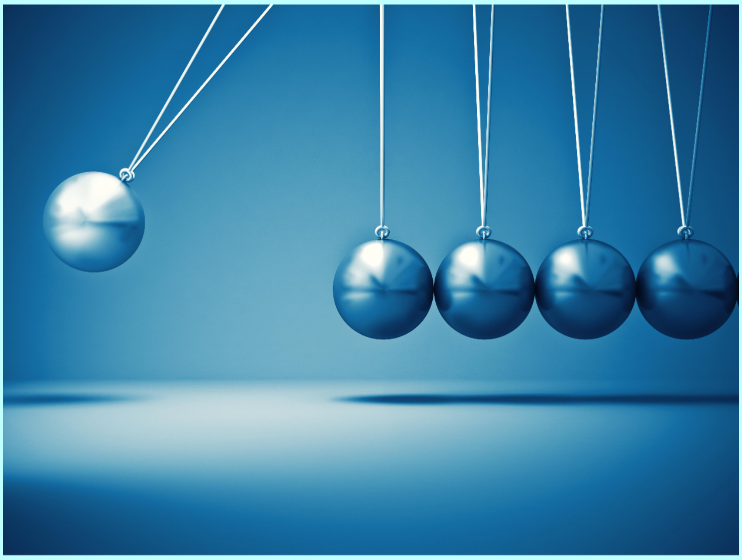
Unit 2 : Energy and Momentum

Lesson Content
Introduction to Energy and Momentum
- goal is to be able to tell the difference between energy and momentum
how to calculate each quantity



Feb 27-12:57 AM

Recall from Grade 11

Energy is the capacity to do work

Work : the energy transferred to an object by an applied force over a measured distance.

$$W = \Delta E = \vec{F} \Delta \vec{d}$$

work & energy → scalars

units → $N \cdot m$ = Joule.

$1N = kg\,m/s^2$ → $kg\,m/s^2 \cdot m$

→ $kg\,m^2/s^2$

Momentum

Momentum is a property of the mass and velocity of an object

$$\vec{p} = m\vec{v}$$

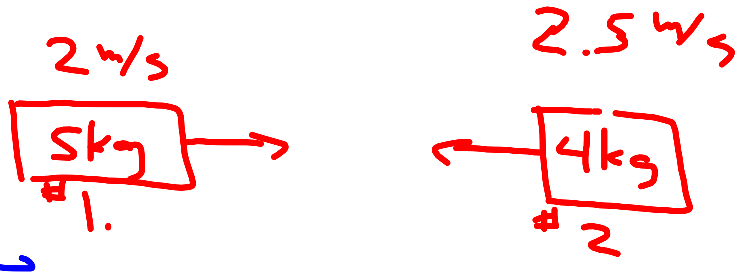
$\vec{p}$ - symbol for momentum

Characteristics of Momentum:

1. vector → magnitude, units, direction
2. units → kg m/s (compare to energy $kg(m/s)^2$)
3. property of an object or a system of objects.

Momentum of a system is the vector sum of the individual momenta.

Example:



$$\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2$$

$$= 10\,kg\,m/s\,[E] + 10\,kg\,m/s\,[W]$$

$$= \phi$$

momentum $\vec{p} = m\vec{v}$

1. What is the magnitude of the momentum of the following three objects all moving at the same velocity of 5.00 m/s [N]?

i. shotput - mass = 7.26kg 36.3 kg m/s

ii. hockey puck – mass = 160 grams 0.800 kg m/s

iii. ping pong ball – mass = 2.70 grams 1.35×10^{-3} kg m/s

1b. How fast would the hockey puck and the ping pong ball have to be travelling to have the same momentum as the shotput moving at 5.00 m/s [N]?

$p = 36.3 \text{ kg m/s}$

hockey puck $m = 0.160 \text{ kg}$

$$v = \frac{p}{m} = \frac{36.3 \text{ kg m/s}}{0.160 \text{ kg}}$$

$= 227 \text{ m/s}$

p.p. ball $m = 0.00270 \text{ kg}$

$v = 1.34 \times 10^{4} \text{ m/s}$

Impulse

Impulse is defined as the change in momentum:

$$\vec{J} = \Delta\vec{P}$$
$$= \vec{P}_2 - \vec{P}_1$$

How does impulse (change in momentum) relate to net force?

$$\vec{J} = \vec{P}_2 - \vec{P}_1$$
$$\vec{J} = m\vec{v}_2 - m\vec{v}_1$$
$$\vec{J} = m(\vec{v}_2 - \vec{v}_1)$$

divide both sides by Δt

$$\frac{\vec{J}}{\Delta t} = m\frac{(\vec{v}_2 - \vec{v}_1)}{\Delta t}$$
$$\frac{\vec{J}}{\Delta t} = m\vec{a}$$
$$\boxed{\frac{\vec{J}}{\Delta t} = \vec{F}}$$

Newton's 2ND Law.

$$\boxed{\vec{J} = \vec{F}\Delta t}$$

$$\vec{J} = \Delta\vec{P}$$

units

L.S. R.S.

kg m/s N·s

= kg m/s² · s

= kg m/s

Summary

momentum

$$\Delta\vec{P} = \vec{F}\Delta t$$

kg m/s

energy

$$\Delta E = \vec{F}\Delta\vec{d}$$

kg m²/s²

impulse = change in momentum

$$\vec{j} = \Delta\vec{p} = \vec{F}\Delta t$$

2a. What is the impulse given to a golf ball by a club if they are in contact for 5.0x10^{-3} secs and the average force on the ball is 500N[forward]?

$$\vec{j} = \Delta\vec{p} = \vec{F}\Delta t$$
$$= 500N[forward] \times .005\,s$$
$$= 2.5\,N\cdot s\,[forward]$$
$$= 2.5\,kg\,m/s\,[forward]$$

2b. What will the speed of the ball be if the mass is 46 grams (assuming the ball starts at velocity = 0m/s)?

$\Delta p = 2.5\,kg\,m/s$, $P_1 = 0$

$\Delta p = P_2 - P_1$

$\therefore P_2 = \Delta p + P_1$

$= 2.5\,kg\,m/s\,[forward]$

$V_2 = P_2/m = \dfrac{2.5\,kg\,m/s\,[forward]}{0.046\,kg}$

$= 54.3\,m/s\,[forward]$

$V_1 = 0$

$t = .005$

$a = ? = F/m = 500N/0.046kg = 10{,}869\,m/s^2$

$V_2 = V_1 + a\Delta t = 54.3\,m/s$

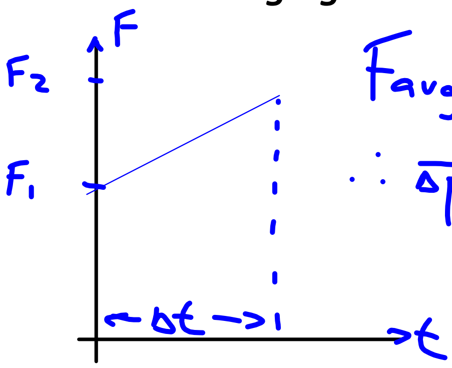
3. What is the impulse given a golf ball (46 grams) if the initial velocity is 0 m/s and the final velocity is 65 m/s [forward].

Calculating Impulse When Force is not Constant

impulse $= \vec{J} = \Delta\vec{p} = \vec{F}_{avg}\Delta t$

$*$ the trick is determining $\vec{F}_{avg}$

Case 1 : Force is changing linearly.

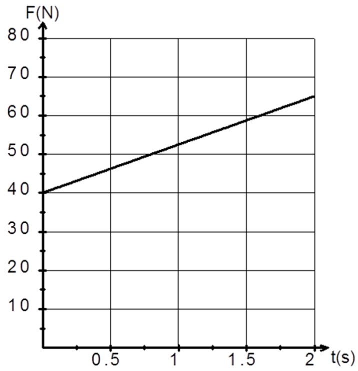


$$F_{avg} = \frac{F_1 + F_2}{2}$$

$$\therefore \Delta\vec{p} = \left(\frac{\vec{F}_1 + \vec{F}_2}{2}\right)\Delta t$$

Example (from handout)

Force increases linearly from 40N to 65N in 2 seconds. Find the impulse.



$$\vec{J} = \Delta\vec{p} = \vec{F}_{avg}\Delta t$$
$$= \left(\frac{40+65}{2}\right)2$$
$$= +105\ N\cdot s$$

b. if a 5 kg object originally moving at 21 m/s was acted on by this force, what is the final velocity?

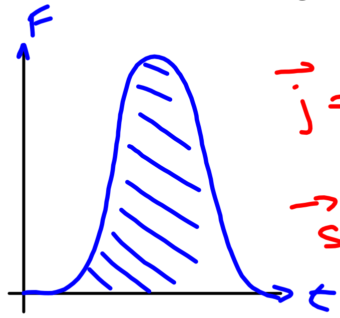
initial momentum

$$P_i = mv = 5kg \times 21 m/s$$
$$= 105 kg\,m/s$$

$$P_f = \Delta\vec{p} + \vec{P}_i$$
$$= 210 kg\,m/s$$

$$V_f = \vec{P}_f / m = 42 m/s$$

Case 2 : Force is changing non-linearly.

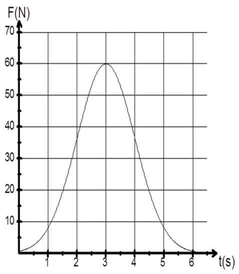


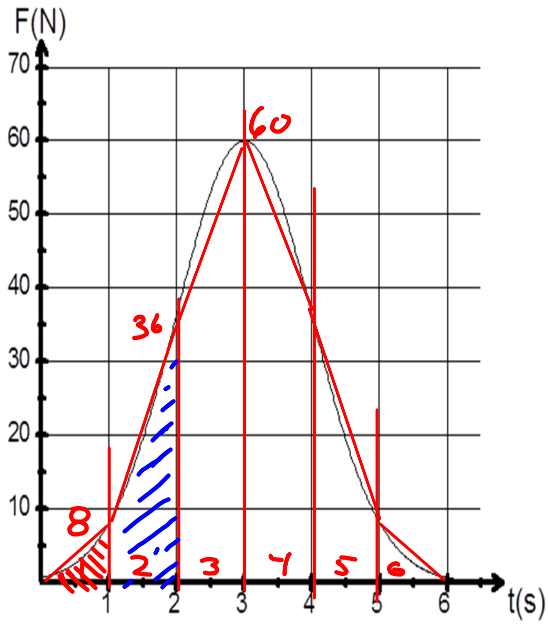
$\vec{J} = \Delta\vec{p}$ = area under F-t graph

→ break area into smaller segments & approximate with straight lines.

Example (from handout)

Force increases from zero to a maximum of 60N and then decreases back to zero in six seconds. Find the impulse.





impulse = $\Delta\vec{p}$ = area under graph

= A1 + A2 + A3 + A4 + A5 + A6

$A1 = \frac{1}{2}bh = \frac{1}{2}(1)8 = 4$

$A2 = \left(\frac{8+36}{2}\right)(1) = 22$

$A3 = \left(\frac{36+60}{2}\right)(1) = 48$

$A4 = 48$

$A5 = 22$

$A6 = 4$

$\Delta p = 148\ N \cdot s$

Conservation of Momentum

If no external forces are applied to a closed system, the total momentum remains constant.

if $\vec{F}_{net} = \phi$, $\Delta\vec{P} = \phi$.

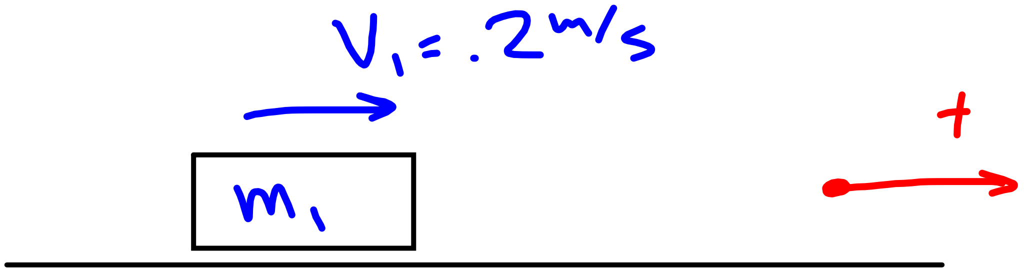
initial momentum = final momentum

$\vec{P}_{tot} = \vec{P}_{tot}'$ prime symbol means final

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 \cdots = \vec{P}_1' + \vec{P}_2' + \vec{P}_3' + \ldots$

Collision Cart Examples:

Example 1 : 1 Object

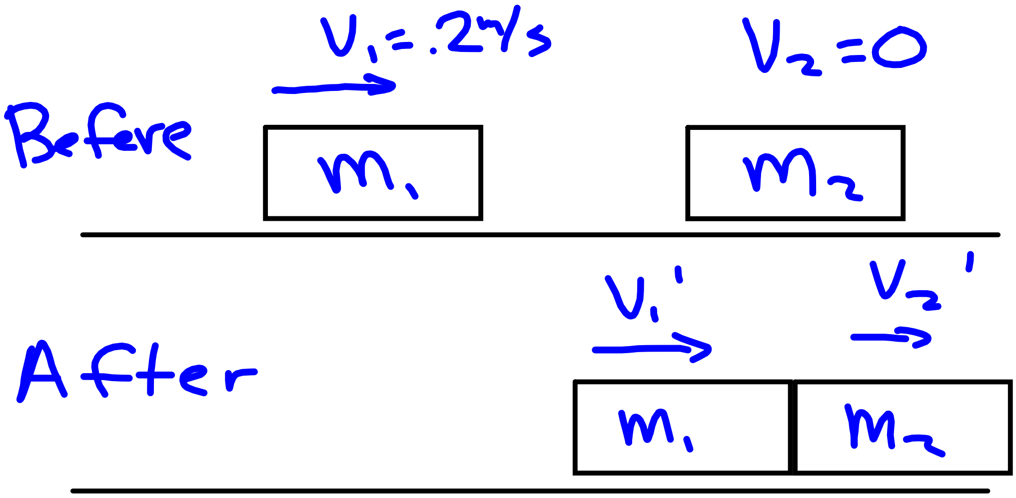


$m_1 = 2kg$

$$\vec{P}_{tot} = \vec{P}_1$$
$$= 0.4\ kg\,m/s$$

Mar 10-2:47 PM

Example 2 : Two Objects - join together after collision



$m_1 = 2\,kg$
$m_2 = 0.5\,kg$

$$\vec{P}_{tot} = \vec{P}_{tot}'$$

$$m_1V_1 + \cancel{m_2V_2} = m_1V_1' + m_2V_2'$$

$$0.4 = 2V_1' + 0.5V_2'$$

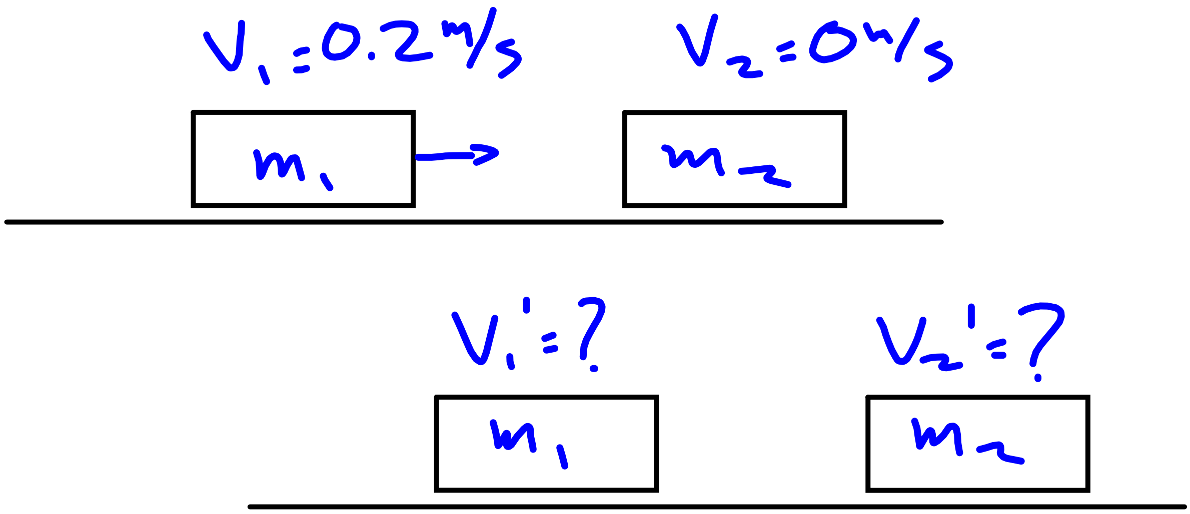
Since join together $V_1' = V_2' = V'$

$$0.4 = 2V' + 0.5V'$$
$$0.4 = 2.5V'$$
$$V' = 0.16\,m/s$$

$$m_1V_1 + m_2V_2 = (m_1 + m_2)V'$$

when stick together

Example 3 : 2 Objects - Bounce off each Other



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$0.4 = 2v_1' + 0.5v_2'$$

Cannot be solved without one additional piece of information.

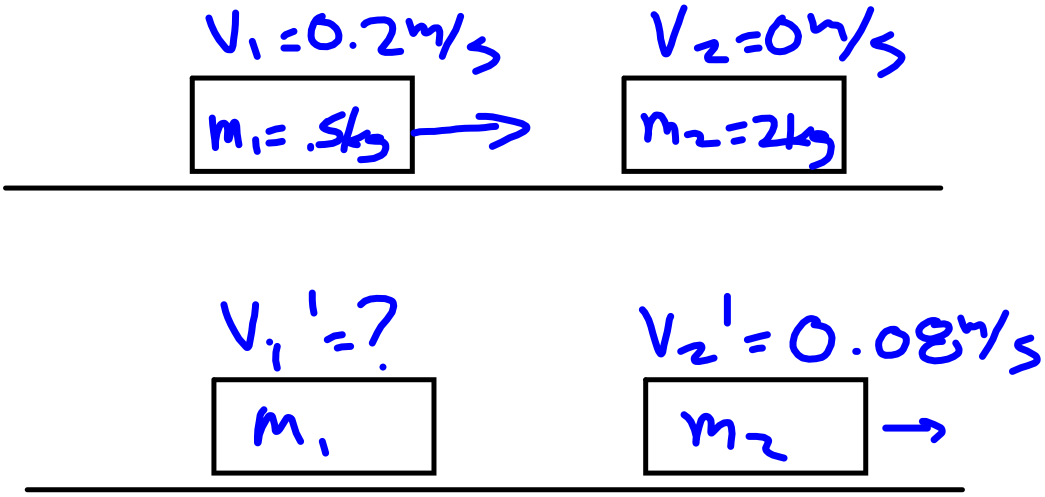
if given $v_1' = 0.12\,m/s$, can then find v_2'

$$0.4 = 2(.12) + .5(v_2')$$

$$0.16 = .5v_2'$$

$$v_2' = 0.32\,m/s$$

Example 4 : 2 Objects - Light hits Heavy



find V_1'

$$m_1 V_1 = m_1 V_1' + m_2 V_2'$$

$$0.1 = .5 V_1' + 0.16$$

$$-0.06 = 0.5 V_1'$$

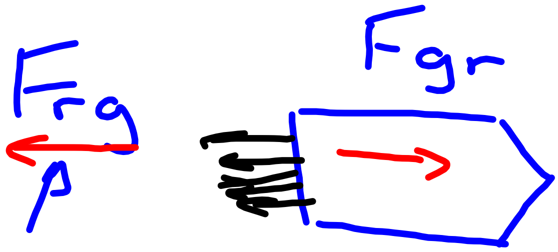
$$\boxed{V_1' = -0.12\,m/s}$$

Thrust

Thrust is a concept using Newton's 2nd & 3rd Laws in conjunction with Conservation of Momentum.

2^{ND} $F = ma$

3^{RD} equal & opposite.



← force gas on rocket

force of rocket on gas

$$F = ma$$

$$F = m\left(\frac{\Delta V}{\Delta t}\right)$$

$$* \quad F = \left(\frac{m}{\Delta t}\right)\Delta V \quad *$$

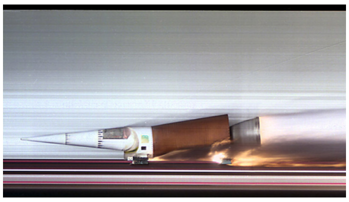
exhaust rate in kg/s.

exhaust speed in m/s.

* Thrust Equation. *

Jan 22-10:14 AM

6. An experimental rocket sled on a level frictionless track has a mass of 1.4×10^4 kg. For propulsion, it expels gases from its' rocket engines at a rate of 10 kg/s and at an exhaust speed of 2.5×10^4 m/s relative to the rocket. For how many seconds must the engines burn in order that the sled acquire a velocity of 50 m/s starting from rest? (You may ignore the small decrease in mass of the sled and the small speed of the rocket compared to the exhaust gas).



10 kg/s
25 000 m/s

1. analyze force

$$F = \left(\frac{\Delta m}{\Delta t}\right)\Delta v$$
$$= (10\ \text{kg/s})(25000\ \text{m/s})$$
$$= 2.5\times10^5\ \text{kg m/s}^2\ (\text{N})$$

2. analyze motion of rocket

$V_1 = 0$
$V_2 = 50\ \text{m/s}$
$\Delta t = ?$
$\Delta d = x$

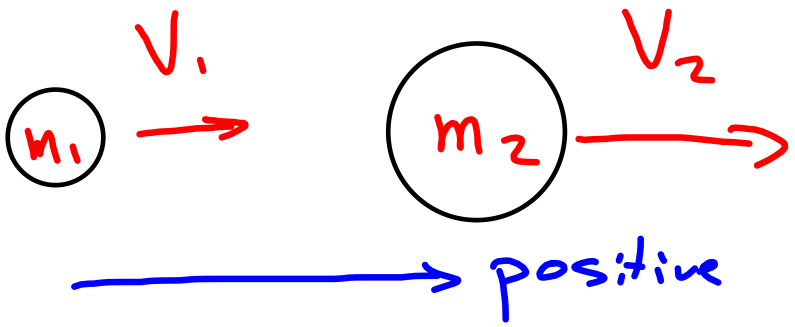
$$a = \frac{F}{m}$$
$$= \frac{2.5\times10^5\ \text{N}}{14000\ \text{kg}}$$
$$= \underline{\qquad}$$

②
$$\Delta t = \frac{V_2 - V_1}{a}$$
$$= 2.8\ \text{s}$$

$$\Delta p = F\Delta t$$
$$m\Delta v = F\Delta t$$
$$\Delta t = \frac{m\Delta v}{F}$$
$$= \frac{(14000\ \text{kg})(50\ \text{m/s})}{2.5\times10^5\ \text{N}}$$
$$= 2.8\ \text{s}$$

= 28 kg of fuel.

Elastic vs Inelastic Collisions



Collisions

momentum → momentum is always conserved.

① $m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$

energy → energy is always conserved.

→ in an elastic collision kinetic energy is conserved

② $\cancel{\tfrac{1}{2}}m_1V_1^2 + \cancel{\tfrac{1}{2}}m_2V_2^2 = \cancel{\tfrac{1}{2}}m_1V_1'^2 + \cancel{\tfrac{1}{2}}m_2V_2'^2$

$m_1V_1^2 + m_2V_2^2 = m_1V_1'^2 + m_2V_2'^2$

knowns - m_1, m_2, V_1, V_2

unknowns - V_1', V_2'

2 equations with 2 unknowns.

① $m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$
② $m_1V_1^2 + m_2V_2^2 = m_1V_1'^2 + m_2V_2'^2$

Simplify the problem by setting $V_2 = 0$.

① $m_1V_1 = m_1V_1' + m_2V_2'$
② $m_1V_1^2 = m_1V_1'^2 + m_2V_2'^2$

① $m_1(V_1 - V_1') = m_2V_2'$
② $m_1(V_1^2 - V_1'^2) = m_2V_2'^2$

② ÷ ①

$$\frac{\cancel{m_1}(V_1^2 - V_1'^2)}{\cancel{m_1}(V_1 - V_1')} = \frac{\cancel{m_2}V_2'^2}{\cancel{m_2}V_2'}$$

$$\frac{(V_1^2 - V_1'^2)}{(V_1 - V_1')} = \frac{V_2'^2}{V_2'}$$

$$\frac{\cancel{(V_1 - V_1')}(V_1 + V_1')}{\cancel{(V_1 - V_1')}} = V_2'$$

③ $\boxed{V_1 + V_1' = V_2'}$

plug ③ into ① for V_2'

$$m_1V_1 = m_1V_1' + m_2(V_1 + V_1')$$

$$m_1V_1 - m_2V_1 = m_1V_1' + m_2V_1'$$

$$V_1(m_1 - m_2) = V_1'(m_1 + m_2)$$

* $\boxed{V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1}$

we can also show that

* $\boxed{V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1}$

Apr 7-1:38 PM

Practice Problem #1

An air track glider of mass 0.050 kg, moving at 1.0 m/s collides elastically with another glider of mass 0.200 kg, which is initially at rest. What are the velocities of each glider after the collision?

$m_1 = 0.050\text{kg}$
$v_1 = 1.0\text{m/s}$
$m_2 = 0.200\text{kg}$
$v_2 = 0$

$v_1' = -0.60\text{m/s}$
$v_2' = +0.40\text{m/s}$

Practice Problem #2

An 20.0 kg red curling rock is travelling at 2.3 m/s [E] when it collides head-on elastically with the opponent's yellow rock of the same mass. What is the velocity of both rocks after the collision?

$m_1 = 20.0\text{kg}$
$v_1 = 2.3\text{m/s}$
$m_2 = 20.0\text{kg}$
$v_2 = 0$

$v_1' = 0$
$v_2' = 2.3\text{m/s}$

Collisions

Momentum is conserved
$p_i = p_f$

Energy is Conserved
$E_i = E_f$

Elastic Collision
Kinetic Energy is Conserved

Inelastic Collision
Kinetic Energy is NOT Conserved

Special Case : Head On Collision
- no angular components
- one object at rest at beginning

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1$$

$$V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1$$

Three Scenarios for V_1'

1. $m_1 > m_2$ – (a big mass hits a smaller mass), then $m_1 - m_2$ is positive and V_1' is positive, meaning m_1 keeps going in the same direction but at a lesser speed
2. $m_1 < m_2$ – (a smaller mass hits a bigger mass), then $m_1 - m_2$ is negative and V_1' is negative, meaning m_1 bounces back in the opposite direction to what it was going before the collision
3. $m_1 = m_2$, (equal masses) then $m_1 - m_2$ is zero and V_1' is zero.

Scenario for V_2'

The second equation shows that m_2 rebounds in the same direction that m_1 came into the collision (i.e. V_2' and V_1 are in the same direction).

If m1=m2, then $V_2' = V_1$ (m_2 rebounds at the same speed that m_1 started with).

May 23-9:59 PM