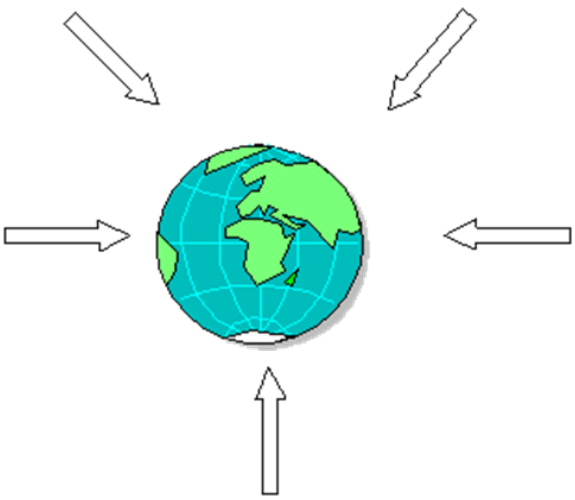
Gravitational Force

``REVIEW``

- Gravity is a force between two objects.
- The direction of the force is towards the centre of each object.
- The further away objects are the lower the gravitational force.
- The strength of the gravitational force varies directly with the mass of the objects (as masses get smaller the gravitational force gets weaker).

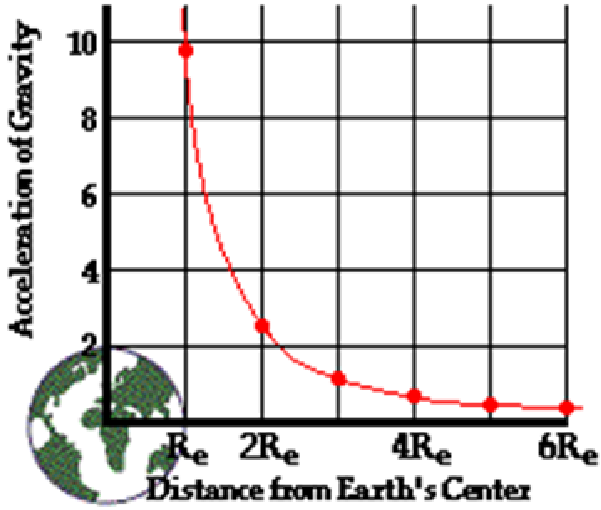


Acceleration Due to Gravity

- Acceleration due to gravity is constant near the earth's surface,

  a = 9.8m/s^2 (sometimes called g= 9.8m/s^2)

- As you move further and further away from the surface of the earth, the acceleration due to gravity decreases.



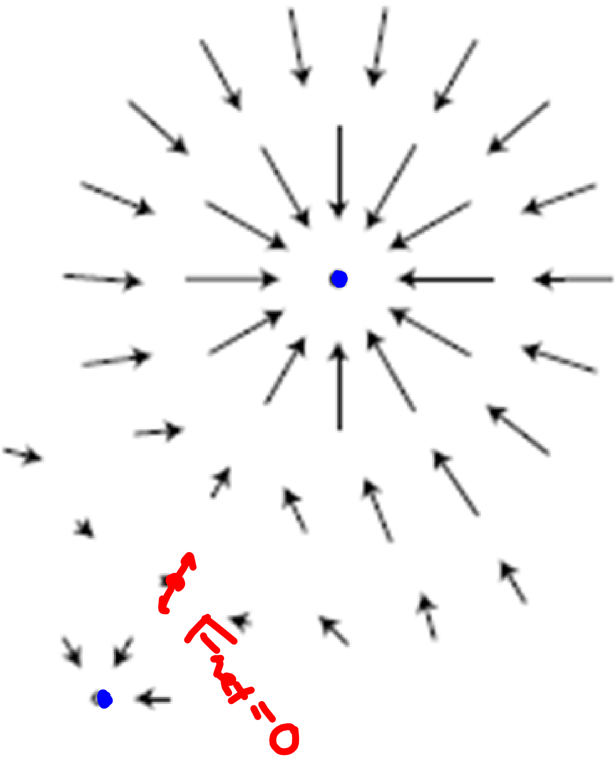
… because the gravitational force decreases…

Gravitational Force Field

- Surrounding the earth is a gravitational force field that attracts objects towards the earth.

- The gravitational field strength of a planet is measured as the amount of force per unit mass at a given distance from planet.

N/kg.



At the earth's surface
gravitational field strength = 9.8 N/kg [down]

$9.8\ N/kg$

$= 9.8\ (kg \cdot m/s^2)/kg$

$= 9.8\ m/s^2$

$1N = 1\ kg \cdot m/s^2$

m
kg
s

$F = ma$

$= kg \cdot m/s^2$

Mass versus Weight

Definitions - mass vs. weight

- Mass is the amount of matter in an object (measured in kg)
- Weight is the force of gravity on an object (measured in N)

mass never changes, weight varies depending on location.

Example #1 :

The gravitational field strength on the surface of the earth is 9.8 N/kg and on the surface of the moon 1.6 N/kg. Calculate the weight of a 70kg man on the earth and on the moon.

↳ mean F_g.

Earth

$g = 9.8\ N/kg$

$F_g = mg$

$= (70\,kg)(9.8\ N/kg)$

$= 686\ N$

Moon

$g = 1.6\ N/kg$

$F_g = mg$

$= (70\,kg)(1.6\ N/kg)$

$= 112\ N$

Universal Gravitation

How do we calculate the force of gravity in situations not on the earth's surface?

Newton's Law of Universal Gravitation

The force of gravitational attraction between any two objects is directly proportional to the product of the masses, and inversely proportional to the square of the distance between their centres

proportionality statement $F_g \propto \frac{m_1 m_2}{r^2}$ r = distance

Law of Universal Gravitation
as a formula

* $$F_g = \frac{G m_1 m_2}{r^2}$$

m_1, m_2 – masses of two objects (kg)

r – distance between centres of two objects (m)

G – Universal Gravitational Constant

= 6.67×10^{-11} Nm^2/kg^2

Example #2 :

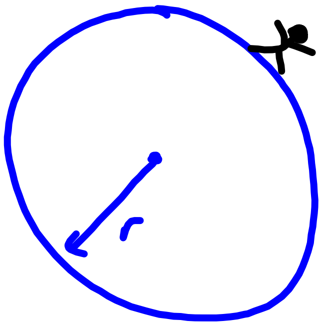
Using Newton's Law of Universal Gravitation, calculate the force of gravity between a 100 kg man and the earth (assuming the man is standing on the surface of the earth)

useful variables :

radius of earth - 6,380km

mass of earth - 5.98×10^{24} kg

$G = 6.67\times10^{-11} N\cdot m^2/kg^2$



$m_1 = 100kg$

$m_2 = 5.98\times10^{24} kg$

$r = 6{,}380km$

$= 6{,}380{,}000m$

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{(6.67\times10^{-11} Nm^2/kg^2)(100kg)(5.98\times10^{24}kg)}{(6{,}380{,}000\ m)^2}$$

$= 979.9N$

$= 980N$

on the surface of earth

$F_g = mg$

$= 100kg \cdot 9.8 N/kg$

$= 980N$

Recall:

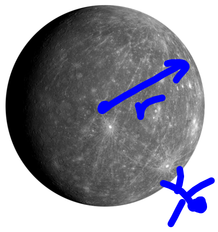
$9.8\ m/s^2$ is equivalent to $9.8\ N/kg$.

Universal Gravitation

#1 : Calculate the force of gravity on a 70 kg person on each of the 8 planets, Pluto, the sun and the moon.

#2 – What would the acceleration due to gravity be on the surface of each of the 8 planets, Pluto, the sun and the moon?

Example : Mercury



$m_1 = 70\,kg$

$m_2 = 0.329 \times 10^{24}\,kg$

$r = 2440\,km$
$= 2\,440\,000\,m$
$= 2.44 \times 10^{6}\,m$

$G = 6.67 \times 10^{-11}\,N \cdot m^2/kg^2$

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.67\times10^{-11})(70)(0.329\times10^{24})}{(2.44\times10^{6})^2} = 258\,N$$

$F_g = mg$ $a = g$

$$g = \frac{F_g}{m} = \frac{258\,N}{70\,kg} = 3.7\,m/s^2$$

6052 km = 6052000 m
= 6.052×10^6 m

Planetary Data Sheet

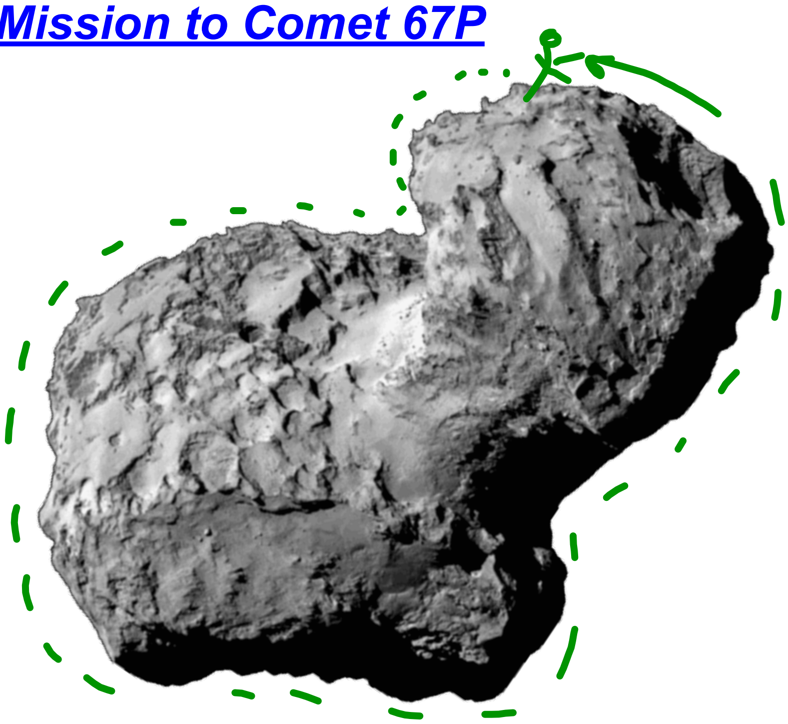
	Mass (10^{24}kg)	Diameter (km)	Radius (km)	Distance from Sun (10^6 km)	Force Gravity (70 kg person)	Acceleration (m/s^2)
MERCURY	0.329	4,879	2,440	57.9	258	3.7
VENUS	4.87	12,104	6,052	108.2		
EARTH	5.97	12,756	6,378	149.6	685	9.8
MARS	0.642	6,792	3,396	227.9		
JUPITER	1,899	142,984	71,492	778.6		
SATURN	568	120,536	60,268	1433.5		
URANUS	86.8	51,118	25,559	2872.5		
NEPTUNE	102	49,528	24,764	4495.1		
PLUTO	0.0125	2,390	1,195	5870		
SUN	1,980,000	1,390,000	695,000	na		
MOON	0.073	3,475	1,738	0.384		
				dist from earth		

http://nssdc.gsfc.nasa.gov/planetary/factsheet/

last updated July 19, 2013

	Mass (10^{24}kg)	Diameter (km)	Radius (km)	Distance from Sun (10^6 km)	Force on a 70kg person	Acceleration (m/s^2) N/kg
MERCURY	0.329	4,879	2,440	57.9	258	3.7
VENUS	4.87	12,104	6,052	108.2	621	8.9
EARTH	5.97	12,756	6,378	149.6	685	9.8
MARS	0.642	6,792	3,396	227.9	260	3.7
JUPITER	1,899	142,984	71,492	778.6	1735	25
SATURN	568	120,536	60,268	1433.5	730	10
URANUS	86.8	51,118	25,559	2872.5	620	8.9
NEPTUNE	102	49,528	24,764	4495.1	777	11
PLUTO	0.0125	2,390	1,195	5870	41	0.58
SUN	1,980,000	1,390,000	695,000	na	19139	270
MOON	0.073	3,475	1,738	0.384	113	1.6
Comet 67P	9.982 $\times 10^{12}$	–	1.65	na	0.017	0.00024

Rosetta Mission to Comet 67P



$m_2 = 9.982 \times 10^{12}$ kg
$r = 1.65$ km
$m_1 = 70$ kg
$F_g = 0.017$ N

$a = 0.00024$ m/s^2
2.4×10^{-4} m/s^2

Practice :

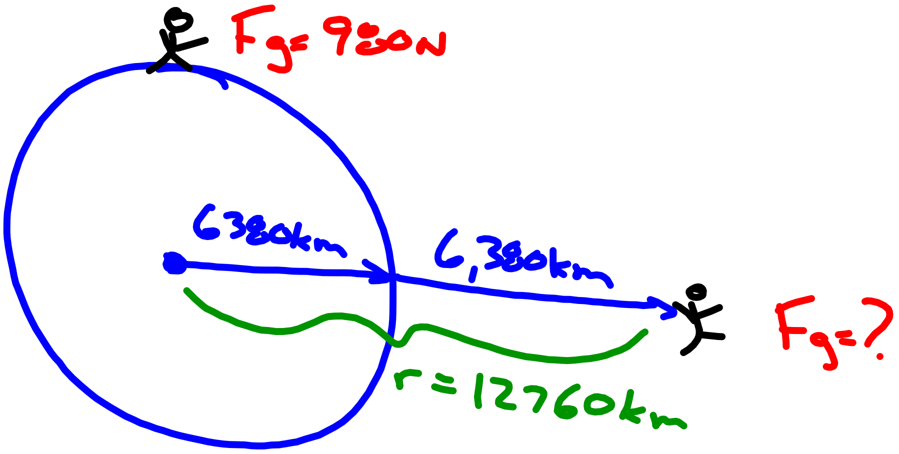
What would the force of gravity be on a 100kg man if he was 6,380 km above the surface of the earth?

$m_m = 100kg$

r_e=6,380km

M_e=5.97x10^{24}kg

G=6.67x10^{-11} Nm^2/kg^2



$$F_g = \frac{G m_1 m_2}{r^2}$$

$$= \frac{(6.67\times10^{-11})(100)(5.97\times10^{24})}{(1.276\times10^{7})^2}$$

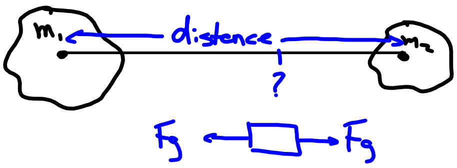
$$= 245\ N$$

r doubles → $F_g \div 4$

r triples (x3) → $F_g \div 9$

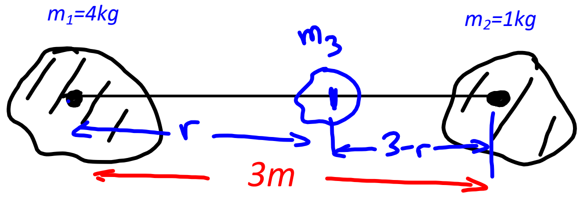
2 Body Problem

Finding the point where the net force of gravity is zero between two objects.



$F_{net} = \phi$

Example 1 :



$$F_{g_{13}} = F_{g_{23}}$$

$$\frac{G m_1 m_3}{r^2} = \frac{G m_2 m_3}{(3-r)^2}$$

$$\frac{4}{r^2} = \frac{1}{(3-r)^2}$$

$$4(3-r)^2 = r^2$$

$$4(9-6r+r^2) = r^2$$

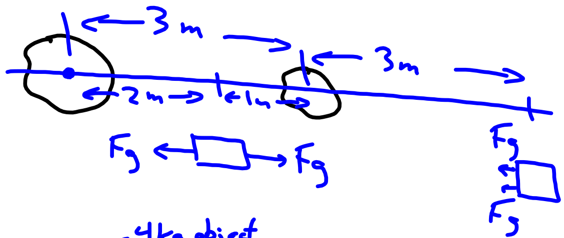
$$36-24r+4r^2 = r^2$$

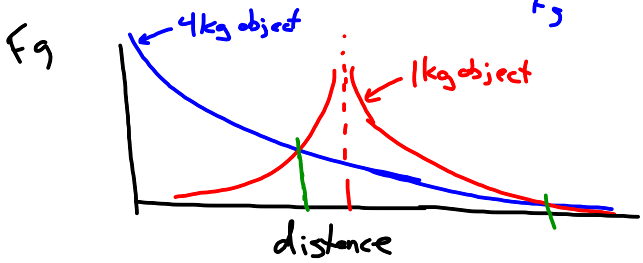
$$\div 3 \quad 3r^2-24r+36=0 \quad \div 3$$

$$r^2-8r+12=0$$

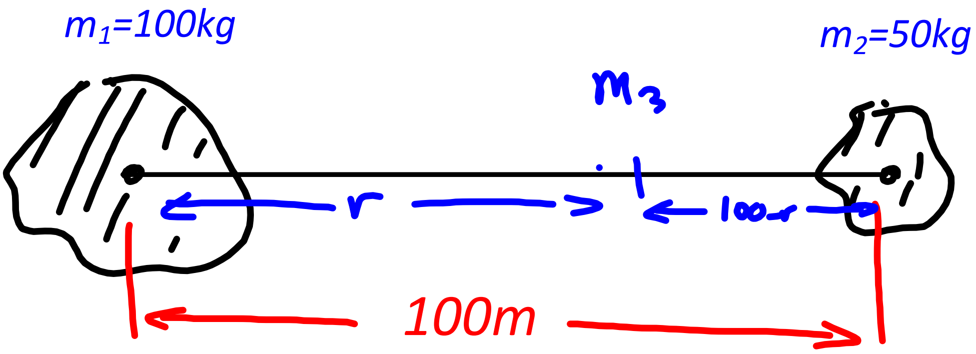
$$(r-6)(r-2)=0$$

$$\therefore r = 6, 2$$





Example 2 :



$$F_{g_{13}} = F_{g_{23}}$$

$$\frac{G m_1 m_3}{r^2} = \frac{G m_2 m_3}{(100-r)^2}$$

$$\frac{m_1}{r^2} = \frac{m_2}{(100-r)^2}$$

$$\frac{100}{r^2} = \frac{50}{(100-r)^2}$$

$\div 50$ $\quad 100(100-r)^2 = 50r^2 \quad \div 50$

$$2(100-r)^2 = r^2$$

$$2(10\,000 - 200r + r^2) = r^2$$

$$20\,000 - 400r + 2r^2 = r^2$$

$$r^2 - 400r + 20\,000 = 0$$

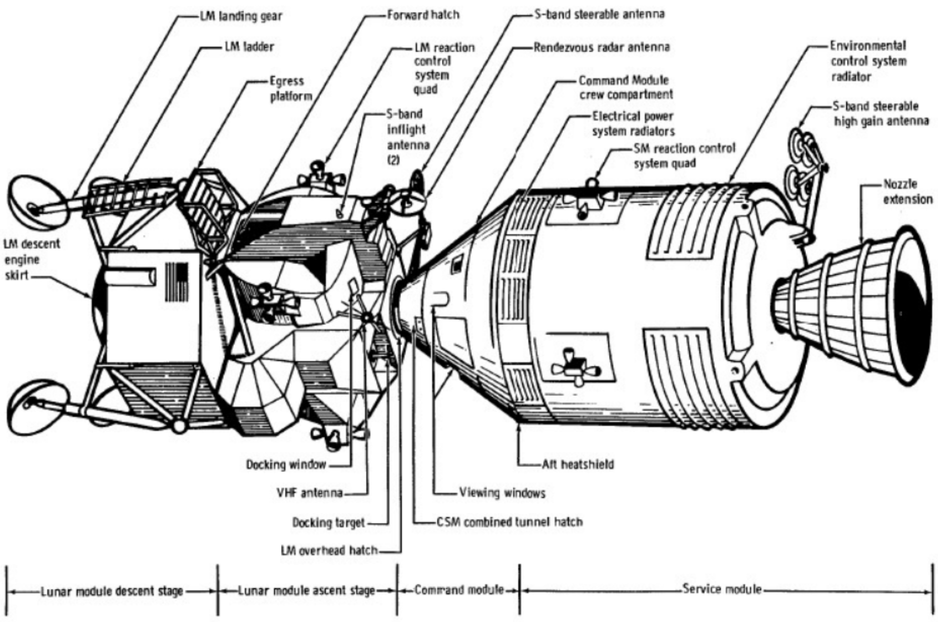
$$r = 341.4\text{m}, 58.6\text{m}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

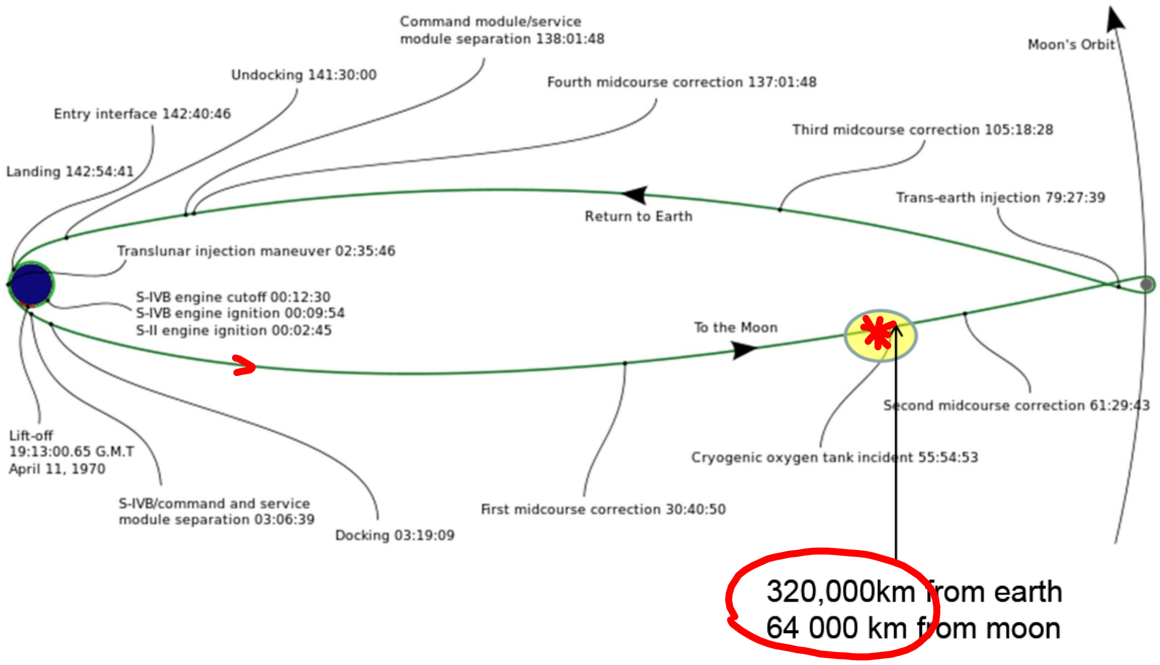
Apollo 13

Key Facts

- Mass
  - Command module 5,609 kg
  - Lunar module 15,192 kg
  - Service Module 23,181 kg
- Timeframes
  - Cryogenic oxygen tank incident – 55:54 (320,000 km from earth)
  - Return from far side of moon – 79:27
  - Total mission length – 142:54



Apollo 13 space vehicle configuration.


Command module/service module separation 138:01:48
Undocking 141:30:00
Entry interface 142:40:46
Fourth midcourse correction 137:01:48
Moon's Orbit
Landing 142:54:41
Third midcourse correction 105:18:28
Return to Earth
Trans-earth injection 79:27:39
Translunar injection maneuver 02:35:46
S-IVB engine cutoff 00:12:30
S-IVB engine ignition 00:09:54
S-II engine ignition 00:02:45
To the Moon
Second midcourse correction 61:29:43
Lift-off
19:13:00.65 G.M.T
April 11, 1970
Cryogenic oxygen tank incident 55:54:53
S-IVB/command and service module separation 03:06:39
Docking 03:19:09
First midcourse correction 30:40:50
320,000km from earth
64 000 km from moon

Apollo 13 Problem

Calculate the net force of gravity on the Apollo 13 spacecraft when the explosion occurred (320,000 km from earth).

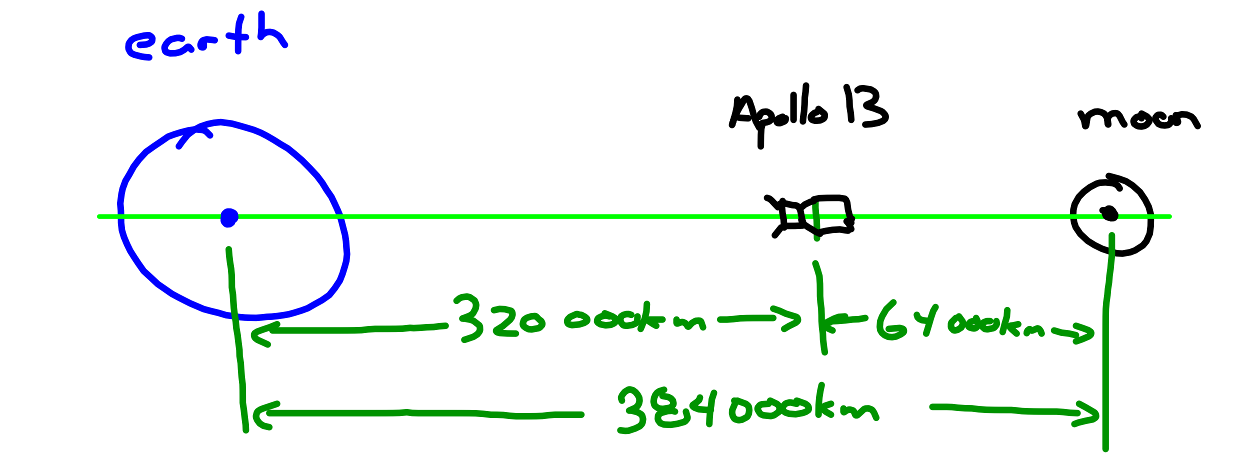
useful variables :

distance to moon - 384,000 km

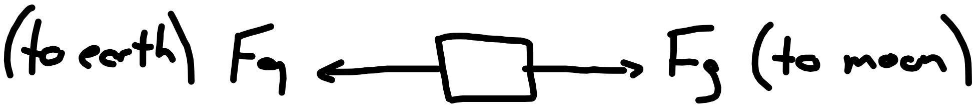
mass of Apollo 13 spacecraft - 43,000 kg m_A

mass of earth - 5.97×10^{24} kg m_e

mass of moon - 7.35×10^{22} kg m_m



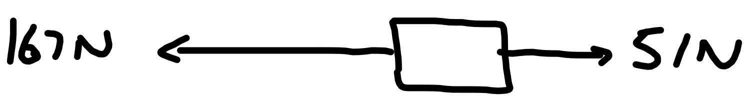
Free Body Diagram



using $F_g = \frac{Gm_1m_2}{r^2}$

F_g (to earth) = 167N

F_g (to moon) = 51N



F_{net} = 116N [towards earth]

Apollo 13 problem - at what distance from the earth is the net force of gravity zero (in other words where is the force of gravity from the moon and the earth equal).