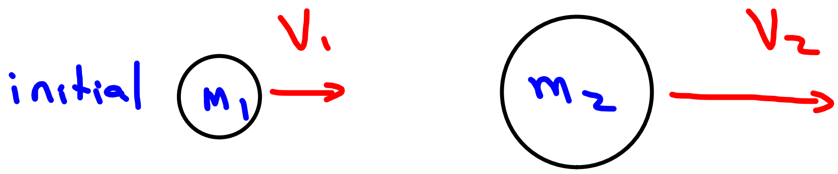
Elastic vs Inelastic Collisions (linear)



unknowns → v_1' & v_2'

momentum is always conserved

1. $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

energy is always conserved

if kinetic energy is conserved then the collision is called elastic (if kinetic energy is lost (or gained) then it is inelastic)

2. Elastic Collision.

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2$$

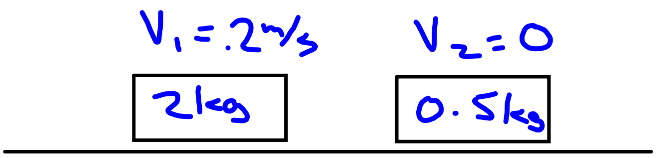
$$m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$$

given m_1, m_2, v_1, v_2
can solve for v_1', v_2'
(2 eq'ns with 2 unknowns)

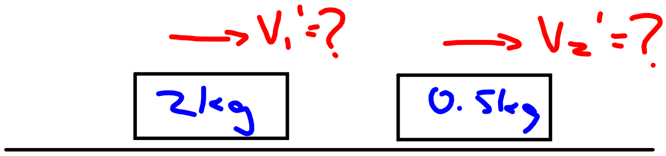
Collision Cart Examples:

problem from our first day doing collisions

Example 1 : 2 Objects (1 object moving, 1 object stationary)



find V_1' & V_2' (assume elastic)



mom $0.4 = 2V_1' + 0.5V_2'$ ①

E_k $0.08 = 2V_1'^2 + 0.5V_2'^2$ ②

① x2 $0.8 = 4V_1' + V_2'$

$V_2' = 0.8 - 4V_1'$

sub into ② for V_2'

② $0.08 = 2V_1'^2 + 0.5(0.8 - 4V_1')^2$

$0.08 = 2V_1'^2 + 0.5(0.64 - 6.4V_1' + 16V_1'^2)$

$0.08 = 2V_1'^2 + 0.32 - 3.2V_1' + 8V_1'^2$

$10V_1'^2 - 3.2V_1' + 0.24 = 0$

$V_1' = \underline{0.2}$ or 0.12

initial velocity

$\therefore V_1' = 0.12$ m/s

Sub into eq'n ① to find V_2'

$V_2' = 0.32$ m/s.

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Collision Cart Examples:

Example 2 : 2 Objects - both moving

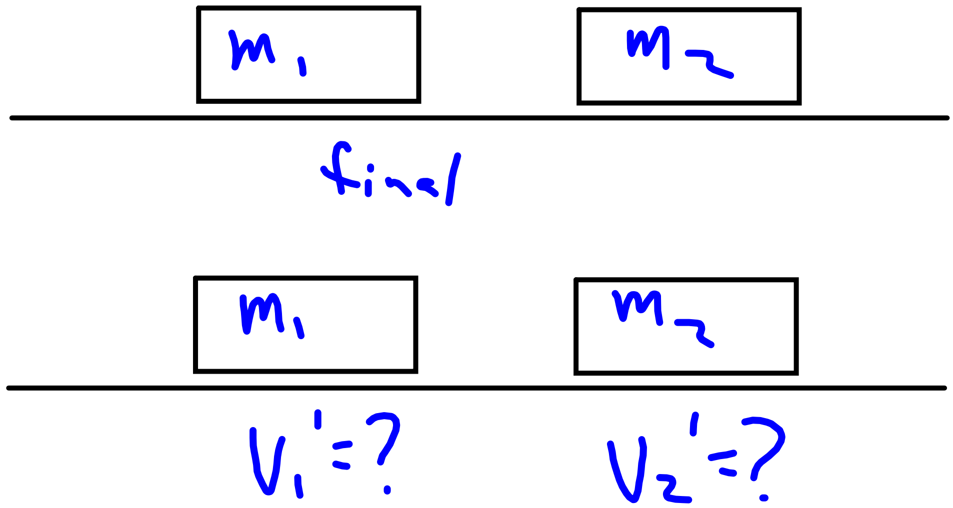
+

initial

$m_1 = 2\,kg$

$m_2 = 0.5\,kg$

$V_1 = 0.25\,m/s$ $V_2 = -0.50\,m/s$



Solving Elastic Collision Problems

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$
$$m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$$

let $v_2 = 0$

$$m_1v_1 = m_1v_1' + m_2v_2'$$
$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1(v_1 - v_1') = m_2v_2' \quad (1)$$

$$m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2$$
$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2$$
$$m_1(v_1^2 - v_1'^2) = m_2v_2'^2$$
$$m_1(v_1 - v_1')(v_1 + v_1') = m_2v_2'^2 \quad (2)$$

(2) ÷ (1)

$$\frac{m_1(v_1 - v_1')(v_1 + v_1')}{m_1(v_1 - v_1')} = \frac{m_2v_2'^2}{m_2v_2'}$$

$$v_1 + v_1' = v_2' \quad (3)$$

sub (3) into (1) & solve for v_1'

$$m_1(v_1 - v_1') = m_2(v_1 + v_1')$$

rearrange & solve for v_1'

$$m_1v_1 - m_1v_1' = m_2v_1 + m_2v_1'$$
$$m_1v_1 - m_2v_1 = m_2v_1' + m_1v_1'$$
$$\frac{v_1(m_1 - m_2)}{m_1 + m_2} = \frac{v_1'(m_1 + m_2)}{m_1 + m_2}$$

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$$
$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

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Practice Problem #1

An air track glider of mass 0.050 kg, moving at 1.0 m/s collides elastically with another glider of mass 0.200 kg, which is initially at rest. What are the velocities of each glider after the collision?

$m_1 = 0.050\,kg$
$v_1 = 1.0\,m/s$
$m_2 = 0.200\,kg$
$v_2 = 0$

$v_1' = -0.60\,m/s$
$v_2' = +0.40\,m/s$

Practice Problem #2

An 20.0 kg red curling rock is travelling at 2.3 m/s [E] when it collides head-on elastically with the opponent's yellow rock of the same mass. What is the velocity of both rocks after the collision?

$m_1 = 20.0\,kg$
$v_1 = 2.3\,m/s$
$m_2 = 20.0\,kg$
$v_2 = 0$

$v_1' = 0$
$v_2' = 2.3\,m/s$

Collisions

momentum is conserved

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

energy is conserved

$E_i = E_f$

Elastic Collision

$E_{k\,TOT} = E_{k\,TOT}'$

Inelastic

E_k is not conserved

Special Case

linear (no angles)

$V_2 = 0$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1 \qquad V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1$$

Three Scenarios for V_1'

1. $m_1 > m_2$, V_1' will be +'ve
2. $m_2 > m_1$, V_1' will be -'ve
3. $m_1 = m_2$, $V_1' = 0$

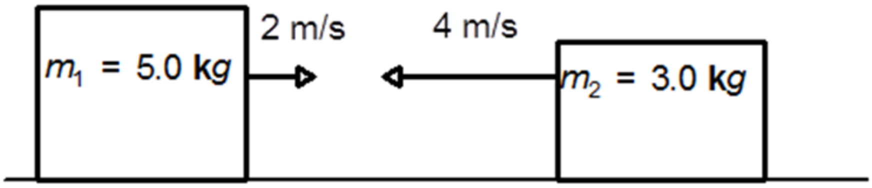
Scenario for V_2'

V_2' will always be +'ve.

What if Both Objects are Moving?

Collisions Involving Two Moving Objects

$V_1 = 2\,m/s$ $V_2 = -4\,m/s$



→ +'ve

Mass 1 is 5.0 kg and is moving to the right at 2.0 m/s.
Mass 2 is 3.0 kg and is moving to the left at 4.0 m/s.
They collide in an elastic collision.

a. Determine their velocities after the collision.

mom.

$$(5)(2)+(3)(-4)=5V_1'+3V_2'$$

① $-2 = 5V_1' + 3V_2'$

kin energy

$$\tfrac{1}{2}(5)(2)^2+\tfrac{1}{2}(3)(4)^2 = \tfrac{1}{2}(5)V_1'^2 + \tfrac{1}{2}(3)V_2'^2$$

$$10 + 24 = 2.5V_1'^2 + 1.5V_2'^2$$

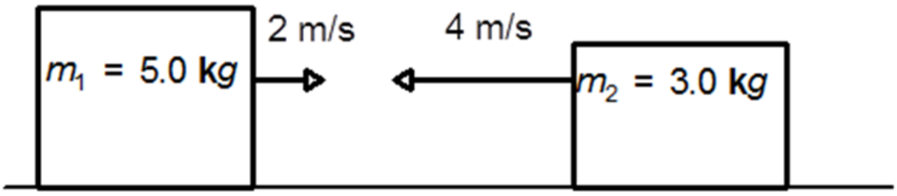
② $34 = 2.5V_1'^2 + 1.5V_2'^2$

sub ① into ② & solve for V_1' or V_2'

Collisions Involving Two Moving Objects

Shifting Frame of Reference

When two objects are both moving, we can shift our frame of reference to one of the two masses. This mass then has a relative velocity of zero.



Original Frame of Reference

$V_1 = 2\,m/s$ $\quad$ $V_2 = -4\,m/s$

Shift the frame of reference (for) by adding 4 m/s to each object

$V_1 = 6\,m/s$ $\quad$ $V_2 = 0\,m/s$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1 = \left(\frac{5-3}{5+3}\right)6 = 1.5\,m/s$$

$$V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1 = \left(\frac{2(5)}{5+3}\right)6 = 7.5\,m/s$$

Shift back to original f.o.r. by subtracting 4.0 m/s

$$V_1' = 1.5\,m/s - 4.0\,m/s = -2.5\,m/s$$

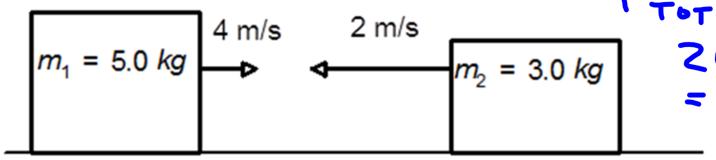
$$V_2' = 7.5\,m/s - 4.0\,m/s = 3.5\,m/s$$

Example 1b **: (same initial conditions as example #1, but with velocities reversed):**

Mass 1 is 5.0 kg and is moving to the right at 4.0 m/s.
Mass 2 3.0 kg and is moving to the left at 2.0 m/s.
They collide in an elastic collision.

a. Determine their velocities after the collision.
~~**b. Determine the potential energy stored in the bumpers at the point of minimum separation.**~~

$P_{TOT} = 20 - 6 = 14$



Answer

Shift our frame of reference to m_2.

initial velocities

$V_1 = 4\,m/s \quad V_2 = -2\,m/s$

$m_1 = 5\,kg \quad m_2 = 3\,kg$

shift f.o.r by adding 2 m/s

$V_1 = 6\,m/s \quad V_2 = 0$

$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1 = 1.5\,m/s \qquad V_2' = \left(\frac{2m_1}{m_1 + m_2}\right) V_1 = 7.5\,m/s$

shift back to original f.o.r by subtracting 2 m/s from each.

$V_1' = 1.5\,m/s - 2\,m/s = -0.5\,m/s \qquad V_2' = 7.5\,m/s - 2\,m/s = 5.5\,m/s$

Check

$P_{TOT} = 14 \qquad P_{TOT}' = (5)(-.5) + (3)(5.5) = -2.5 + 16.5 = 14$

Mar 30-10:01 AM

Warmup

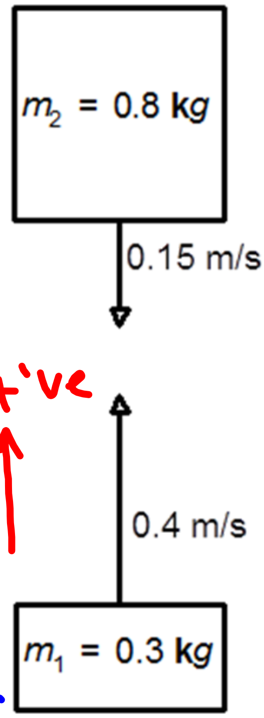
Example #2:

A 0.30kg object moving north at 0.40 m/s collides **elastically** with a 0.80kg object moving south at 0.15m/s.

a. What is velocity of the two objects after the collision?

~~b. What is the total kinetic energy at the point of minimum separation?~~

~~c. What is the potential energy stored in the bumpers at the point of minimum separation?~~



+'ve ↑

$V_1 = 0.4$ $V_2 = -0.15$

+0.15 shift f.o.r.

$V_1 = 0.55$ $V_2 = 0$

$V_1' = -0.25$ $V_2' = 0.3$

-0.15 shift back

$V_1 = -0.4$ $V_2 = 0.15$

$= 0.4$ [S] $= 0.15$ [N]