Energy and Elastic Collisions

one mass at rest

1. A 3 kg mass (mass 1) is travelling at a constant velocity of 4 m/s. It strikes a stationary 5 kg mass (mass 2) head-on. The collision is a perfectly elastic one.
   Determine the velocity of each mass after the collision.

both masses moving

2. A 10 kg cart (mass 1) is moving east at 6 m/s and strikes a 8 kg cart (mass 2) head - on. The 8 kg cart was moving west at 3 m/s. The collision is cushioned by a 15 cm spring bumper.

a) Switch into the mass 2 frame of reference by setting v_2 to 0 m/s. Find the shifted v_1.

b) Find the velocity of each cart after the collision.

c) **Calculate the velocity of the carts at minimum separation.**

d) Determine the maximum loss of kinetic energy during the collision.

e) The bumper exerts a constant force while compressing. It compresses a maximum of 9 cm during the collision. Find the force that the bumper exerts on the carts while compressing. (Hint: Use your answer from (d) to determine this.)

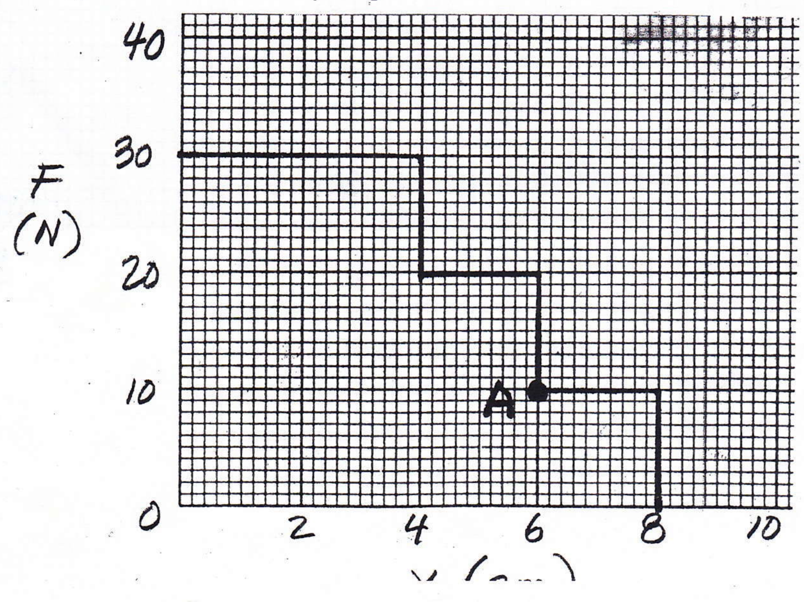
3. Two air track gliders of mass 500.0 g and 300.0 g are moving towards each other in opposite directions with speeds of 50.0 cm/s and 70.0 cm/s respectively. Assume that the direction of the 500.0 g glider is positive.

a) If the collision is perfectly elastic, find the velocity of each glider after the collision.

b) The most inelastic collision would happen if the two gliders stuck together on impact. If this were the case, find (i) the velocity of the pair after the collision and (ii) the kinetic energy lost as a result of the collision.

4. A mass of 3.75 kg travelling at 0.80 m/s collides with a spring anchored against a wall. The force separation graph for this spring bumper is shown below. Calculate:

a) the kinetic energy of the mass before the collision.
b) the velocity of the mass when it reaches point A.
c) the maximum compression of the spring.

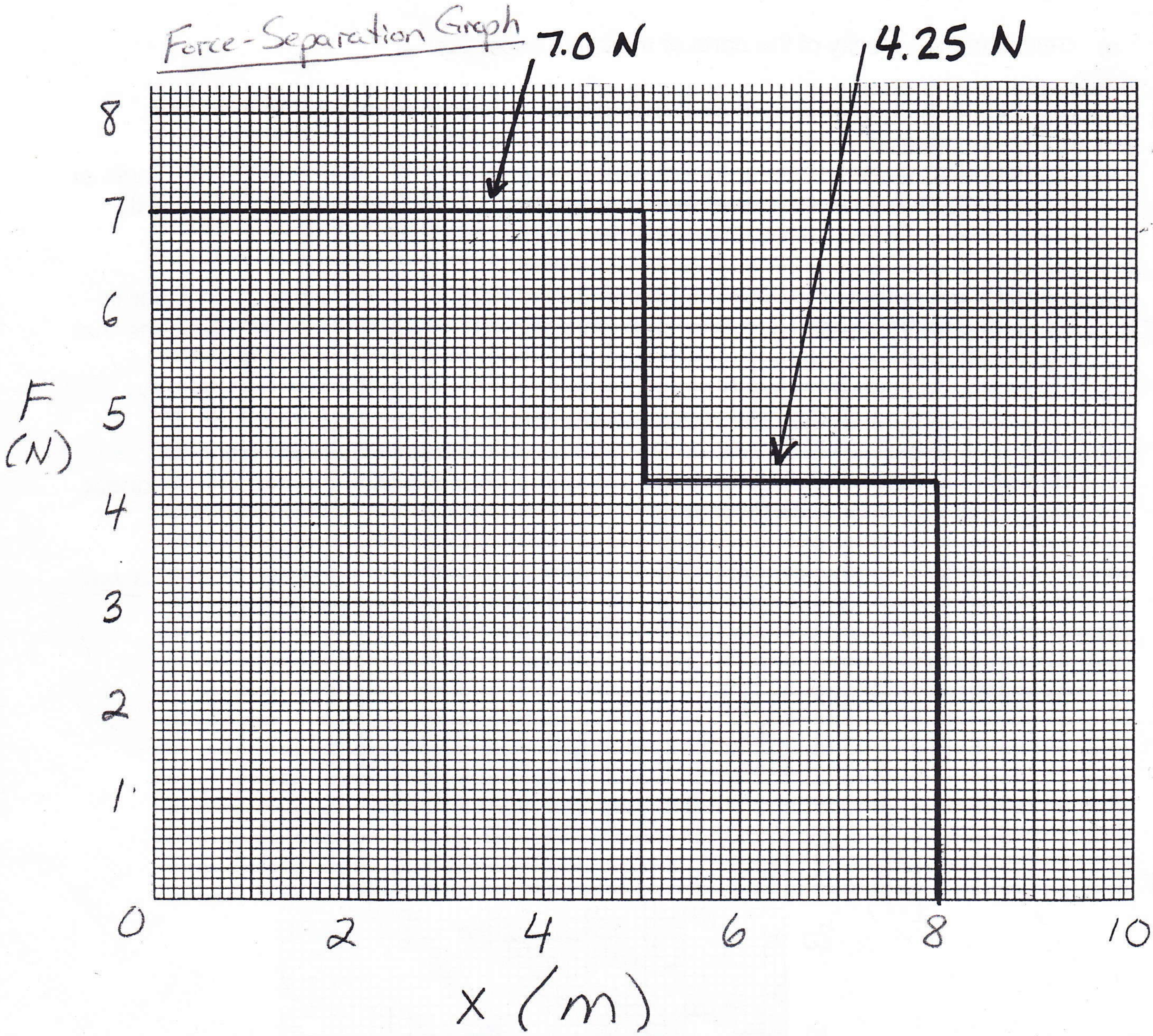


5. Two air track gliders of mass 3.0 kg and 5.0 kg are moving towards each other in opposite directions with speeds of 4.0 m/s and 2.0 m/s respectively. Assume that the direction of the 3.0 kg glider is positive.

a) If the collision is perfectly elastic and cushioned by a spring bumper, find the velocity of each glider after the collision.

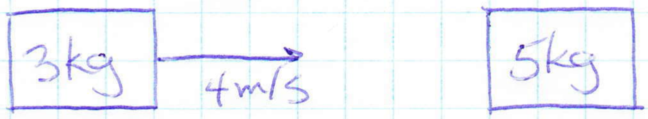
b) Given the F-x graph below for the spring bumper, find out the minimum separation of the two gliders (ie. how short the bumper becomes).

*Show your calculations clearly. *



Energy & Elastic Collisions

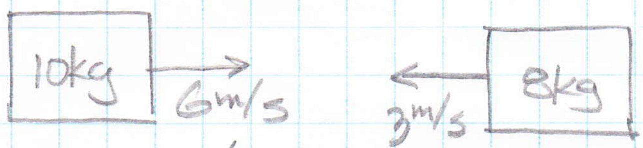
1.



$v_1 = 4\,m/s$ $\qquad$ $v_2 = 0$

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 = -1\,m/s \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 = 3\,m/s$$

2.



$v_1 = 6\,m/s$ $\qquad$ $v_2 = -3\,m/s$

a. Shifted

$V_1 = 9\,m/s$ $\qquad$ $V_2 = 0\,m/s$ (added 3 to each)

b. $$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1 = 1\,m/s \qquad V_2' = \left(\frac{2m_1}{m_1 + m_2}\right) V_1 = 10\,m/s$$

shift back by subtracting $3\,m/s$ to each.

$V_1' = -2\,m/s$ $\qquad$ $V_2' = 7\,m/s$

c. $m_1 V_1 + m_2 V_2 = m_1 V_0 + m_2 V_0$

$10kg \times 6\,m/s + 8kg \times (-3\,m/s) = 10kg\, V_0 + 8kg\, V_0$

$V_0 = 2.0\,m/s$

d. $\Delta E = E_{k\,min\,sep} - E_{k\,i} = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 - \frac{1}{2} m_1 V_0^2 - \frac{1}{2} m_2 V_0^2$

$= -180\,J$

e. loss in kinetic = gain in potential

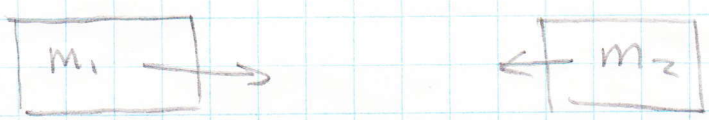
$\therefore E_p = +180J$

$F\Delta x = 180J$ $\qquad F = 180J/\Delta x$

$= 180J/.09$

$= 2000N$

3.



a. $m_1 = .500$ $\qquad m_2 = .300$

$v_1 = 50\ cm/s$ $\qquad v_2 = -70.0\ cm/s$

shifted $(+70\ cm/s)$

$v_1 = 120\ cm/s$ $\qquad v_2 = 0\ cm/s$

$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1$ $\qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$

$= 30\ cm/s$ $\qquad = 150\ cm/s$

"undo" the shift $(-70\ cm/s)$

$v_1' = -40\ cm/s$ $\qquad v_2' = 80\ cm/s$

b. $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_0$

$$v_0 = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{0.500 \times 50\ cm/s + 0.300(-70.0\ cm/s)}{0.500 + 0.300}$$

$= 5\ cm/s$

$= 0.05\ m/s$

$\Delta E_K = E_{Kf} - E_{Ki} = \frac{1}{2}(0.800kg)(0.05\ m/s)^2 - \frac{1}{2}(0.500kg)$
$\times (.50\ m/s)^2 - \frac{1}{2}(0.300kg)(0.70\ m/s)^2$

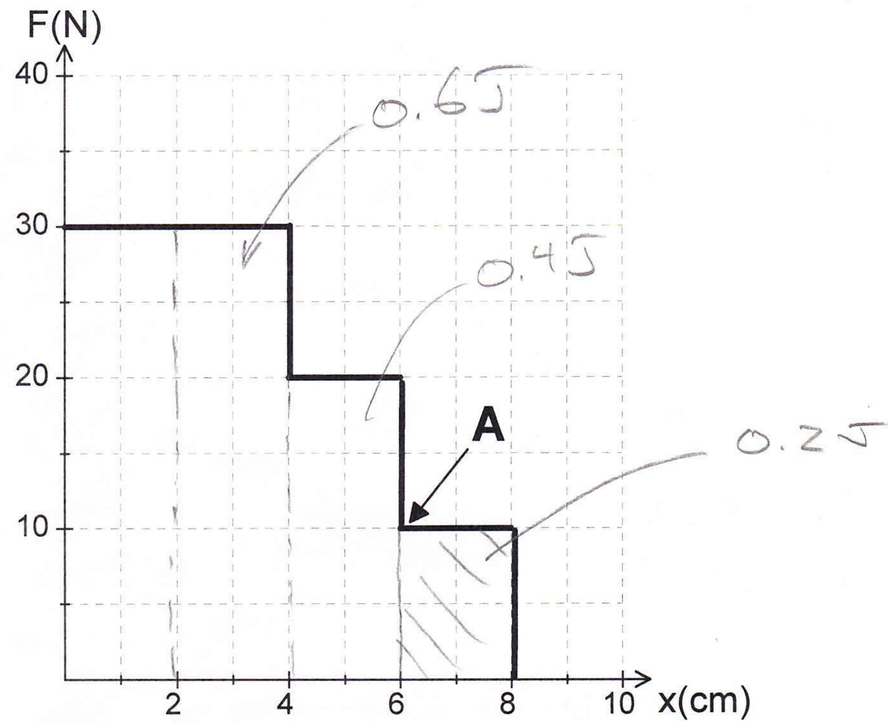
$= -0.135\ J$

4. A mass of 3.75 kg travelling at 0.80 m/s collides with a spring anchored against a wall. The force separation graph for this spring is shown below. Calculate

a. the kinetic energy of the mass before the collision

b. the velocity of the mass when it reaches point A.

c. the maximum compression of the spring.



4\. a. $E_K = \frac{1}{2}mv^2$

$= \frac{1}{2}(3.75\,kg)(0.80\,m/s)^2$

$= 1.2\,J$

b. point A $E_p = F\Delta x$

$= 10N \times 0.02\,m$

$= 0.2\,J$

$\therefore E_K = 1.0\,J$ $V = \sqrt{2E_K/m}$

$= 0.73\,m/s$

c. $E_K = 0$ when $E_p = 1.2\,J$

Area 1 = 0.2 J

Area 2 = 0.4 J

$\therefore$ Area 3 = 0.6 J

$0.6\,J = 30N \times \Delta x$

$\Delta x = 0.02\,m = 2\,cm$

$\therefore$ total compression = 2 + 2 + 2 cm

= 6 cm.

5.

$m_1 = 3.0\,kg$ $\quad$ $m_2 = 5.0\,kg$

$V_1 = 4.0\,m/s$ $\quad$ $V_2 = -2.0\,m/s$.

shift to make $V_2 = 0$ (add $2.0\,m/s$)

$V_1 = 6.0\,m/s$ $\quad$ $V_2 = 0\,m/s$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1 \qquad V_2' = \left(\frac{2m_1}{m_1 + m_2}\right) V_1$$

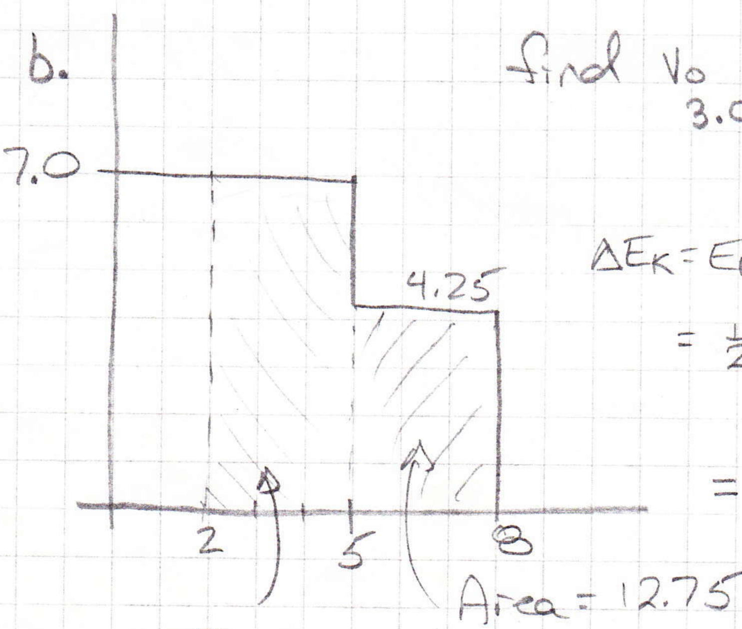
$$= \left(\frac{3.0 - 5.0}{3.0 + 5.0}\right) 6.0\,m/s \qquad V_2' = \left(\frac{2 \times 3.0}{3.0 + 5.0}\right) 6.0\,m/s$$

$$= -1.5\,m/s \qquad = 4.5\,m/s$$

shift back to earth's f.o.r. (subtract $2.0\,m/s$)

$V_1' = -3.5\,m/s$ $\quad$ $V_2' = 2.5\,m/s$.

b.



find V_o $\quad$ $m_1 V_1 + m_2 V_2 = m_1 V_o + m_2 V_o$

$3.0\,kg \times 4.0\,m/s + 5.0\,kg \times (-2.0\,m/s) = 8.0\,kg\, V_o$

$V_o = 0.25\,m/s$

$\Delta E_K = E_{K\,min\,sep} - E_{Ki}$

$= \frac{1}{2}(3.0\,kg)(0.25\,m/s)^2 + \frac{1}{2}(5.0\,kg)(0.25\,m/s)^2$
$\quad - \frac{1}{2}(3.0)(4.0\,m/s)^2 - \frac{1}{2}(5.0)(2.0)^2$

$= -33.75\,J$ = gain in Potential

area = 21J

$33.75 - 12.75 = 21\,J$

$3 \times 7 = 21\,J$ $\quad$ $\therefore$ total compression = 6m