquad k = 28\,m/s$

#2

frequency (Hz)	5	10	20	50	75	100
period (s)	0.2	0.1	0.05	0.02	0.01333	0.01

$P = \frac{1}{f} \qquad k = 1 \quad s \cdot Hz$

$= \cancel{s} \cdot \frac{1}{\cancel{s}}$

$= \underline{\qquad}$

#3

y	250	750	2500	5000
x	3	9	30	60

$y = \frac{250}{3} x$

#4

a	20	80	180	2000
b	14	28	42	140

$b = \frac{7}{\sqrt{5}} \sqrt{a}$

#5

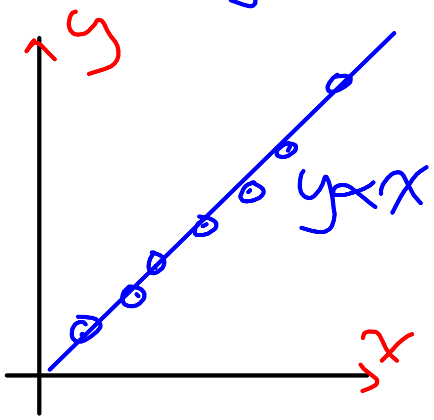
f	900	225	36	14	1
r	1	2	5	8	30

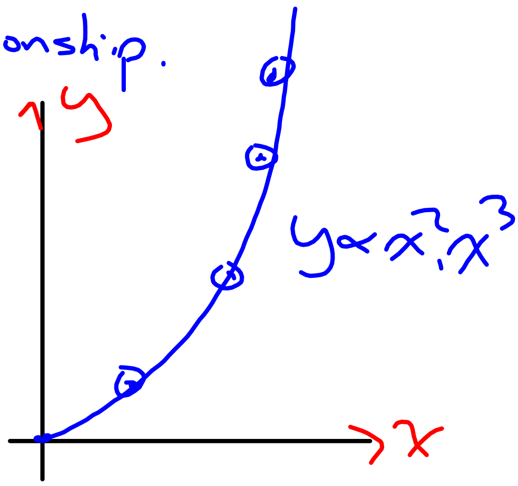
$r = \frac{30}{\sqrt{f}}$

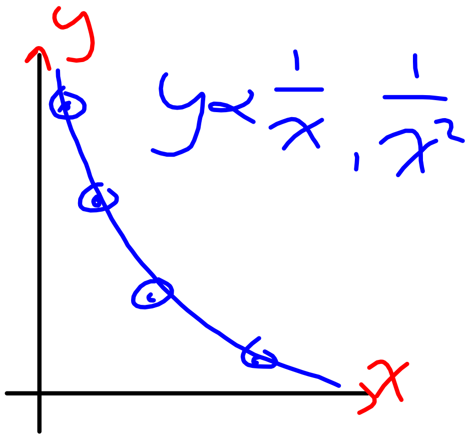
Analyzing Relations : The graphing method

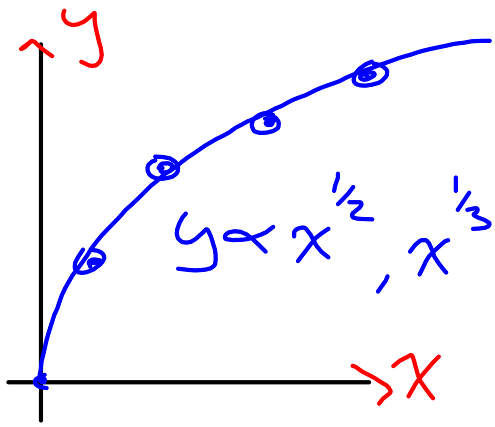
Steps:

1. Graph data
2. Hypothesize (guess) the relationship
3. Verify the relationship.









Others:

$y = a^x$

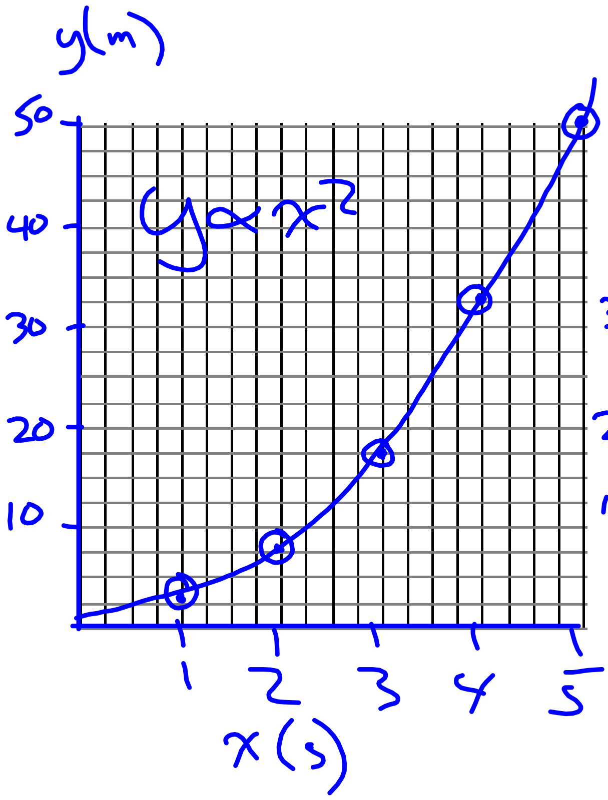
$y = \log x$

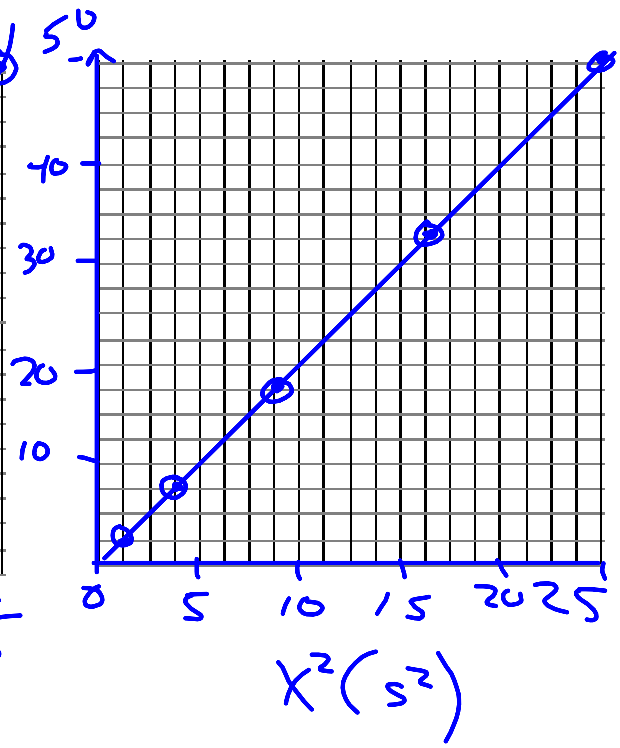
$y = \sin x$

Graphing Technique : Example #1

X (s)	Y (m)
1	2
2	8
3	18
4	32
5	50

X^2 (s)	Y (m)
1	2
4	8
9	18
16	32
25	50





$y \propto x^2$

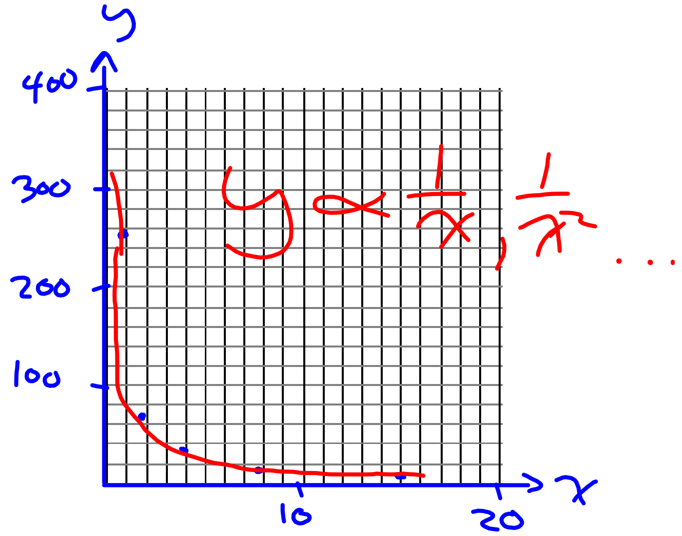
$y = kx^2$

$k = \frac{y}{x^2}$ = slope of y vs x^2 graph

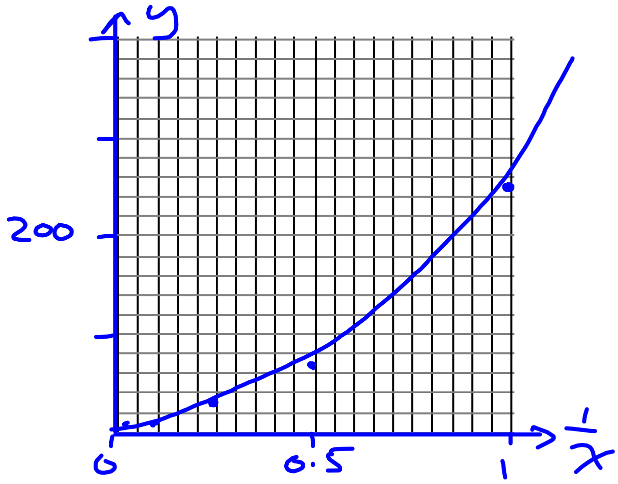
$= 2\ m/s^2$

Example #2

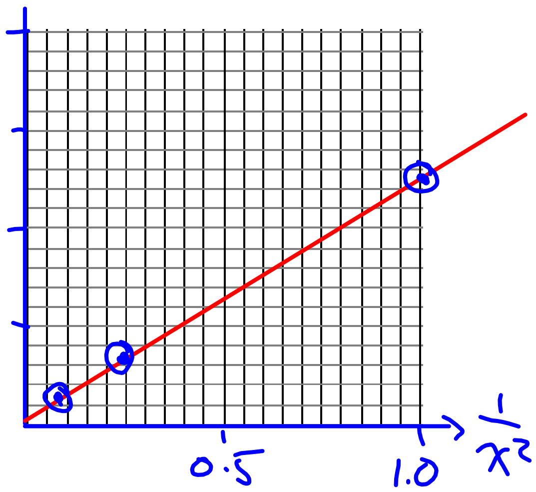
x	y
1	256
2	64
4	16
8	4
16	1



$\frac{1}{x}$	y
1	256
0.5	64
0.25	16
0.125	4
0.0625	1



$\frac{1}{x^2}$	y
1	256
.25	64
.0625	16
⋮	4
	1



$\therefore y \propto \frac{1}{x^2}$

$k = \text{slope} = 256$

$\therefore y = \frac{256}{x^2}$

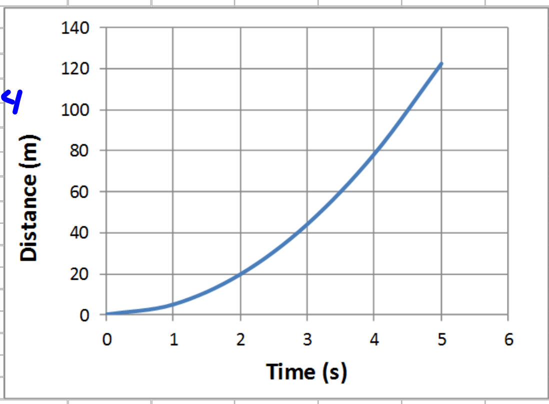
Homework

Find the equation relating the two variables for Example #1 and Example #2. Be sure to show the units on the constant of proportionality (i.e. - k)

Example #1 (Distance vs Time Experiment)

t (s)	D (m)
0	0
1	4.9
2	19.6
3	44.1
4	78.4
5	122.5

×2 ×4 ×4 ×16



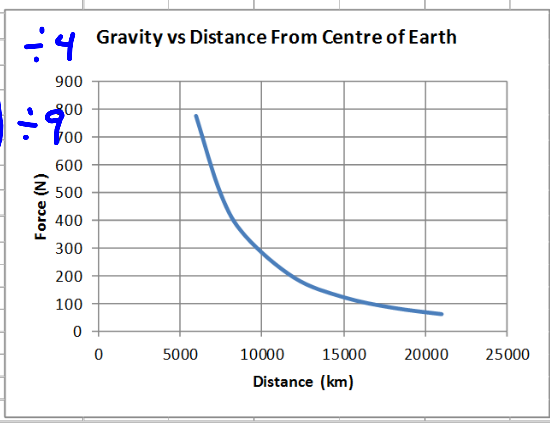
$D \propto t^2$

$D =$ ______

Example #2 (Force vs Distance Experiment)

d (km)	F (N)
6000	776
7500	496
9000	345
12000	194
15000	124
18000	86
21000	63

×2 ×3 ÷4 ÷9



$F =$ ______

$F \propto \frac{1}{d^2}$

Example #1 (Distance vs Time Experiment)

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0	0
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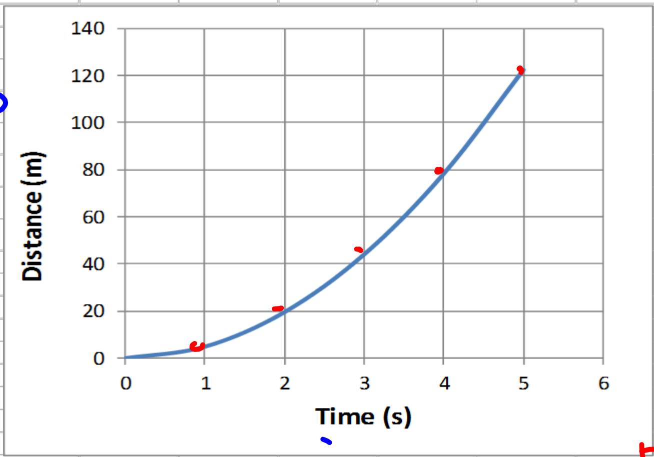


Numeric

$D \propto t^2$

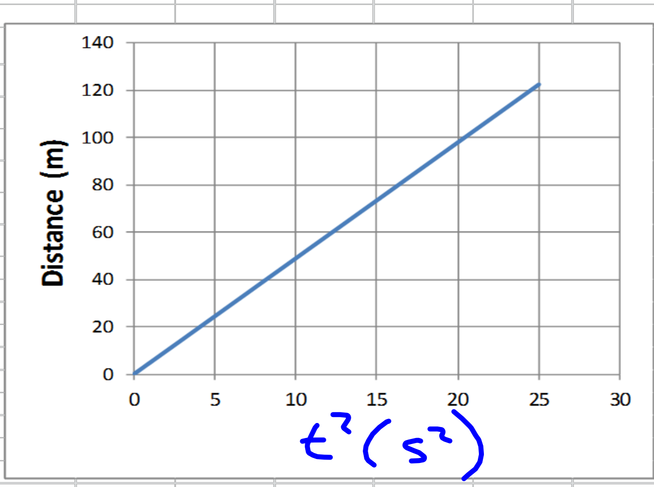
$D = kt^2$

$4.9\,m = k(1\,s)^2$

$k = 4.9\,m/s^2$

$\boxed{D = 4.9\,m/s^2\, t^2}$

$t (s)^2$	D (m)
0	0
1	4.9
4	19.6
9	44.1
16	78.4
25	122.5



Graphing

hypothesize $D \propto t^2$

straight line

$k = \text{slope}$

$= \frac{122.5\,m}{25\,s^2}$

$= 4.9\,m/s^2$

$\boxed{D = 4.9\,m/s^2\, t^2}$

$\Delta d = V_1 \Delta t + \frac{1}{2} a \Delta t^2$

$V_1 = 0$

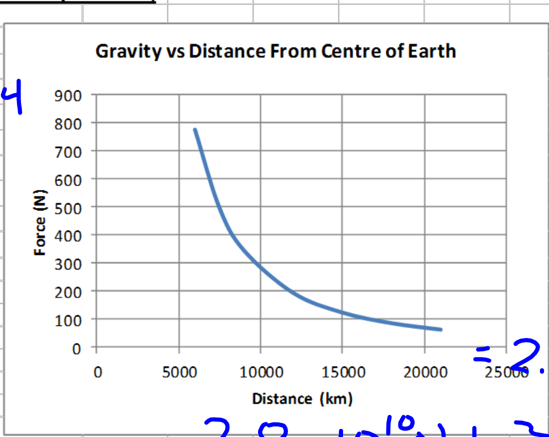
$a = 9.8\,m/s^2 \quad d = 4.9\,t^2$

Example #2 (Force vs Distance Experiment)

d (km)	F (N)
6000	776
7500	496
9000	345
12000	194
15000	124
18000	86
21000	63

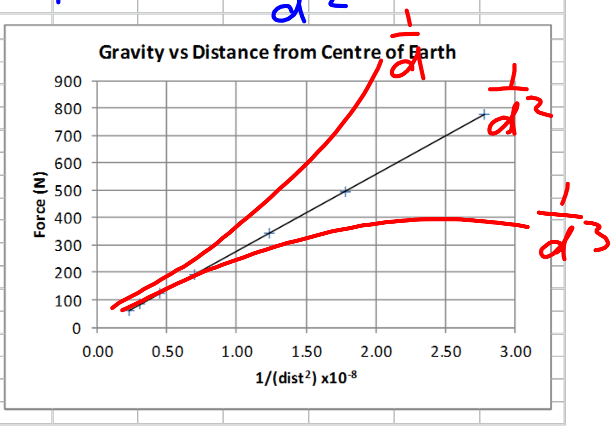
$\times 2$ $\div 4$ $\times 3$ $\div 9$

$\therefore F \propto \frac{1}{d^2}$



$\therefore F = \frac{2.8\times10^{10}\,N\cdot km^2}{d^2}$

$1/(d\ (km))^2$	F (N)
2.78E-08	776
1.78E-08	496
1.23E-08	345
6.94E-09	194
4.44E-09	124
3.09E-09	86
2.27E-09	63



Numeric

$F \propto \frac{1}{d^2}$

$F = k\,\frac{1}{d^2}$

$k = F\cdot d^2$

$= 776\,(N) \times 6000\,(km)^2$

$= 2.8\times10^{10}\ N\cdot km^2$

Graphing:

hypothesize $F \propto \frac{1}{d^2}$

plot F vs $\frac{1}{d^2}$

$\text{slope} = \frac{776\,N}{2.78\times10^{-8}\,\frac{1}{km^2}}$

$= 2.8\times10^{10}\,N\cdot km^2$

$\therefore F = \frac{2.8\times10^{10}\,N km^2}{d^2}$

$F_g = \frac{G m_1 m_2}{r^2}$

Going from Equations to Proportionality Statements and Back

example : Ideal Gas Law

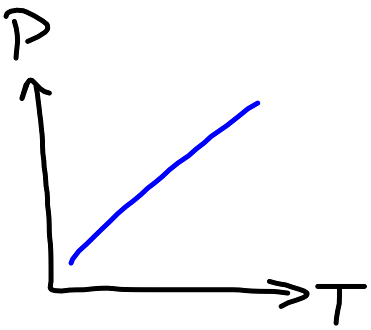
$$PV = nRT$$

1. how does Pressure (P) vary with Temperature (T)?

$P \propto T$

assume all other variables constant

$$P = \underbrace{\left(\frac{nR}{V}\right)}_{\propto} T$$



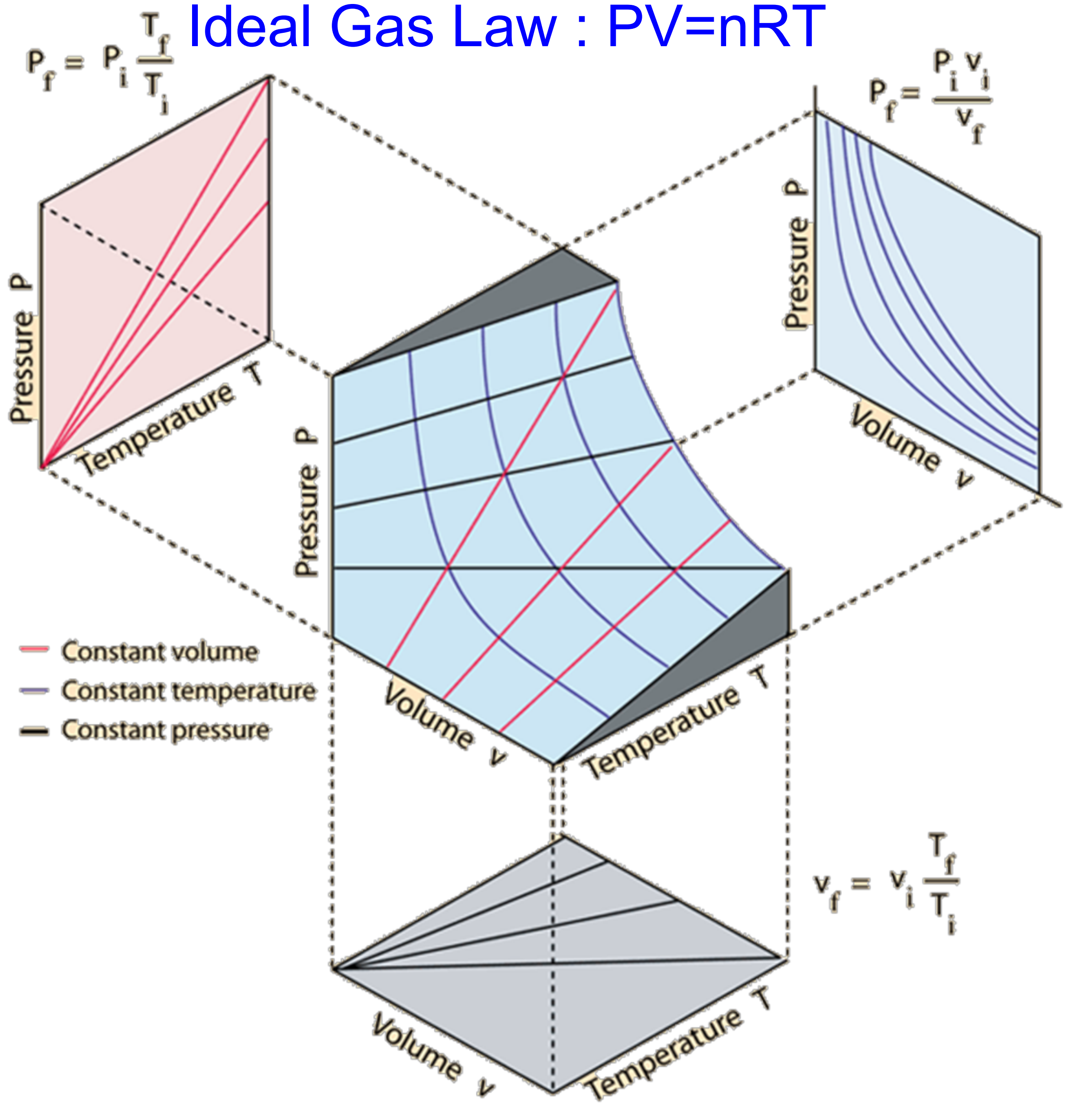
2. how does Pressure (P) vary with volume (V)?

$$PV = nRT$$

$$P = \underbrace{nRT}_{\propto}\left(\frac{1}{V}\right)$$

$$P \propto \frac{1}{V}$$

Ideal Gas Law : PV=nRT



Solving proportionality statements:

If Energy (E) varies with velocity squared (v^2) and E=100J when v=5 m/s, what is E when v=25 m/s

Formal Method

$E \propto v^2$

$E = kv^2$

$k = \frac{E}{v^2}$ *

$= \frac{100}{5^2} = 4 \frac{Js^2}{m^2}$

$E = 4v^2$

when $v=25$

$E = 4 \cdot 25^2$

$= 2500 J$

Ratio Method

$\frac{E}{v^2} = \text{constant}$

$\frac{100}{5^2} = \frac{E}{25^2}$

$E = 2500 J$

Equations from Proportionality Statements... and back

In $y \propto x$, the $\propto$ symbol is replaced by $= k$ to make an equation.

equation ---> proportionality statement

To go from an equation to a proportionality statement, replace all the constants with a proportionality symbol i.e $y = kx$ becomes $y \propto x$.

1. Write the proportionality statement in your notes for each of the following equations:
   (a) $V = \frac{4}{3}\pi r^3$;the relationship between V and r (b) $F = \frac{mv^2}{r}$; between F and v

   (c) same equation as (b); between F and r (d) $F = \frac{Gm_1m_2}{r^2}$; between F and m_1

   (e) same equation as (d); between F and r (f) $K = \frac{R^3}{T^2}$; between R and T

proportionality statement ----> equation

To go from a proportionality statement to an equation, replace the $\propto$ symbol with $= k$. Substitute in a set of values and solve for k. Now use the equation to solve other similar cases.

2. If $a \propto b^3$ and a = 4 when b = 3.5, what is a when b = 7 ?

3. If $d \propto at^2$ and a = 2 when t = 4 and d = 16, what will d be when a = 4 and t = 12 ?

4. If $p \propto \frac{q^3}{r^2}$ and p = 400 when q = 5 and r = 3, what will p be when q = 15 and r = 5 ?

5. If $E \propto mv^2$ and E = 98 when m = 4 and v = 7, what will E be when m = 10 and v = 42 ?

combining the Multiplier Method

6. a) Determine the proportionality statement that describes the data below using the Multiplier Method.

x	0.2	0.4	0.6	0.8	1
y	200	50	22.2	12.5	8

b) Write an equation relating y and x. c) If x = 0.55, what would x be?

Practice

Determine the proportion for each of the following tables of values.

A	B
2	100
8	200
50	500
200	1000

C	D
3	120
6	60
9	40
12	30

E	F
2	90
54	270
16	180
250	450

G	H
6	5
12	20
18	45
42	245

K	L
7	800
35	32
28	50
70	8

M	N
2	3
4	24
6	81
8	192

2. A slider that starts from rest and slides down an inclined air track covers the distances *d* in the times *t*. Using graphical methods, determine the relationship between *d* and *t*.

t	0	0.8	1.0	1.2	1.4
d	0	12.8	20.0	28.8	39.2

3. An experiment is performed to find the relationship between two physical quantities, B and A. The following data is obtained.

A	100	64	49	36	25	16
B	1.99	1.59	1.39	1.19	1.00	0.80

Determine the relationship between B and A.

/5

Quiz Warmup

X (cm)	Y (N)
6	432
12	3456
18	11 664
24	27648
30	54000
36	93 312

Y= ______

predict what Y will equal when X = 1.00m

$Y \propto x^3$ ✓

$Y = kx^3$

$k = \frac{Y}{x^3}$ ✓

$= \frac{432\,N}{(6\,cm)^3}$ ✓

$k = 2\ N/cm^3$

$Y = 2\ N/cm^3 \cdot x^3$ ✓

$Y = 2\ N/cm^3 (100\,cm)^3$

$= 2.00 \times 10^6\ N$ ✓

(2 million N)

2,000,000 N