

Potential Energy (a deeper examination)

Hooke's Law (for an ideal spring)



The force needed to extend (or compress) a spring a certain distance 'x' is proportional to that distance.

$$F \propto x$$

$$F = kx$$

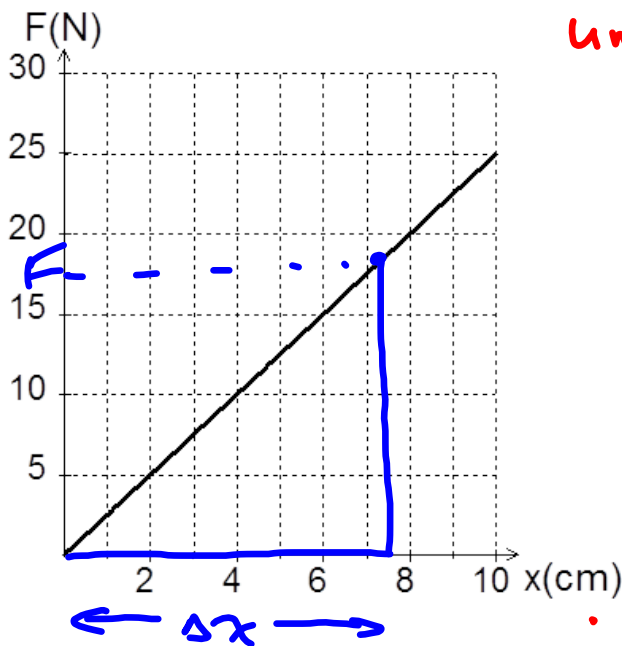
$k \rightarrow$ Hooke's constant
Spring constant

units on k .

$$F = kx$$

$$k = \frac{F}{x} \Rightarrow \text{N/m}$$

(slope)



$\Delta E =$ area under graph

$$= \frac{1}{2} \text{ base} \cdot \text{height}$$

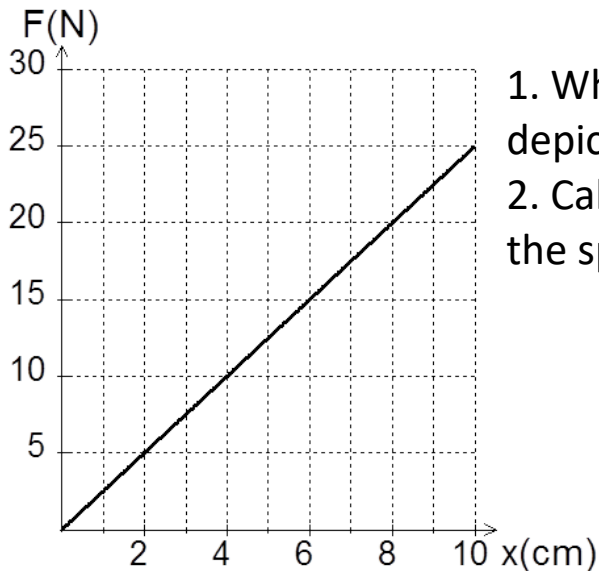
$$= \frac{1}{2} \Delta x F$$

$$= \frac{1}{2} \Delta x k \Delta x$$

recall $F = kx$
 $= k \Delta x$

$$E_e = \frac{1}{2} k \Delta x^2$$

↑
elastic potential energy stored in a spring.

Example 1: An ideal spring (i.e. obey's Hooke's Law)

1. What is the spring constant of the spring depicted in the diagram to the left?
2. Calculate how much energy is stored in the spring if it is compressed 6 cm.

$$1. \quad F = kx$$

$$k = \frac{F}{x}$$

$$= \frac{25\text{N}}{0.1\text{m}}$$

$$2. \quad \Delta x = 6\text{cm} = 0.06\text{m} \quad = 250\text{N/m}$$

$$E_e = \frac{1}{2}kx^2 = \frac{1}{2}(250)(0.06)^2$$

$$= 0.45\text{N}\cdot\text{m}$$

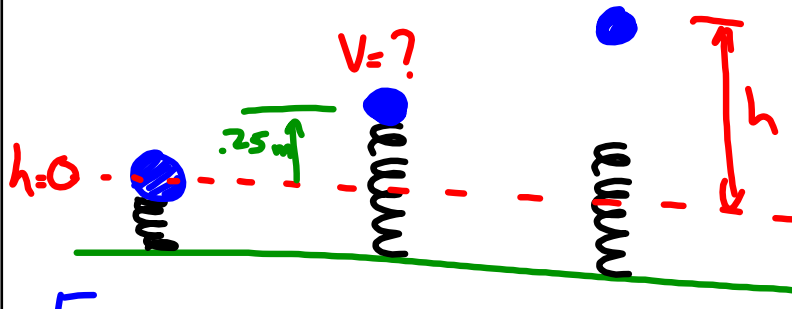
$$= 0.45\text{J}$$

$$\frac{\text{N}}{\text{m}} \cdot \text{m}^2 = \text{N}\cdot\text{m}$$

Example 2 : Spring Launcher

A vertical compression spring with a spring constant of 275N/m is compressed 25.0cm down. A 2.5kg metal ball is placed on the spring and it is launched upwards.

- What is the launch velocity of the ball?
- How high from the ground does the ball soar?



$E_k = 0$	$* E_k = 2.46$	$E_k = 0$
$E_p = 0$	$E_p = 6.13$	$E_p = 8.59$
$E_e = 8.59$	$E_e = 0$	$E_e = 0$
<hr/>	<hr/>	<hr/>
$E_T = 8.59$	8.59	8.59

$$E_e = \frac{1}{2} k x^2$$

$$= \frac{1}{2} (275 \text{ N/m}) (0.25 \text{ m})^2$$

$$= 8.59 \text{ N}\cdot\text{m}$$

$$E_p = mgh$$

$$= (2.5 \text{ kg}) (9.8) (0.25 \text{ m})$$

$$= 6.13$$

$$E_k = 2.46$$

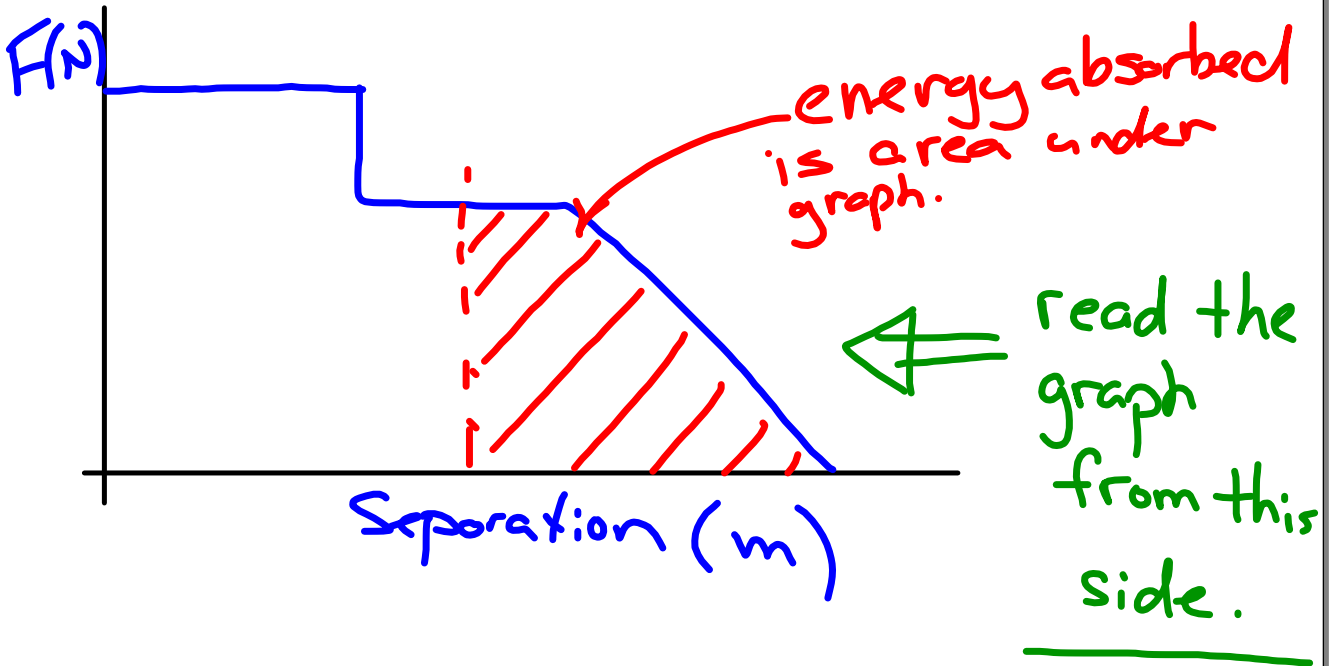
$$V = \sqrt{\frac{2E_k}{m}}$$

$$= 1.4 \text{ m/s}$$

$$h = \frac{E_p}{mg}$$

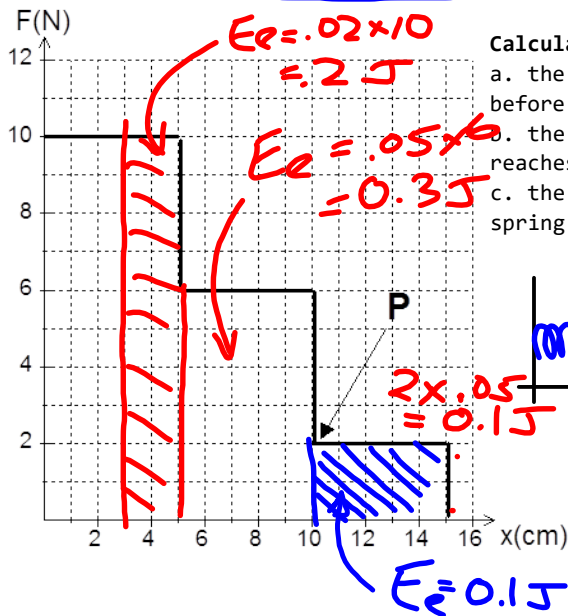
$$= 0.35 \text{ m}$$

Force Separation Graphs (non ideal springs)



Example 1:

A mass of 4.8 kg travelling at 0.5 m/s collides with a spring anchored against a wall. The force-separation graph for this spring is shown below.



Calculate

- the kinetic energy of the mass before the collision
- the velocity of the mass when it reaches point P.
- the maximum compression of the spring.

a. $E_{ki} = \frac{1}{2}mv^2 = 0.60\text{ J}$

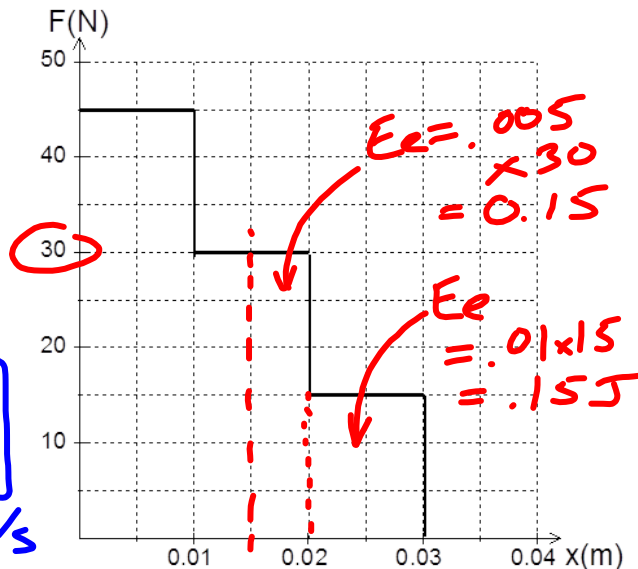
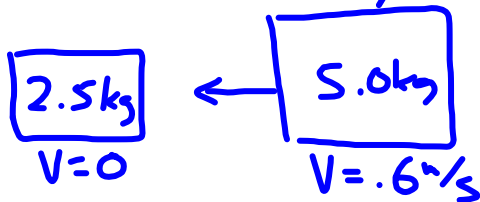
b. at point P. $E_k = 0.60\text{ J} - 0.1\text{ J} = 0.5\text{ J}$

$$v = \sqrt{\frac{2E_k}{m}} = 0.46\text{ m/s}$$

c. max compression
 → when all of E_k is converted to E_e . (elastic potential)
 → find where area under graph = 0.60 J.
 from graph $E_e = 0.6\text{ J}$ at 3 cm
 ∴ maximum compression = 15 cm - 3 cm = 12 cm.

Example 2: A 2.5 kg mass at rest, is approached head-on by a 5.0 kg mass moving at 0.60 m/s. The force-separation graph for the ensuing collision is shown to the right.

(assume elastic)



- What is the total kinetic energy before the collision? After?
- What is the velocity of each mass at minimum separation?
- What is the total kinetic energy at minimum separation?
- How much energy is stored at minimum separation?
- What is the minimum separation distance?
- What is the magnitude of the force acting on each mass at minimum separation?

- $E_{ki} = 0.90 \text{ J} = E_{kf}$
- $V_0 = 0.40 \text{ m/s}$ $m_1 v_1 + m_2 v_2 = (m_1 + m_2) V_0$
- $E_{k \text{ min sep}} = 0.60 \text{ J}$ $\frac{1}{2} (m_1 + m_2) V_0^2$
- $E_e = 0.90 - 0.60 = 0.30 \text{ J}$
(lost kinetic at minimum sep)
- find area under graph = 0.30 J
min sep dist = 0.015 m
- from the graph
 $F = 30 \text{ N}$ at 0.015 m