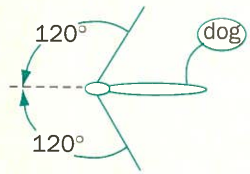
CHAPTER 10

Geometry

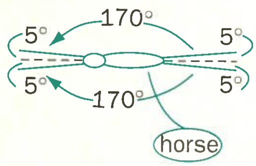
How Can We Model Your Field of Vision?

Your field of vision, or your peripheral vision, includes everything you can see when you look straight ahead *with your eyes fixed*. Your side-to-side, or horizontal, peripheral vision is the horizontal angle through which you can see. Your up-and-down, or vertical, peripheral vision refers to the two angles, one upward and the other downward, through which you can see.

The field of vision varies from one creature to another. For example, a dog has an angle of horizontal peripheral vision of about 240°, or about 120° on each side of the centre line.



For a horse, the angle of horizontal peripheral vision is about 340°, or 170° on each side. When a horse looks straight ahead, it cannot see an area within an angle of 10° directly in front of it or 10° directly behind it.



1. How does the placement of the eyes give dogs and horses different angles of horizontal peripheral vision?

2. Why is it not a good idea to approach a horse silently from directly in front or directly behind?

3. Among mammals, the angles of horizontal peripheral vision are smaller for dogs and other predators than for horses and other prey. What are the advantages to each type of mammal?

4. Use your research skills to find out

a) the field of vision for another type of creature, such as insects, fish, or birds

b) whether racehorses are running blind when they wear blinkers

5. Estimate your horizontal, upward, and downward angles of peripheral vision when you look straight ahead with your eyes fixed.

In Modelling Math — Biology on pages 555 to 557, you will experiment with the mathematical model to determine your own field of vision.

GETTING STARTED

Seeing Shapes

1 Length

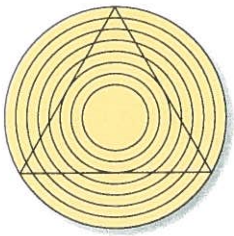
1. Horizontal and vertical lines can create an illusion involving length. In the diagram, does the horizontal or the vertical line seem longer?

2. a) Measure the lines and compare their lengths.
b) Explain your results.

2 Perspective

1. Do the sides of the triangle appear to be straight or bent?

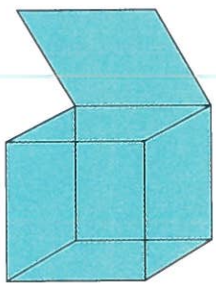
2. Describe the effect the circles have on the sides of the triangle.



3 Reversing Figures

1. Focus on this box until you see it change position.

2. Describe the second position as you perceive it.



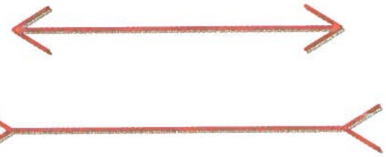
4 Impossible Figures

1. Focus on the figure and describe what you see.

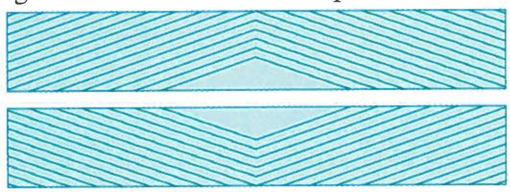
2. Is the figure in question 1 possible in the real world? Explain.

5 More Illusions

1. Which line segment do you think is longer? Check and explain.

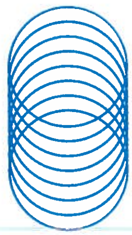


2. Do you think that the horizontal lines are straight or bent? Check and explain.

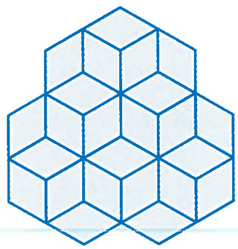


3. Describe the different ways you see each figure.

a)

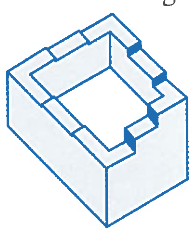


b)



4. What is wrong with each figure?

a)



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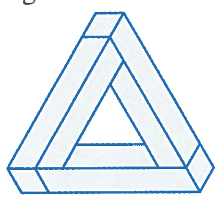
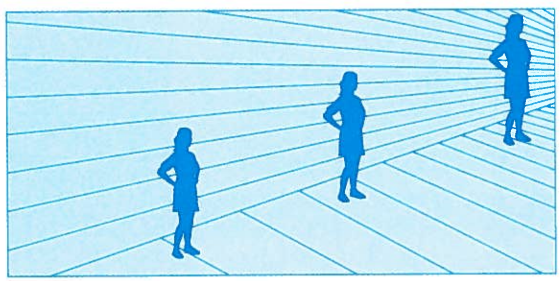
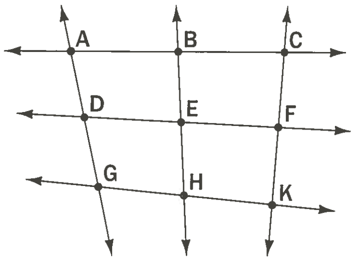


5. Are these 3 girls all the same height? Check and explain.



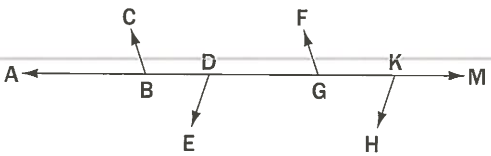
Warm Up

1. List 5 points and 5 lines in the diagram.



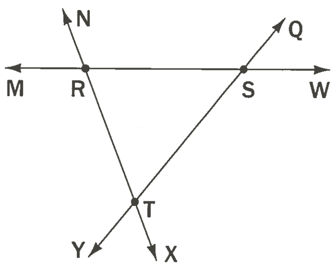
2. Name 5 line segments in this diagram.

3. Name 8 angles in this diagram.



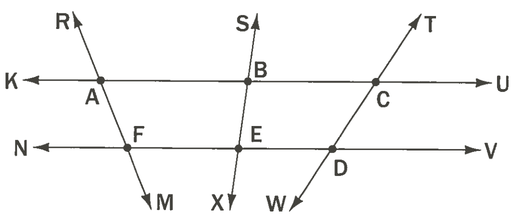
4. Name the following from the figure shown below.

a) 3 points
b) 3 lines
c) 6 angles
d) 3 rays
e) 3 line segments



5. In how many different ways can 3 lines intersect? Draw diagrams to illustrate your answer.

6. How many line segments are in this diagram? Name them.



Mental Math

Performing Operations and Solving Equations

Calculate.

1. $90 - 14$
2. $90 - 39$
3. $90 - 61$
4. $180 - 44$
5. $180 - 98$
6. $180 - 137$
7. $180 - 35 - 10$
8. $180 - 88 - 33$
9. $180 - (43 + 47)$
10. $180 - (29 + 77)$
11. $180 - 2(35)$
12. $180 - 2(67)$

Solve for x.

13. $x + 48 = 79$
14. $x + 91 = 156$
15. $x + x + x = 180$
16. $x + x + 2x = 180$
17. $x + 2x + 3x = 180$
18. $x + x + x + x = 360$

Multiplying by Multiples of 12

To multiply a number by 12, first multiply the number by 10, and then add twice the number.
For 12×34,
multiply by 10: $10 \times 34 = 340$
add 2×34, or 68: $340 + 68 = 408$
So, $12 \times 34 = 408$

Calculate.

1. 12×18
2. 12×21
3. 12×33
4. 12×48
5. 12×130
6. 12×450
7. 12×1.5
8. 12×2.6
9. 12×4.1

To multiply by 1.2, 120, and so on, first multiply by 12, and then place the decimal point.
Thus, $1.2 \times 34 = 40.8$, and $120 \times 34 = 4080$.

Calculate.

10. 120×16
11. 120×31
12. 1200×22
13. 1.2×25
14. 1.2×32
15. 1.2×44

16. Explain why the rule for multiplying by 12 works.

17. Modify the rule for multiplying by 12 to write a rule for multiplying by 22.

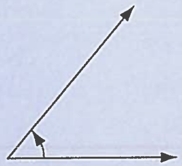
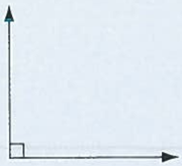
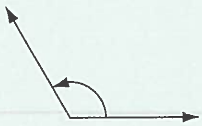
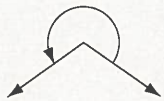
Calculate using your rule from question 17.

18. 22×14
19. 22×22
20. 22×43

Angles, Intersecting Lines, and Triangles

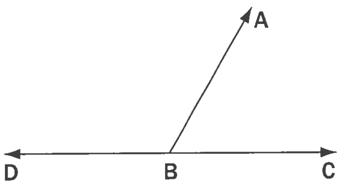
1 Angles

Recall that angles are classified according to their sizes in degrees.

Acute Angle	Right Angle	Obtuse Angle	Straight Angle	Reflex Angle
				
Less than 90°	Equal to 90°	More than 90°	Equal to 180°	Between 180° and 360°

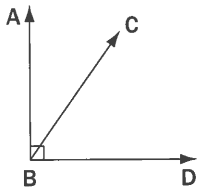
Some pairs of angles have special names.

Supplementary angles add to 180°.



∠ABC and ∠ABD are supplementary because ∠ABC + ∠ABD = 180°.

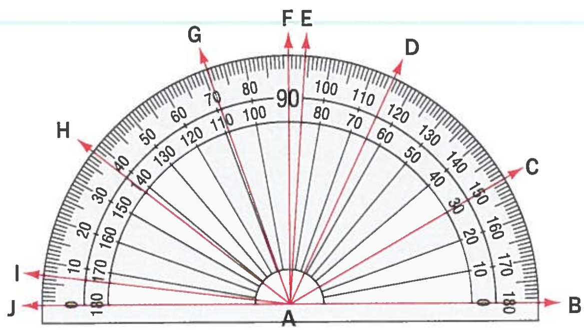
Complementary angles add to 90°.



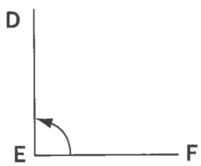
∠ABC and ∠CBD are complementary because ∠ABC + ∠CBD = 90°.

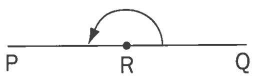
1. Determine the size of each of the following angles drawn on the protractor. Then, classify each angle as acute, right, obtuse, or straight.

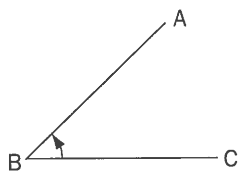
a) ∠BAC **b)** ∠BAD **c)** ∠BAE
d) ∠BAF **e)** ∠HAJ **f)** ∠GAJ
g) ∠IAJ **h)** ∠FAJ **i)** ∠DAJ
j) ∠BAH **k)** ∠BAI **l)** ∠BAJ

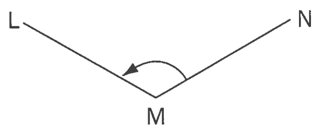


2. Estimate the size of each angle, and then classify each angle.

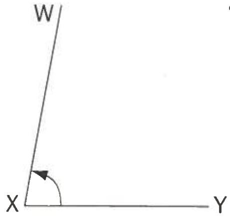
a)


b)


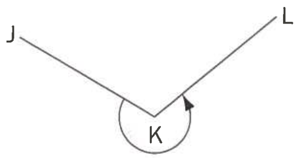
c)


d)


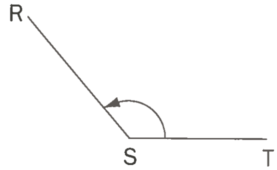
e)



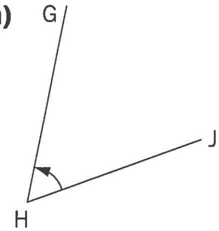
f)



g)



h)



3. Write the complement of each angle.

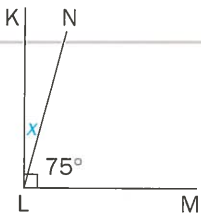
a) 30° **b)** 42° **c)** 75° **d)** 9°
e) 89° **f)** 66° **g)** 15° **h)** 1°

4. Write the supplement of each angle.

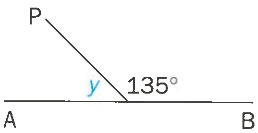
a) 40° **b)** 70° **c)** 85° **d)** 150°
e) 125° **f)** 8° **g)** 110° **h)** 179°

5. Find each unknown angle measure.

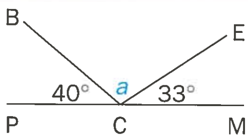
a)



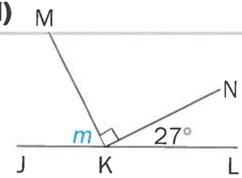
b)



c)

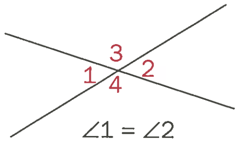


d)



2 Intersecting Lines

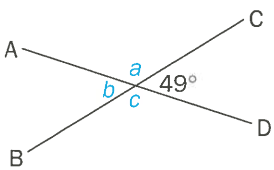
Recall that, when two lines intersect, the opposite angles are equal.



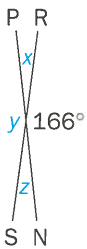
∠1 = ∠2
∠3 = ∠4

1. Find each unknown angle measure.

a)



b)



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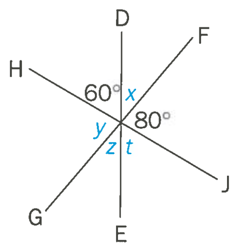
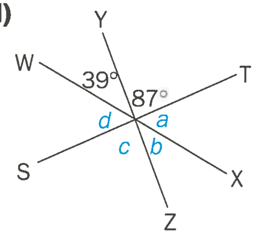
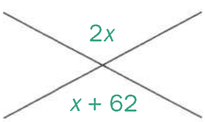


d)



2. **Algebra** Find the measures of all the acute and obtuse angles in each diagram.

a)



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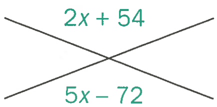

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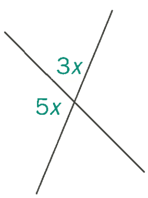

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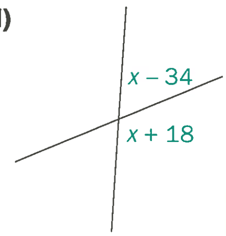
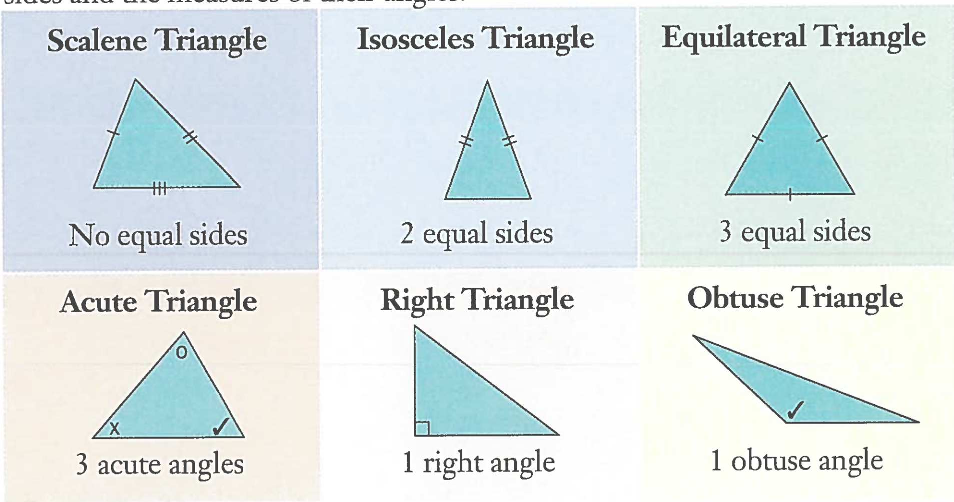


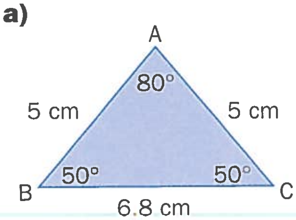
3 Triangles

Triangles can be classified according to the lengths of their sides and the measures of their angles.

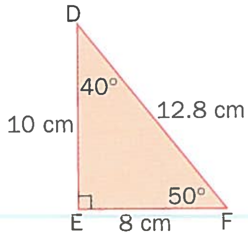


1. Classify each triangle according to its sides and angles.

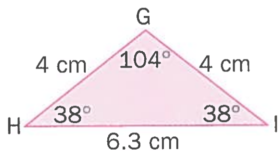
a)



b)



c)



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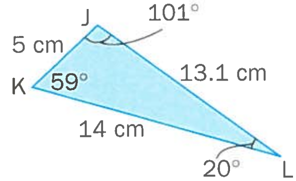
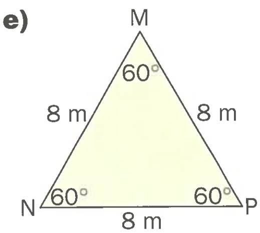


e)



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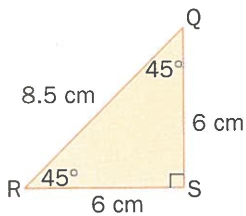
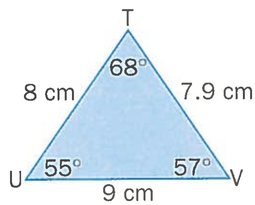
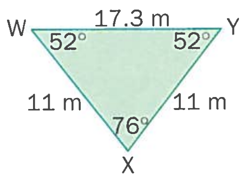


g)



h)



2. Flags Triangles are found in the flag designs of many countries. Classify each triangle in each of the following flags in two ways.

a)

Antigua and Barbuda

b)

Jamaica

c)

Marshall Islands

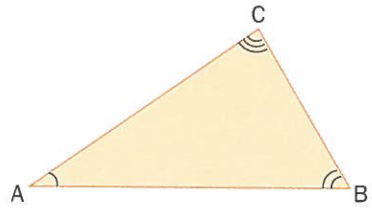
d)

Seychelles

Exploring Interior and Exterior Angles Using Geometry Software

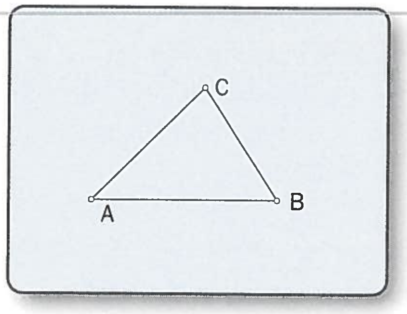
Complete the following explorations using geometry software or a graphing calculator with geometry capabilities. If suitable technology is not available, complete the equivalent paper-and-pencil explorations on pages 518–519.

1 Interior Angles of a Triangle



The three angles inside a triangle are known as **interior angles**. In △ABC, ∠A , ∠B, and ∠C are interior angles. ∠A is opposite side BC, ∠B is opposite side AC, and ∠C is opposite side AB.

1. a) Construct a scalene triangle.
b) Measure the lengths of the sides.
c) Measure the interior angles.
d) Calculate the sum of the interior angles.
e) List the sides from longest to shortest length and the angles from greatest to least measure.

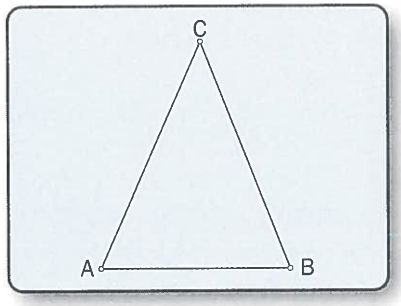


2. a) Drag a vertex to change the size and the shape of the triangle.
b) Calculate the sum of the interior angles.
c) List the sides from longest to shortest length and the angles from greatest to least measure.
d) Repeat parts a) to c) at least twice, or until you see a pattern in the sum of the interior angles.

3. Write a statement to describe the relationship between the measures of the three interior angles of a triangle.

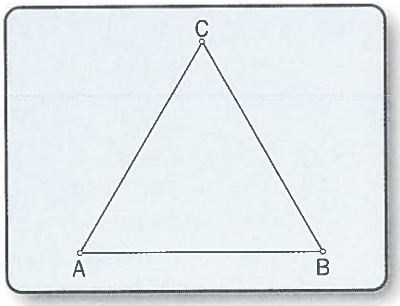
4. a) In a scalene triangle, how is the position of the greatest angle related to the position of the longest side?
b) In a scalene triangle, how is the position of the smallest angle related to the position of the shortest side?
c) Write a statement to describe the relationship between the measures of the angles and the lengths of the sides in a scalene triangle.

5. a) Construct an isosceles triangle.
b) Measure the angles opposite the equal sides.
c) Drag the vertex of the equal sides to change the size and the shape of the triangle.
d) Compare the angle measures.
e) Repeat parts c) and d) at least twice, or until you see a pattern.



6. Write a statement to describe the relationship between the measures of the angles opposite the equal sides in an isosceles triangle.

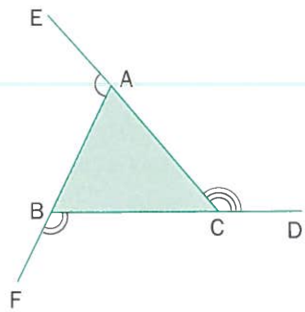
7. a) Construct an equilateral triangle.
b) Measure the interior angles.
c) Drag a vertex to change the size of the triangle.
d) Compare the angle measures.
e) Repeat parts c) and d) at least twice, or until you see a pattern.



8. Write a statement to describe the relationship between the measures of the angles in an equilateral triangle.

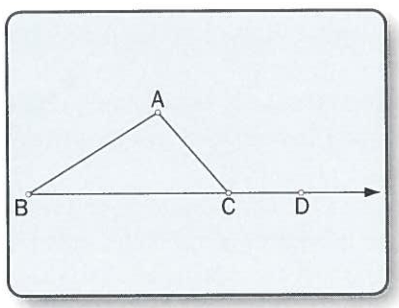
2 Exterior Angles of a Triangle

When each side of a triangle is extended in one direction, three angles are created outside the triangle. ∠BAE, ∠CBF, and ∠ACD are **exterior angles** of △ABC.



There are two interior angles opposite each exterior angle. ∠BAC and ∠ABC are opposite ∠ACD. ∠ABC and ∠BCA are opposite ∠BAE. ∠BCA and ∠BAC are opposite ∠CBF.

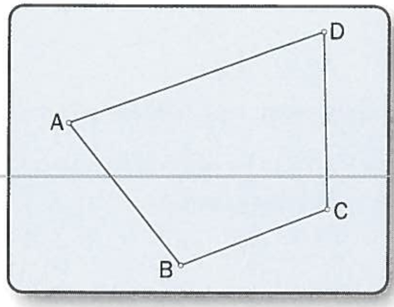
1. a) Construct a triangle.
b) Extend one side to create an exterior angle.
c) Measure the exterior angle.
d) Measure the two interior angles opposite the exterior angle.
e) Calculate the sum of the two interior and opposite angles, and compare the sum to the measure of the exterior angle.



2. a) Drag a vertex of the triangle.
b) Calculate the sum of the two interior and opposite angles, and compare the sum to the measure of the exterior angle.
c) Repeat parts a) and b) at least twice, or until you see a pattern.

3. Write a statement to describe the relationship between the measure of an exterior angle and the measures of the two interior and opposite angles.

3 Interior Angles of a Quadrilateral

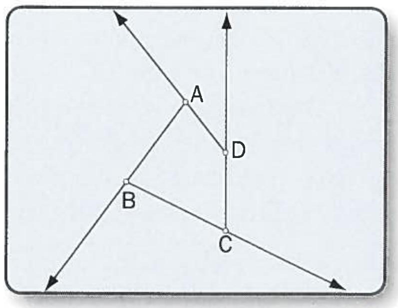


1. a) Construct a quadrilateral.
b) Measure the interior angles.
c) Calculate the sum of the interior angles.

2. a) Drag a vertex of the quadrilateral.
b) Calculate the sum of the interior angles.
c) Repeat parts a) and b) at least twice, or until you see a pattern.

3. Write a statement to describe the relationship between the measures of the four interior angles of a quadrilateral.

4 Exterior Angles of a Quadrilateral



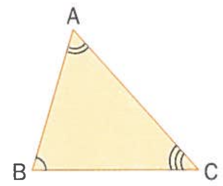
1. a) Construct a quadrilateral.
b) Extend each side in one direction, as shown, creating four exterior angles.
c) Measure the exterior angles.
d) Calculate the sum of the exterior angles.

2. a) Drag a vertex of the quadrilateral.
b) Calculate the sum of the exterior angles.
c) Repeat parts a) and b) at least twice or until you see a pattern.

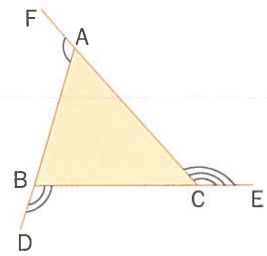
3. Write a statement to describe the relationship between the measures of the exterior angles of a quadrilateral.

Exploring Interior and Exterior Angles

The three angles inside a triangle are known as **interior angles**. In the diagram, ∠A, ∠B, and ∠C are the interior angles.



When the sides of the triangle are extended as shown, three more angles are created, ∠BAF, ∠CBD, and ∠ACE. These angles outside the triangle are called **exterior angles**.



1 Interior Angles of a Triangle

Draw a triangle of each of the following types in your notebook. Name the vertices.

a) isosceles **b)** equilateral **c)** scalene

1. Measure the three sides and the three angles in each triangle.

2. Calculate the sum of the interior angles in each triangle. Compare your findings with your classmates.

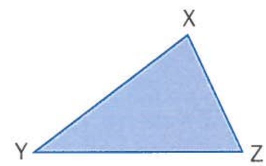
3. Write a statement to describe the relationship between the three interior angles of a triangle.

4. What is the measure of each angle in an equilateral triangle?

5. In △XYZ, ∠X is opposite side YZ, ∠Y is opposite side XZ, and ∠Z is opposite side XY.

a) For the scalene triangle you drew, list the side lengths from greatest to least and the angle measures from greatest to least.

b) Write a statement to describe the relationship between the measures of the angles and the lengths of the sides.



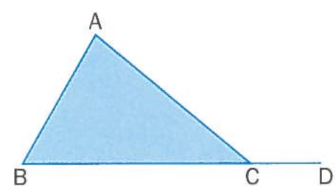
6. For the isosceles triangle you drew, how are the measures of the angles opposite the equal sides related?

2 Exterior Angles of a Triangle

In the diagram, ∠ACD is an exterior angle of △ABC. In your notebook, draw a diagram like the one shown.

1. Measure ∠ACD.

2. Measure ∠A and ∠B, which are the angles that are interior and opposite to ∠ACD.



3. Find the sum of these two interior angles. How does the sum compare with the measure of ∠ACD? Compare your findings with a classmate's.

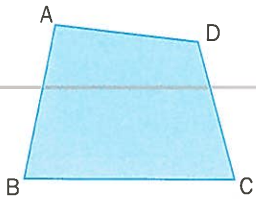
4. Repeat questions 1–3 for other exterior angles drawn at points A and B. In each case, compare the measure of the exterior angle with the sum of the measures of the interior and opposite angles.

5. Write a statement to describe the relationship between the measure of an exterior angle of a triangle and the measures of the two interior and opposite angles.

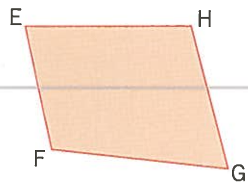
3 Interior Angles of a Quadrilateral

1. In your notebook, draw two quadrilaterals like the ones shown. Then, measure the four interior angles in each quadrilateral.

a)



b)



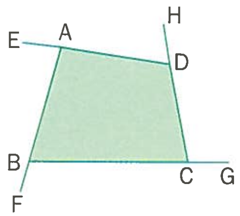
2. What is the sum of the measures of the interior angles in each quadrilateral? Compare your findings with a classmate's.

3. Write a statement to describe the relationship between the interior angles of a quadrilateral.

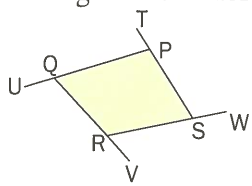
4 Exterior Angles of a Quadrilateral

1. In your notebook, draw two quadrilaterals like the ones shown. Then, measure the four exterior angles shown for each quadrilateral.

a)



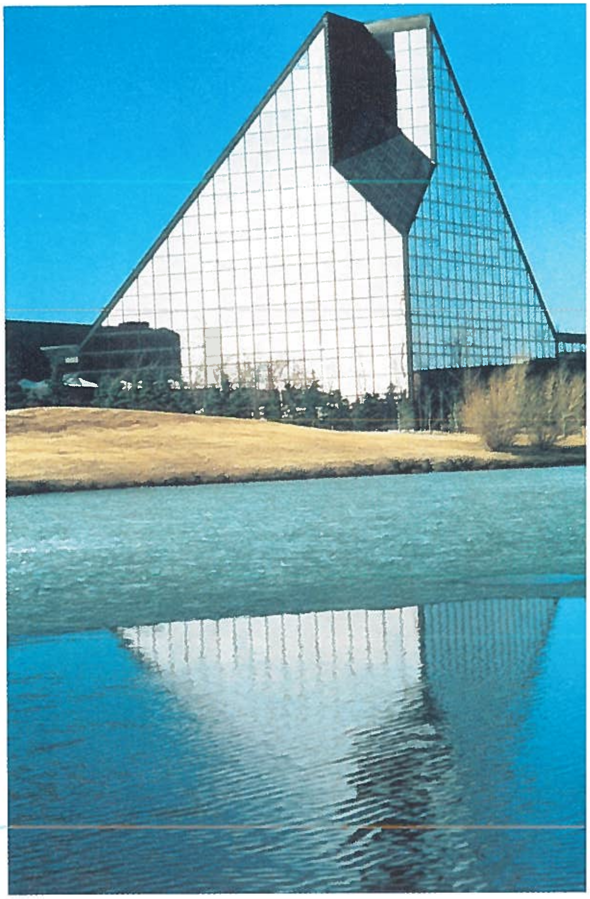
b)



2. What is the sum of the measures of the exterior angles for each quadrilateral? Compare your findings with a classmate's.

3. Write a statement to describe the relationship between the exterior angles of a quadrilateral.

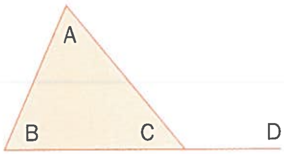
10.1 Interior and Exterior Angles of Triangles and Quadrilaterals



Triangles and quadrilaterals are seen in many types of construction. How many different types of triangles and quadrilaterals do you see in this photograph of the Canadian Mint in Winnipeg?

Explore: Perform an Experiment

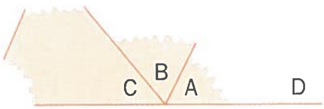
a) Draw an acute triangle, $\triangle ABC$, on a piece of paper. Label the vertices inside the triangle, as shown.



b) Extend BC to D to make an exterior angle, $\angle ACD$.

c) Cut out the triangle and the exterior angle, as shown.

d) Tear off $\angle A$ and $\angle B$, which are the interior and opposite angles to $\angle ACD$. Place them in the exterior angle, as shown.



e) Repeat steps a) to d) for a $\triangle ABC$ where $\angle B = 90°$; where $\angle B$ is obtuse.

Inquire

1. Write a statement about the sum of the interior angles of a triangle.

2. Is it possible for a triangle to have two 90° angles? Explain.

3. Is it possible for a triangle to have more than one obtuse angle? Explain.

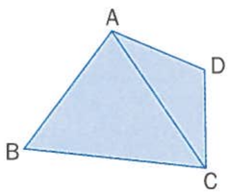
4. Write a statement about the relationship between an exterior angle of a triangle and the sum of the two interior and opposite angles. Compare your statement with a classmate's.

Explore: Use a Diagram

In quadrilateral ABCD, the diagonal AC creates two triangles, $\triangle ABC$ and $\triangle ADC$. What is the sum of the measures of the interior angles of

a) each triangle? **b)** the two triangles?

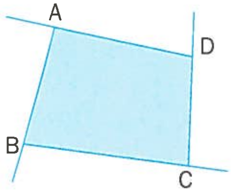
c) the quadrilateral?



Inquire

1. Write a statement about the sum of the measures of the interior angles of a quadrilateral. Compare your statement with a classmate's.

2. In the diagram, the sides of quadrilateral ABCD have been extended to form 4 exterior angles. At each vertex, there is one interior angle and one exterior angle. Each pair of interior and exterior angles forms a straight angle.



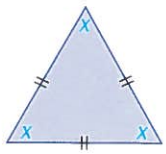
a) What is the measure of a straight angle?
b) What is the sum of the measures of the 4 straight angles?
c) What is the sum of the measures of the 4 interior angles?
d) What is the sum of the measures of the 4 exterior angles?

3. Write a statement about the angle sum of the exterior angles of a quadrilateral.

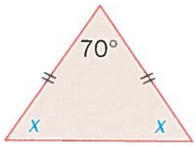
Example 1 Interior Angles of Triangles

Classify each triangle and find the missing angle measures.

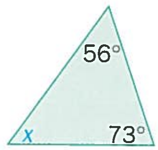
a)



b)



c)



Solution

a) The triangle has 3 equal sides and 3 equal angles.
It is an equilateral triangle.
$3x = 180°$
$x = 60°$

b) The triangle has 2 equal sides and 2 equal angles.
It is an isosceles triangle.
$2x + 70° = 180°$
$2x = 180° - 70°$
$2x = 110°$
$x = 55°$

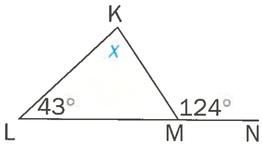
c) The triangle has no equal parts. It is a scalene triangle.

$x + 73° + 56° = 180°$
$x + 129° = 180°$
$x = 180° - 129°$
$x = 51°$

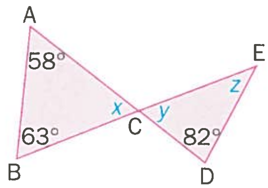
Example 2 Angles and Triangles

Find the missing angle measures.

a)



b)



Solution

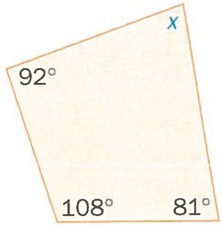
a) ∠KMN is an exterior angle of △KLM.
∠K and ∠L are the interior and opposite angles.
$124° = 43° + x$
$124° - 43° = x$
$81° = x$

b) In △ABC, $x + 58° + 63° = 180°$
$x + 121° = 180°$
$x = 180° - 121°$
$x = 59°$
Since x and y are opposite angles,
$y = x$
$y = 59°$
In △CDE, $59° + 82° + z = 180°$
$141° + z = 180°$
$z = 180° - 141°$
$z = 39°$

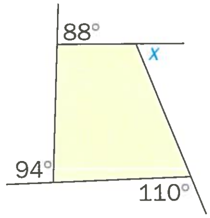
Example 3 Angles and Quadrilaterals

Find the missing angle measures.

a)



b)



Solution

a) The sum of the interior angles is 360°.

$$x + 92° + 108° + 81° = 360°$$
$$x + 281° = 360°$$
$$x = 360° - 281°$$
$$x = 79°$$

b) The sum of the exterior angles is 360°.

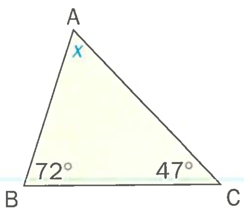
$$x + 88° + 94° + 110° = 360°$$
$$x + 292° = 360°$$
$$x = 360° - 292°$$
$$x = 68°$$

Practice

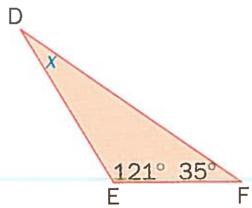
A

Find the missing angle measures.

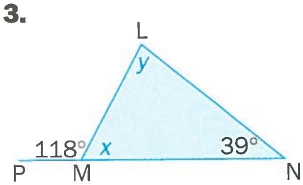
1.



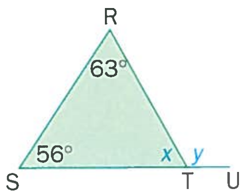
2.



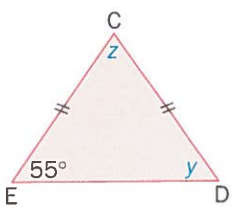
3.



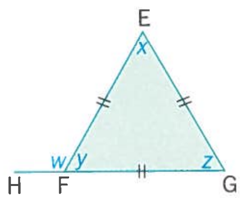
4.



5.



6.



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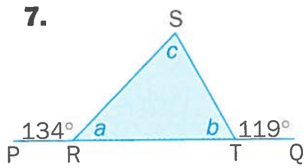

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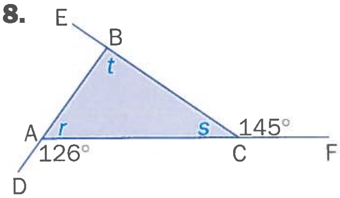

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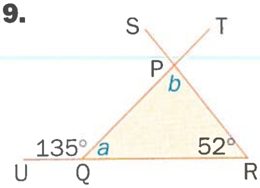

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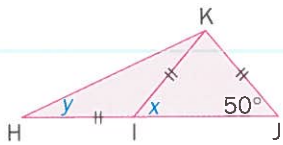

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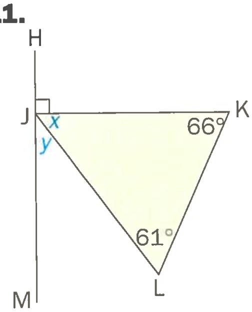

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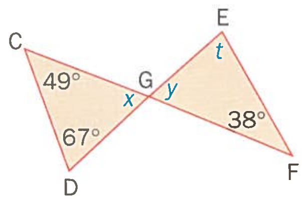
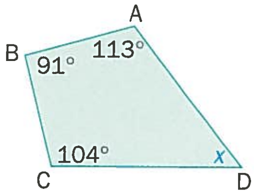


Find the missing angle measures.

13.



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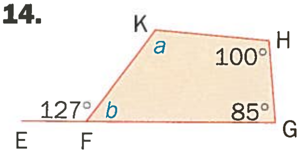

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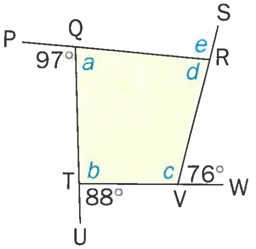
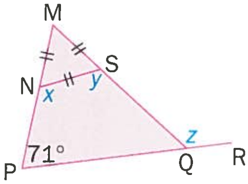


16.

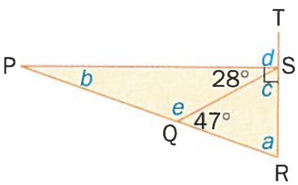


Applications and Problem Solving

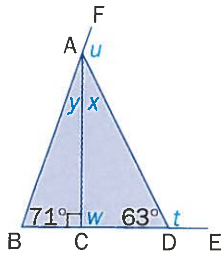
B

17. Find the measures of the indicated angles.

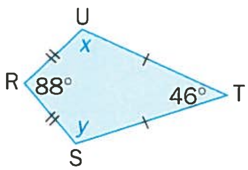
a)



b)

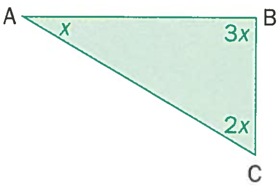


18. Find the measures of the indicated angles.

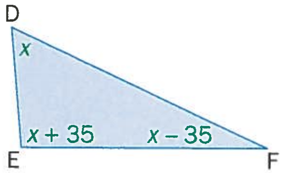


19. **Algebra** Find the value of x. Then, find the measures of all the angles.

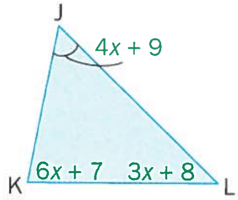
a)



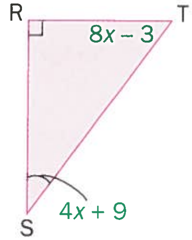
b)



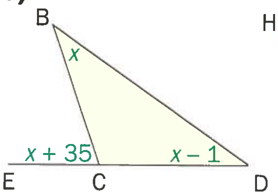
c)



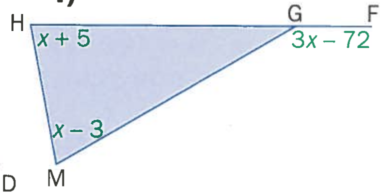
d)



e)

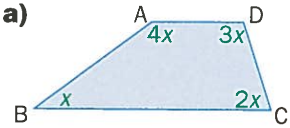


f)

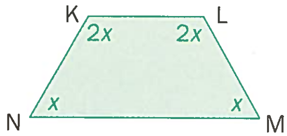


20. **Algebra** Find the value of x. Then, find the measures of all the angles.

a)



b)



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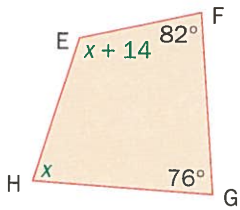

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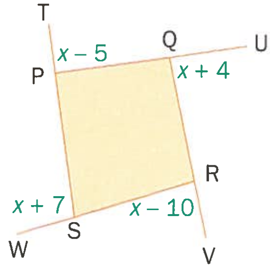
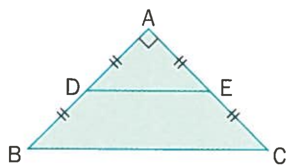
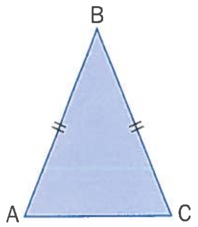


21. The midpoints of the legs of the isosceles right triangle have been joined to form a quadrilateral and a smaller triangle.

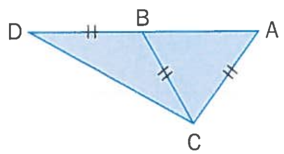


Find the measures of the interior angles of the quadrilateral and of the triangle.

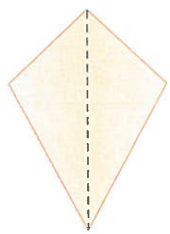
22. If you know the measure of ∠A, how can you find the measures of ∠B and ∠C?



23. If you know the measure of ∠D, how can you find the measure of ∠A?



24. A kite has 1 line of symmetry. A line of symmetry is an imaginary line that divides a figure into two identical pieces. How many lines of symmetry do the following types of triangles have?



a) equilateral **b)** isosceles
c) scalene

C

25. Can a right triangle have a line of symmetry? Explain.

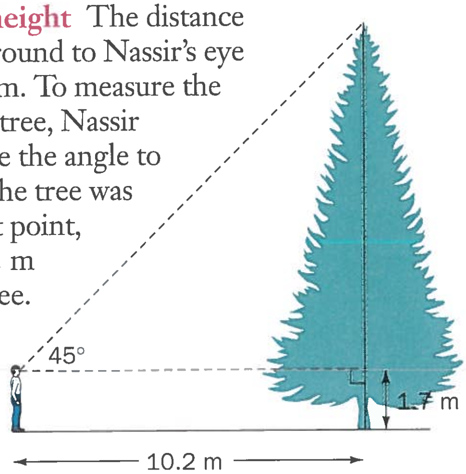
26. One angle of an isosceles triangle has a measure of 40°. What are the possible measures of the other angles?

27. One angle of an isosceles triangle has a measure of 140°. What are the possible measures of the other angles?

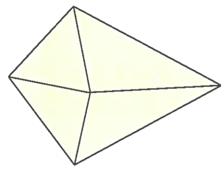
28. Copy and complete each statement with the word always, sometimes, or never. Explain and justify your reasoning.

a) A scalene triangle is ______ isosceles.
b) An obtuse triangle is ______ isosceles.
c) An equilateral triangle is ______ acute.
d) An acute triangle is ______ isosceles.

29. Tree height The distance from the ground to Nassir's eye level is 1.7 m. To measure the height of a tree, Nassir stood where the angle to the top of the tree was 45°. At that point, he was 10.2 m from the tree. How tall is the tree?



30. To show that the sum of the measures of the interior angles of any quadrilateral is 360°, join the four vertices to a point inside the quadrilateral. How can you use the sum of the measures of the interior angles of the four triangles to show that the sum of the interior angles of the quadrilateral is 360°?



31. Can a quadrilateral have each of the following sets of interior angles? If so, show an example in a diagram. Compare your diagrams with a classmate's.

a) four obtuse angles
b) one obtuse angle and three acute angles
c) one acute angle and three obtuse angles
d) two right angles and two acute angles

32. a) Some types of triangles named in the table exist, while others do not. Work with a classmate to draw the types of triangles that exist.

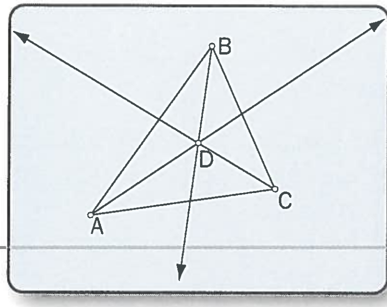
	Scalene	Isosceles	Equilateral
Acute	A	B	C
Right	D	E	F
Obtuse	G	H	I

b) If a triangle does not exist, explain why not. You may wish to use a diagram.

Exploring Angle Bisectors, Medians, and Altitudes Using Geometry Software

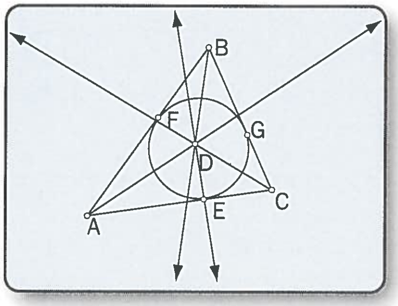
Complete the following explorations using geometry software or a graphing calculator with geometry capabilities. If suitable technology is not available, complete the pencil-and-paper explorations on pages 527–528 and page 529.

1 Angle Bisectors of a Triangle



1. a) Construct an acute triangle.
b) Bisect each interior angle.
c) Describe how the bisectors meet.
d) The point at which the bisectors of the interior angles of a triangle intersect is called the **incentre**. Construct this point.

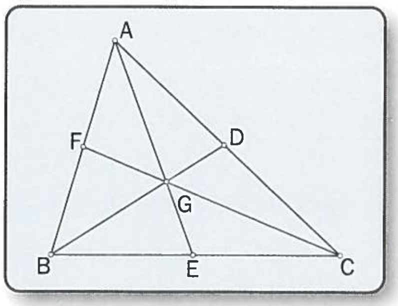
2. a) Construct a perpendicular line through the incentre and one side.
b) Construct the point at the intersection of the perpendicular line and the side.
c) Construct a circle with the incentre as the centre and the point at the intersection of the perpendicular line and the side as the point on the circle.
d) This circle, called the **incircle**, touches each side of the triangle exactly once. Construct the point at the intersection of the circle and each of the other two sides to confirm this.



3. a) Drag one of the vertices to change the triangle to a variety of sizes and shapes.
b) Do other acute triangles have incircles?
c) Do right triangles have incircles?
d) Do obtuse triangles have incircles?

2 Medians of a Triangle

A **median** of a triangle is a line segment that joins a vertex to the midpoint of the opposite side.



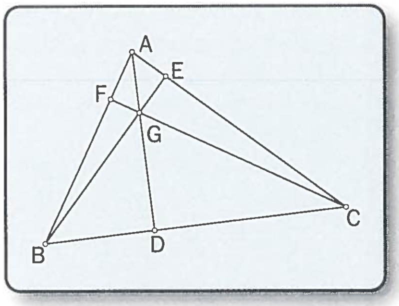
1. a) Construct an acute triangle.
b) Construct the midpoints of the sides.
c) Construct a segment between each midpoint and the opposite vertex.
d) Describe how the segments meet.
e) The point at which the medians of a triangle intersect is called the **centroid**. Construct this point.

2. If you were to print and cut out your triangle, tape it to a piece of cardboard, and turn it upside down with one of your fingers at the centroid, the triangle should be perfectly balanced. Use this information to explain why the centroid of a triangle is sometimes referred to as its *centre of gravity*.

3. a) Drag one of the vertices to change the triangle to a variety of sizes and shapes.
b) Do other acute triangles have centroids?
c) Do right triangles have centroids?
d) Do obtuse triangles have centroids?

3 Altitudes of a Triangle

An **altitude** of a triangle is the perpendicular from a vertex to the opposite side.



1. a) Construct an acute triangle.
b) Construct a perpendicular line through each vertex to the opposite side.
c) Construct the point at the intersection of the perpendicular line and the side.
d) Construct a segment between each vertex and the point at the intersection of the perpendicular line and the side.
e) Hide the perpendicular lines.
f) Describe how the altitudes intersect.
g) The point at which the altitudes of a triangle intersect is called the **orthocentre**. Construct this point.

2. a) Drag one of the vertices to change the triangle to a variety of sizes and shapes. Describe what happens.
b) Do other acute triangles have orthocentres?
c) What happens to the orthocentre when the triangle becomes a right triangle?
d) What happens to the orthocentre when the triangle becomes an obtuse triangle?
e) Construct an obtuse triangle, extend the sides meeting at the obtuse angle, and construct the orthocentre. Where is it?

4 Making and Testing a Conjecture

1. Using the results from the first three explorations, make a conjecture about the three perpendicular bisectors of the sides of a triangle.

2. Test your conjecture. Construct a triangle and the perpendicular bisectors of the sides. Drag one of the vertices to change the triangle to a variety of different sizes and shapes. Describe what happens.

3. Use your research skills to define the terms **circumcentre** and **circumcircle**.

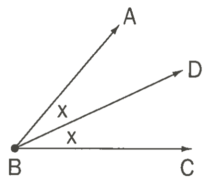
4. Construct the circumcircle of a triangle.

5. Communicate your findings to your classmates.

Geometric Constructions

1 Angle Bisectors

The **bisector** of an angle divides the angle into two equal parts.

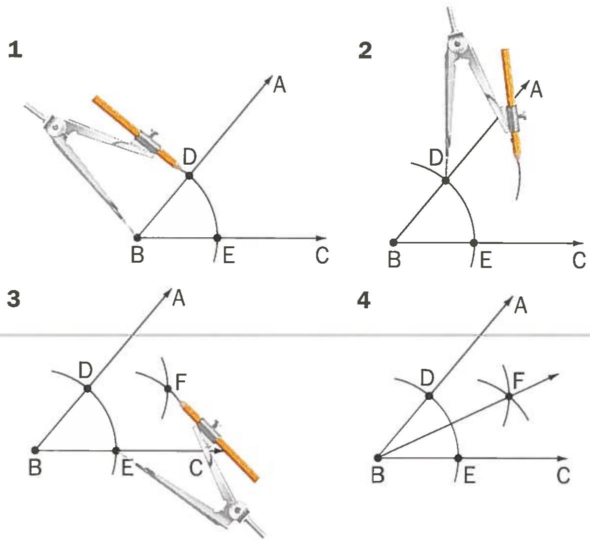


1. Steps 1 to 4 show how to bisect an angle using ruler and compasses. Repeat the construction in your notebook. Work with a partner to write a description of the steps.

2. Describe how to bisect an angle using paper folding.

3. Describe how an angle can be bisected using a Mira.

4. Draw a 60°-angle, a 120°-angle, a right angle, and a straight angle. Then, bisect each of the angles. What is the measure of each new angle?



2 Right Bisectors

The right bisector of a line segment divides the line segment at right angles into two equal parts.

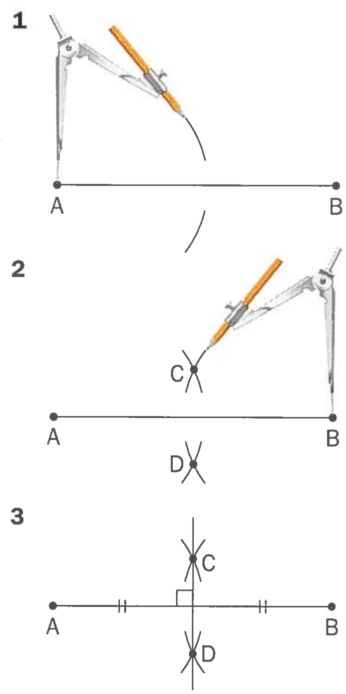
1. Steps 1 to 3 show how to construct the right bisector of a line segment using ruler and compasses. Repeat the construction in your notebook. Work with a partner to write a description of the steps.

2. Describe how to construct the right bisector of a segment by paper folding.

3. Describe how to construct the right bisector of a line segment using a Mira.

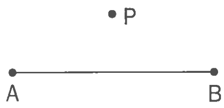
4. Draw a line segment at least 8 cm long. Construct its right bisector.

5. Draw a line segment at least 8 cm long. Divide the line segment into 4 equal parts by constructing 3 right bisectors.

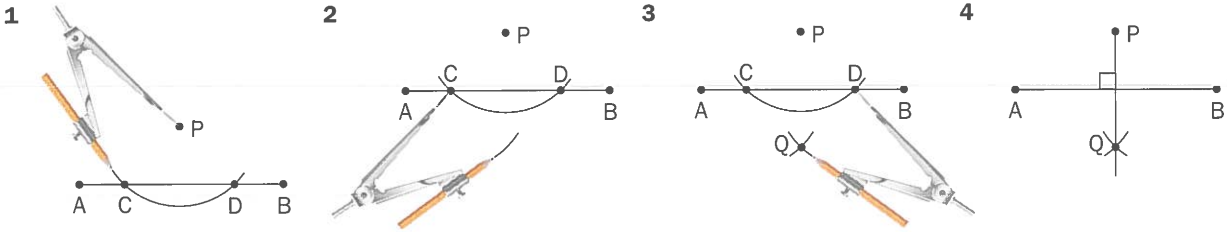


3 Perpendicular Lines

Perpendicular lines meet at 90°. There are several ways to construct a perpendicular to a line from a point not on the line.



1. The following steps 1 to 4 show how to construct a perpendicular to a line from a point not on the line. Repeat the construction in your notebook. Work with a partner to write a description of the steps.



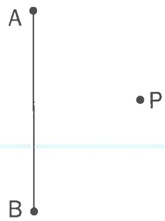
2. Describe how to draw a perpendicular to a line from a point not on the line using paper folding.

3. Describe how to draw a perpendicular to a line from a point not on the line using a Mira.

4. Copy these diagrams and construct perpendiculars from P to AB.

a)

b)



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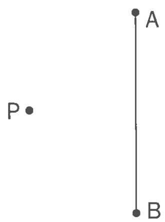


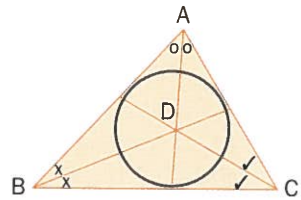
d)

Exploring Angle Bisectors, Medians, and Altitudes

1 Angle Bisectors of a Triangle

1. Draw an acute △ABC. Construct the bisector of each angle. The three bisectors should intersect at a common point, called the **incentre**. Let D be the incentre.

2. Using D as the centre, draw a circle that touches all three sides of the triangle. This circle is the **incircle**.

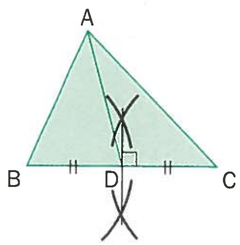


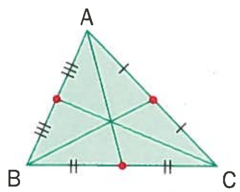
3. a) Repeat steps 1 and 2 for a right triangle and for an obtuse triangle.
b) Does a right triangle have an incircle?
c) Does an obtuse triangle have an incircle?

2 Medians of a Triangle

A **median** of a triangle is a line segment that joins a vertex of a triangle to the midpoint of the opposite side.

1. Draw an acute △ABC. Construct the right bisector of BC. Let D be the midpoint of BC. Join AD, which is a median of △ABC. Also construct the medians from B to AC and from C to AB. The three medians should meet at a common point, called the **centroid**.





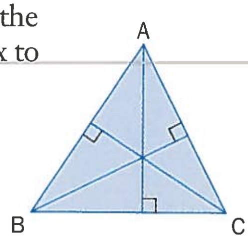
2. If you were to cut out the triangle, tape it to a piece of cardboard, and turn it upside down with one of your fingers at the centroid, the triangle should be perfectly balanced. Use this information to explain why the centroid of a triangle is sometimes known as its *centre of gravity*.

3. a) Repeat step 1 for a right triangle and for an obtuse triangle.
b) Does a right triangle have a centroid?
c) Does an obtuse triangle have a centroid?

3 Altitudes of a Triangle

An **altitude** of a triangle is the perpendicular from a vertex to the opposite side.

1. Draw an acute △ABC. Construct the three altitudes of the triangle. The three altitudes should intersect at a common point, called the **orthocentre**.



2. Repeat step 1 for a right triangle. Where is the orthocentre of a right triangle?

3. Repeat step 1 for an obtuse triangle. Extend the sides, if necessary. Where is the orthocentre of an obtuse triangle?

4 Making and Testing a Conjecture

1. Using the results from the first three explorations, make a conjecture about the three perpendicular bisectors of the sides of a triangle.

2. Test your conjecture for various types of triangles.

3. Use your research skills to define the terms **circumcentre** and **circumcircle**.

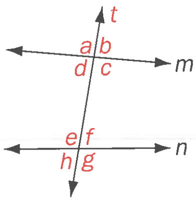
4. Construct the circumcircle of a triangle.

5. Communicate your findings to your classmates.

Exploring Angles and Parallel Lines Using Geometry Software

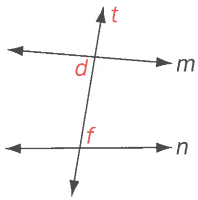
Complete the following explorations using geometry software or a graphing calculator with geometry capabilities. If suitable technology is not available, complete the Explore and Inquire parts of Section 10.2 on page 533 using pencil and paper.

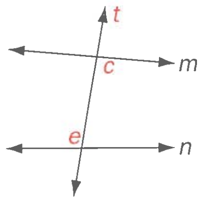
A line that intersects two or more lines is called a **transversal**. When the transversal, t, intersects lines m and n, eight angles are formed.



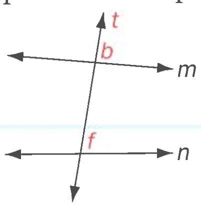
Pairs of these angles are given special names.

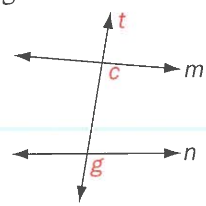
In the diagrams shown, $\angle d$ and $\angle f$ are a pair of **alternate angles**. Another pair of alternate angles is $\angle c$ and $\angle e$.

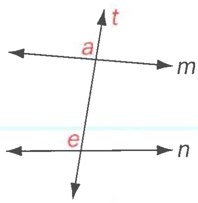


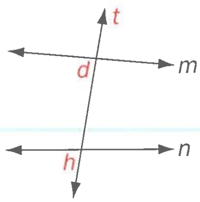


In the following, $\angle b$ and $\angle f$ are a pair of **corresponding angles**. Other pairs of corresponding angles are $\angle c$ and $\angle g$, $\angle a$ and $\angle e$, and $\angle d$ and $\angle h$.

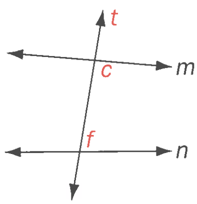


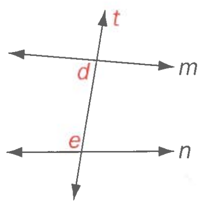






In the diagrams shown, $\angle c$ and $\angle f$ are a pair of **co-interior angles** on the same side of the transversal. Another pair of co-interior angles on the same side of the transversal is $\angle d$ and $\angle e$.

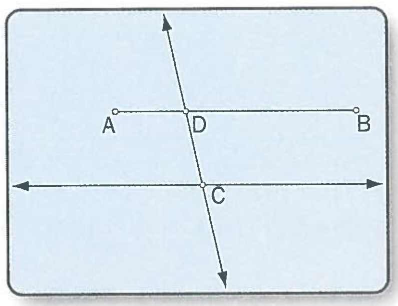




1 Constructing Parallel Lines and a Transversal

Parallel lines are lines in the same plane that do not intersect.

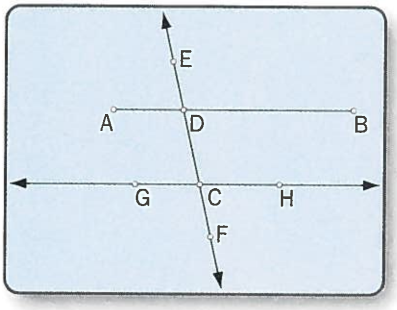
1. Construct a line segment.
2. Construct a point on one side of the line segment.
3. Construct a line parallel to the line segment through the point.
4. Construct a point on the line segment. Then, construct a line through the point on the line segment and the point on the parallel line.



2 Alternate Angles

Use the figure from the previous exploration.

1. On the parallel line and the transversal, construct other points needed to define angles using three points.



2. Measure one pair of alternate angles. How do their measures compare?

3. Measure the other pair of alternate angles. How do their measures compare?

4. Drag the point at which the transversal intersects the line segment to change the position of the transversal. How do the measures of alternate angles compare?

5. Drag the point at which the transversal intersects the parallel line to change the distance between the line segment and the line, and the position of the transversal. How do the measures of alternate angles compare?

6. Write a statement to describe the relationship between pairs of alternate angles formed when a transversal intersects two parallel lines.

3 Corresponding Angles

Use the figure from the previous exploration.

1. Measure one pair of corresponding angles. How do their measures compare?

2. Measure each of the three other pairs of corresponding angles. How do their measures compare?

3. Drag the point at which the transversal intersects the line segment to change the position of the transversal. How do the measures of corresponding angles compare?

4. Drag the point at which the transversal intersects the parallel line to change the distance between the line segment and the line, and the position of the transversal. How do the measures of corresponding angles compare?

5. Write a statement to describe the relationship between pairs of corresponding angles formed when a transversal intersects two parallel lines.

4 Co-Interior Angles

Use the figure from the previous exploration.

1. Measure one pair of co-interior angles. What is the sum of their measures?

2. Measure the other pair of co-interior angles. What is the sum of their measures?

3. Drag the point at which the transversal intersects the line segment to change the position of the transversal. What is the sum of the measures of co-interior angles?

4. Drag the point at which the transversal intersects the parallel line to change the distance between the line segment and the line, and the position of the transversal. What is the sum of the measures of co-interior angles?

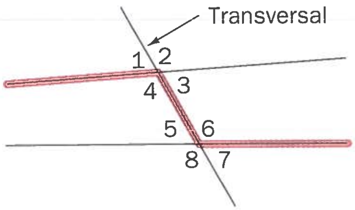
5. Write a statement to describe the relationship between pairs of co-interior angles formed when a transversal intersects two parallel lines.

10.2 Angles and Parallel Lines

A **transversal** is a line that crosses or intersects two or more lines, each at a different point. Some pairs of angles formed by a transversal have special names.

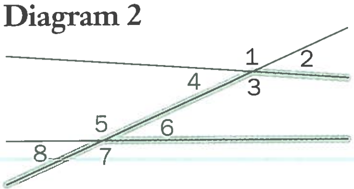
In Diagram 1, ∠4 and ∠6 are **alternate interior angles**, or simply **alternate angles**. Another pair of alternate angles is ∠3 and ∠5. Alternate angles form a Z pattern or a Ƨ pattern.

Diagram 1



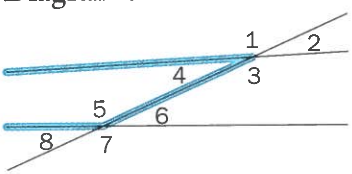
In Diagram 2, ∠3 and ∠7 are a pair of **corresponding angles**. Other pairs of corresponding angles are ∠2 and ∠6, ∠4 and ∠8, and ∠1 and ∠5. Corresponding angles form an F pattern, or an ꟻ, Ⅎ, or ⅎ pattern.

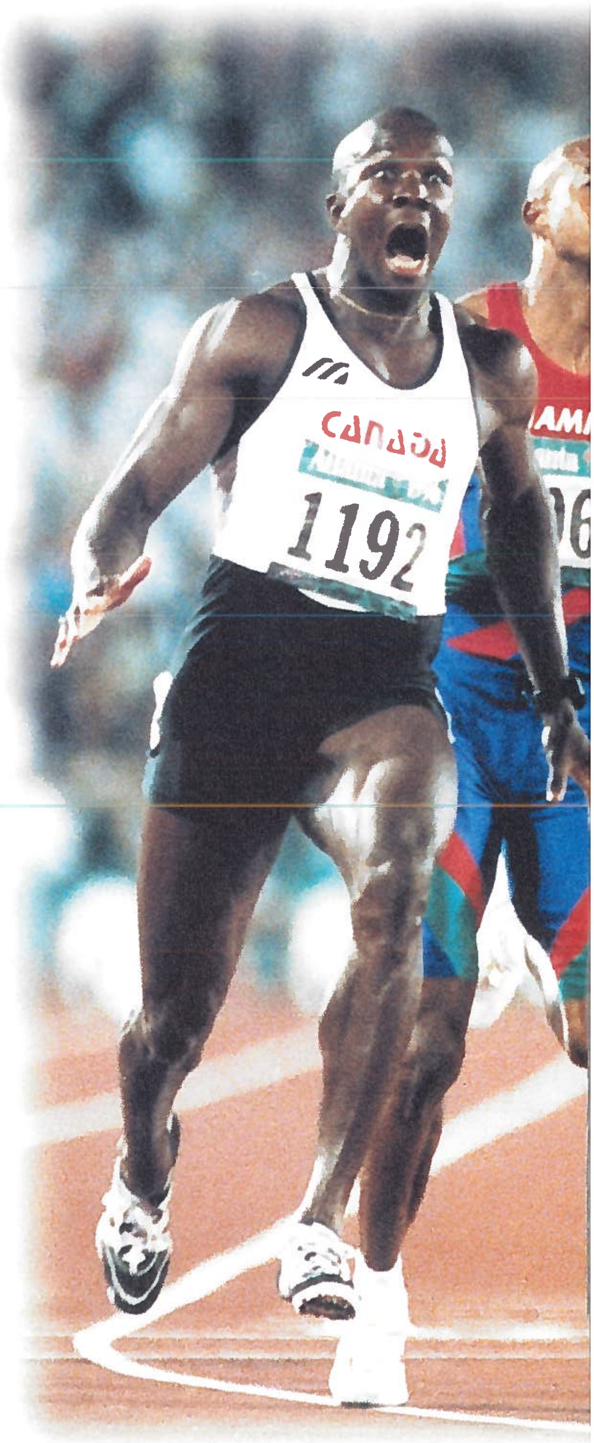
Diagram 2



In Diagram 3, ∠4 and ∠5 are a pair of **co-interior angles**. Co-interior angles are on the same side of the transversal. Another pair of co-interior angles is ∠3 and ∠6. Co-interior angles form a ⊏ pattern or a ⊐ pattern.

Diagram 3

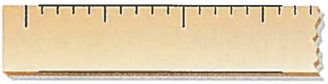


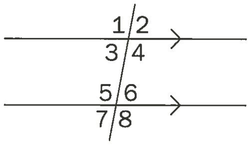


Canada's Donovan Bailey set a new world record of 9.84 s when he won the 100-m race at the Summer Olympics in Atlanta. In a 100-m race, runners compete in lanes separated by parallel markings. In geometry, we often use **parallel lines**, which are lines in the same plane that do not intersect.

Explore: Discover the Relationship

a) Use a ruler to draw 2 parallel lines and a transversal. Label the angles as shown.
b) Measure the 8 angles formed.





Inquire

1. How are the pairs of alternate angles related?

2. How are the pairs of corresponding angles related?

3. How are the pairs of co-interior angles related?

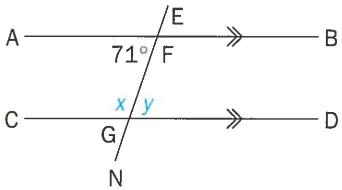
4. Repeat the activity for two more parallel lines and another transversal.

5. Write a statement to describe the relationship between each of the following pairs of angles formed when a transversal intersects two parallel lines.
a) alternate angles
b) corresponding angles
c) co-interior angles

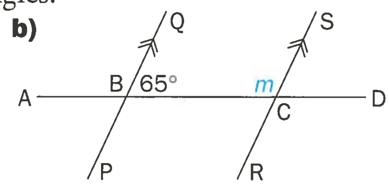
Example 1 Using Alternate and Co-Interior Angles

Find the measures of the indicated angles.

a)



b)



Solution

a) Since AB||CD, alternate angles are equal.

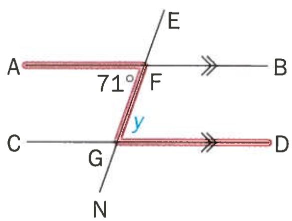
|| means "is parallel to"

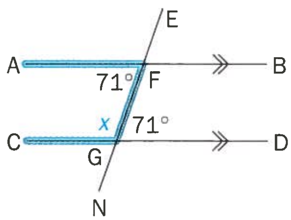
$\angle DGF = \angle AFG$ (alternate angles)
$y = 71°$

Since AB||CD, co-interior angles are supplementary.

$\angle AFG + \angle CGF = 180°$ (co-interior angles)
$71° + x = 180°$
$x = 109°$

The measures are $x = 109°$ and $y = 71°$.

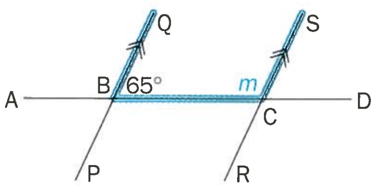




b) Since PQ||RS, co-interior angles are supplementary.

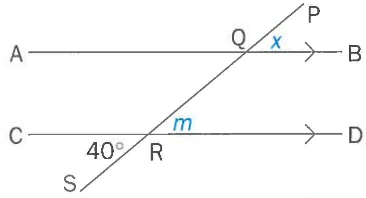
$\angle BCS + \angle CBQ = 180°$ (co-interior angles)
$m + 65 = 180°$
$m = 115°$

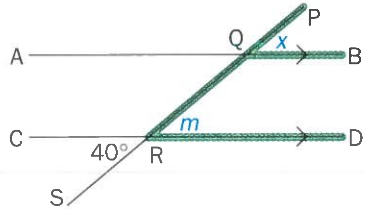
The measure is $m = 115°$.



Example 2 Using Corresponding and Opposite Angles

Find the measures of the indicated angles.





Solution

∠DRQ = ∠CRS (opposite angles)

$m = 40°$

Since AB | | CD, corresponding angles are equal.

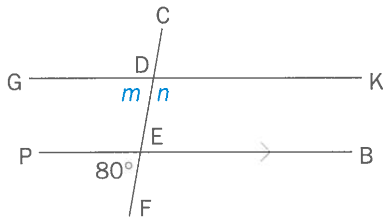
∠BQP = ∠DRQ (corresponding angles)

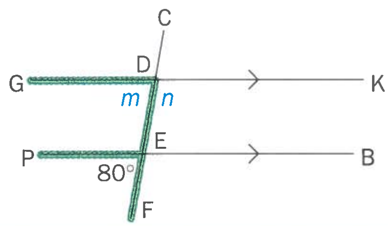
$x = 40°$

The measures are $m = 40°$ and $x = 40°$.

Example 3 Using Corresponding and Straight Angles

What are the measures of m and n in this diagram?





Solution

Since GK | | PB, corresponding angles are equal.

∠EDG = ∠FEP (corresponding angles)

$m = 80°$

∠EDK + ∠EDG = 180° (straight angle)

$n + 80° = 180°$

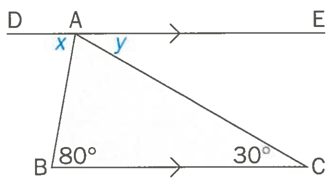
$n = 100°$

The measures are $m = 80°$ and $n = 100°$.

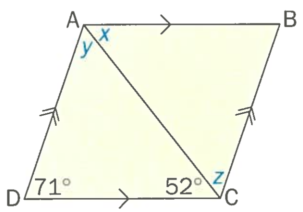
Example 4 Angles in Triangles and Quadrilaterals

Find the measures of the indicated angles.

a)



b)



Solution

a) Since DE | | BC, alternate angles are equal.

∠BAD = ∠ABC (alternate angles)

$x = 80°$

∠CAE = ∠ACB (alternate angles)

$y = 30°$

The measures are $x = 80°$ and $y = 30°$.

b) Since AB | | DC, alternate angles are equal.

∠BAC = ∠ACD (alternate angles)

$x = 52°$

In △ADC, the sum of the angles is 180°.

$y + 71° + 52° = 180°$

$y + 123° = 180°$

$y = 57°$

Since AD | | BC, alternate angles are equal.

∠ACB = ∠CAD (alternate angles)

$z = 57°$

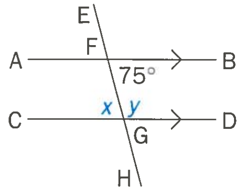
The measures are $x = 52°$, $y = 57°$, and $z = 57°$.

Practice

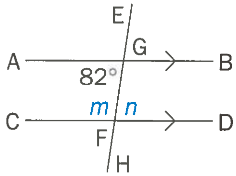
A

Find the indicated angle measures. Give reasons for your answers.

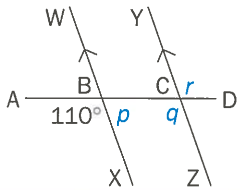
1.



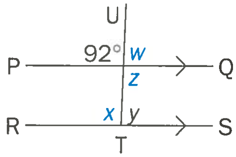
2.



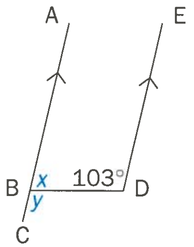
3.



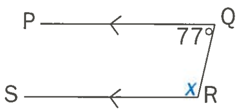
4.



5.

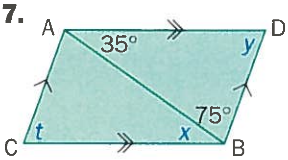


6.

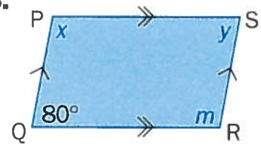


Find the measure of each indicated angle.

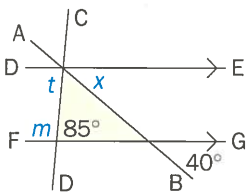
7.



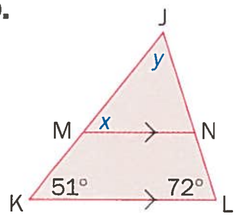
8.



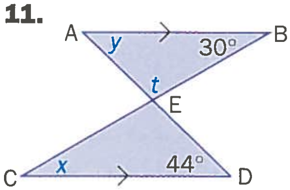
9.



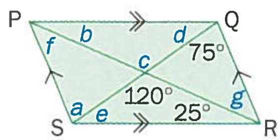
10.



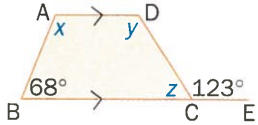
11.



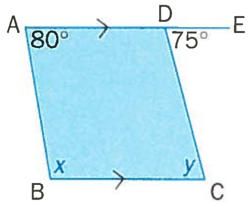
12.



13.



14.



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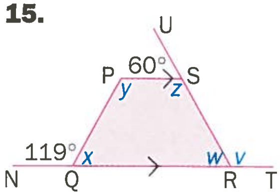

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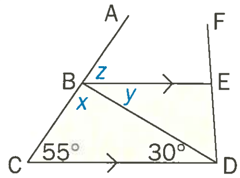

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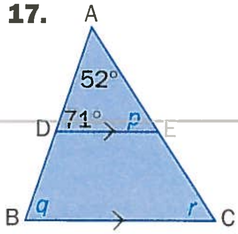

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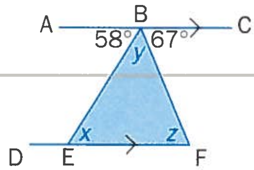
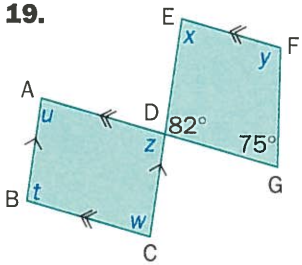


Applications and Problem Solving

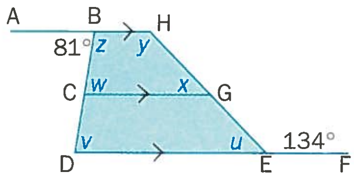
B

Find the measure of each indicated angle.

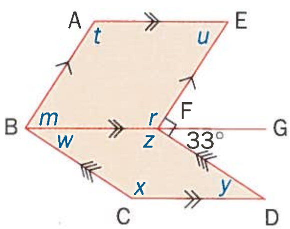
19.



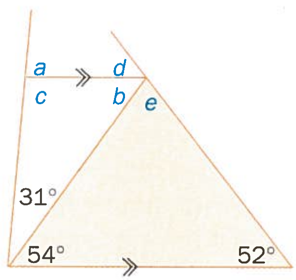
20.



21.

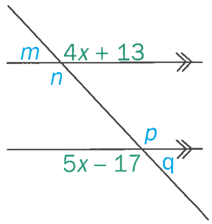


22.

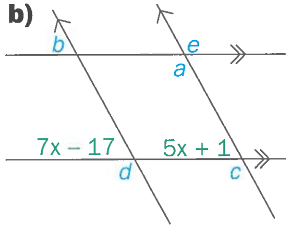


23. Algebra Find the value of x. Then, find the measures of the indicated angles.

a)

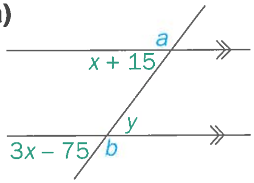


b)

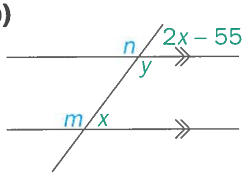


24. Algebra Find the values of x and y. Then, find the measures of the indicated angles.

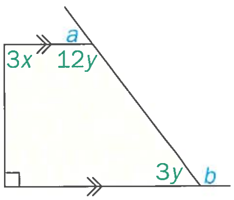
a)



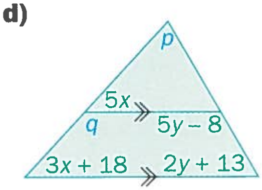
b)



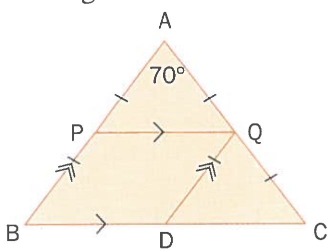
c)



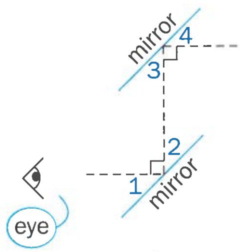
d)



25. List all the angles in this diagram. Then, calculate each angle's measure.



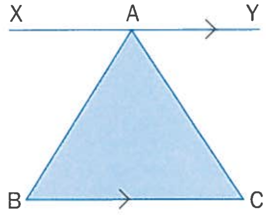
26. Periscope A periscope is an instrument used on submarines to see above the surface of the water. The diagram shows how two parallel mirrors are used in a periscope.



Show that $\angle 1 = \angle 4$. Explain and justify your reasoning

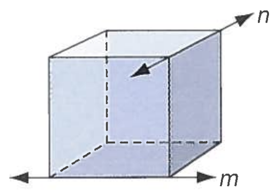
C

27. One way to show the sum of the interior angles of a triangle is to draw a line through one vertex of the triangle parallel to the opposite side. How does this show the following?
$\angle BAC + \angle B + \angle C = 180°$



28. Geography Places on Earth are located using parallels of latitude and meridians of longitude. Are parallels of latitude really parallel lines? Explain why or why not.

29. Lines that do not lie in the same plane are called **skew lines**. Lines m and n in this diagram are skew lines. How are skew lines and parallel lines different? How are they the same?



30. Sports Parallel dividers create lanes for 100-m races. Work with a partner to list other examples of the use of parallel lines in sports.

31. Write a problem that involves angles and parallel lines. Have a classmate solve your problem.

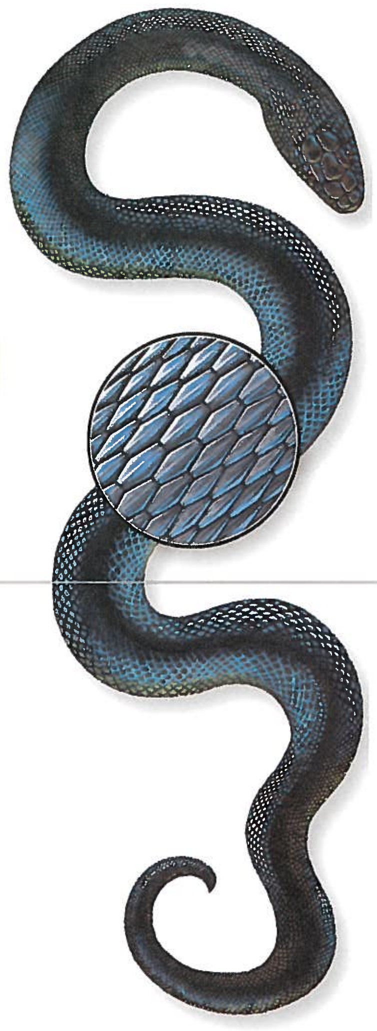
LOGIC POWER

Draw a chessboard in your notebook. Put a marker on each of 8 different squares so that no 2 markers are in the same row, column, or diagonal.

Polygons

A **polygon** is a closed figure formed by joining three or more line segments at their endpoints.

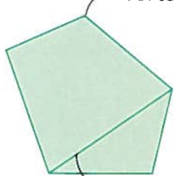
Polygons of many different types appear in nature. On this black rat snake, a reptile native to Ontario, the most prominent type of polygon has 6 sides and is called a hexagon.



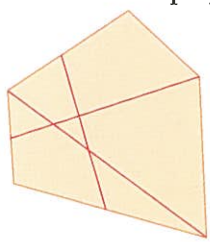
Polygons are classified according to their number of sides.

Polygon	Number of Sides	Polygon	Number of Sides	Polygon	Number of Sides
Triangle	3	Hexagon	6	Nonagon	9
Quadrilateral	4	Heptagon	7	Decagon	10
Pentagon	5	Octagon	8	Dodecagon	12

Each point where two sides of a polygon meet is called a **vertex** of the polygon. A **diagonal** is the line segment that joins any two non-consecutive vertices of a polygon.

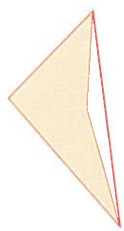


A **convex** polygon is one in which any line segment joining two points on the polygon has no part outside the polygon.



Convex Polygon

A polygon is **concave** if a line segment joining two points on the polygon can be drawn so that a part of the segment lies outside the polygon.

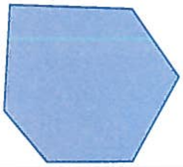


Concave Polygon

In this book, the term *polygon* means a convex polygon, unless otherwise stated.

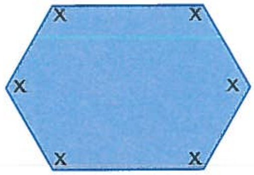
Polygons can be *equilateral* (all sides equal) or *equiangular* (all angles equal). When a polygon is both equilateral and equiangular, it is called a **regular polygon**.

This hexagon is neither equilateral nor equiangular.

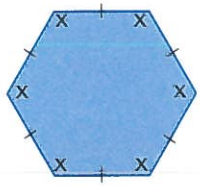


This is an equilateral hexagon.

This is an equiangular hexagon.



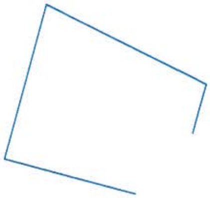
This is a regular hexagon.



1 Equilateral and Equiangular Polygons

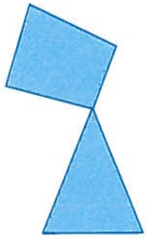
1. Explain why each figure is not a polygon.

a)



b)

c)



2. Sketch each of the following polygons.

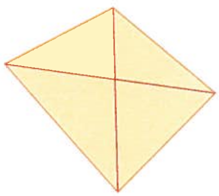
a) a quadrilateral that is equiangular but not equilateral
b) a quadrilateral that is equilateral but not equiangular
c) a pentagon that is equilateral but not equiangular
d) a pentagon that is equiangular but not equilateral
e) a regular pentagon

3. Name a type of quadrilateral that is equilateral but not equiangular.

4. Name a type of quadrilateral that is equiangular but not equilateral.

2 Diagonal Pattern

1. A quadrilateral has 2 diagonals, as shown. Copy and complete the table.



Polygon	Number of Diagonals
Quadrilateral	2
Pentagon	
Hexagon	
Heptagon	

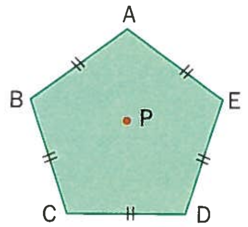
2. Describe the pattern in the number of diagonals.

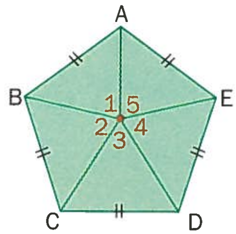
3. Use the pattern to predict the number of diagonals in each of the following polygons.

a) octagon **b)** nonagon **c)** decagon

3 Angles and Regular Polygons

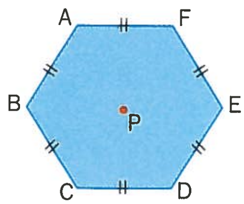
1. ABCDE is a regular pentagon. The point P is the centre of the pentagon. When a line segment is drawn from P to each vertex, five identical triangles are formed.

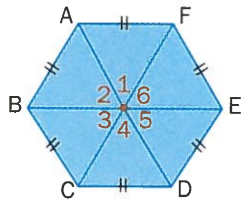




a) What is the sum of the measures of the central angles, $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, and $\angle 5$?
b) What is the measure of each central angle?
c) What type of triangle is formed?
d) What are the measures of the other two angles in each triangle?
e) What is the sum of the measures of the five interior angles of a regular pentagon?
f) What is the measure of each interior angle of a regular pentagon?

2. ABCDEF is a regular hexagon. The point P is the centre of the hexagon. When a line segment is drawn from P to each vertex, six identical triangles are formed.





a) What is the measure of each central angle?
b) What type of triangle is formed?
c) What are the measures of the other two angles in each triangle?
d) What is the sum of the measures of the six interior angles of a regular hexagon?
e) What is the measure of each interior angle of a regular hexagon?

3. a) What is the measure of each central angle of a regular octagon?
b) What is the sum of the measures of the eight interior angles of a regular octagon?
c) What is the measure of each interior angle of a regular octagon?

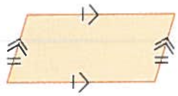
TECHNOLOGY

Exploring the Sides and Diagonals of Polygons Using Geometry Software

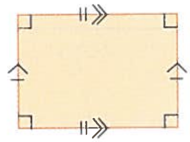
Complete the following explorations using geometry software or a graphing calculator with geometry capabilities. If suitable technology is not available, complete the pencil-and-paper explorations on pages 544–547.

The explorations involve some special types of quadrilaterals, as follows.

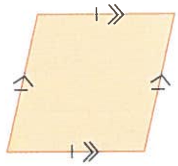
- A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel and equal in length.



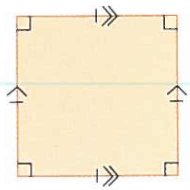
- A **rectangle** is a quadrilateral with both pairs of opposite sides parallel and equal in length, and all angles 90°.



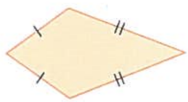
- A **rhombus** is a quadrilateral with both pairs of opposite sides parallel and all sides the same length.



- A **square** is a quadrilateral with both pairs of opposite sides parallel, all sides the same length, and all angles 90°.

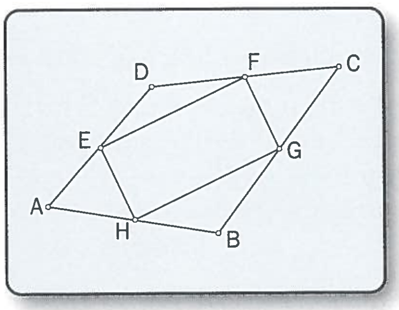


- A **kite** is a quadrilateral with two pairs of adjacent sides equal.



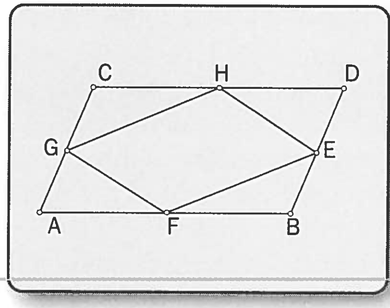
1 Sides

1. a) Write a conjecture about the type of quadrilateral formed when the midpoints of the sides of a quadrilateral are joined.
b) Construct a quadrilateral.
c) Construct the midpoint of each side.
d) Join the midpoints to form a quadrilateral within the quadrilateral.



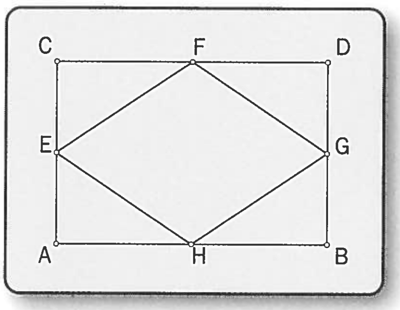
e) Measure the sides of the inner quadrilateral. How do the lengths compare?
f) Measure the interior angles in the inner quadrilateral. How do the angle measures compare?
g) Drag one of the vertices to change the quadrilateral to a variety of sizes and shapes. How do the side lengths compare and the angle measures compare for the inner quadrilateral in each case?
h) Is your conjecture from part a) valid? Explain.

2. a) Write a conjecture about the type of quadrilateral formed when the midpoints of the sides of a parallelogram are joined.
b) Construct a parallelogram.
c) Construct the midpoint of each side.
d) Join the midpoints to form a quadrilateral within the parallelogram.
e) Measure the sides of the inner quadrilateral. How do the lengths compare?
f) Measure the interior angles in the inner quadrilateral. How do the angle measures compare?
g) Drag one of the vertices to make a variety of parallelograms. How do the side lengths compare and the angle measures compare for the inner quadrilateral in each case?
h) Is your conjecture from part a) valid? Explain.

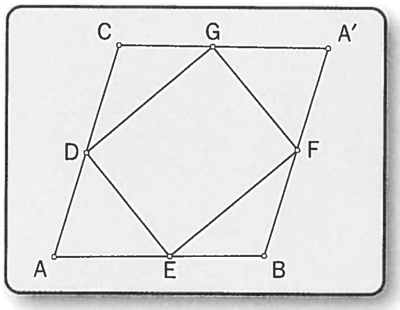


Repeat question 2 starting with each specific quadrilateral.

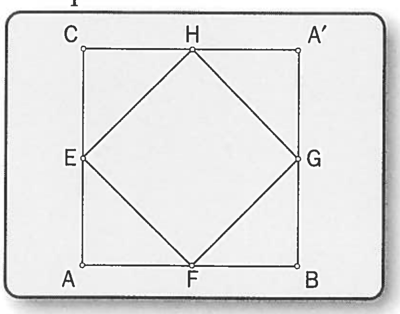
3. a rectangle



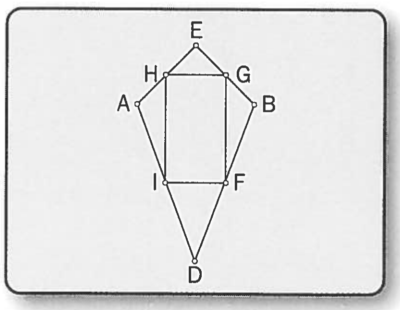
4. a rhombus



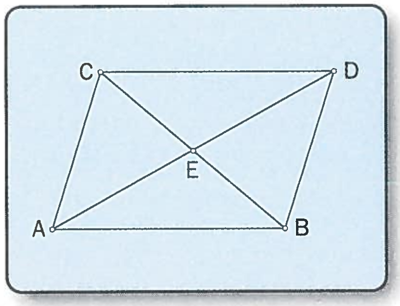
5. a square



6. a kite



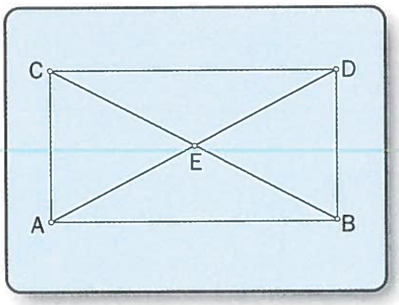
2 Diagonals



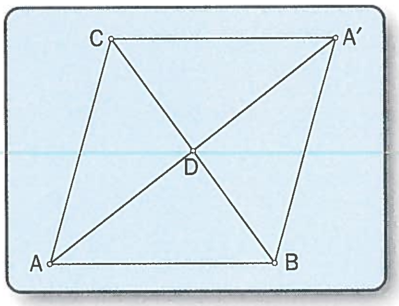
1. a) Construct a parallelogram.
b) Construct the diagonals.
c) Measure the lengths of the diagonals. How do the lengths compare?
d) Construct the point at the intersection of the diagonals.
e) Measure the distance from the point of intersection to each vertex. How do the distances compare?
f) Measure the four angles formed at the point of intersection of the diagonals. How do the angle measures compare?
g) Drag one of the vertices to make a variety of parallelograms. In each case, how do the distances from the point of intersection of the diagonals to the vertices compare? How do the measures of the angles formed at the point of intersection compare?
h) Write a statement describing the relationship between the lengths of the diagonals of a parallelogram, whether they bisect each other, and whether they intersect at 90°.

Repeat question 1 starting with each specific quadrilateral.

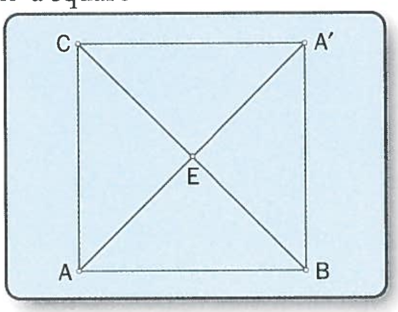
2. a rectangle



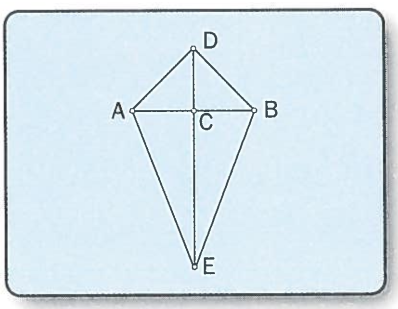
3. a rhombus



4. a square



5. a kite



6. For which of the 5 quadrilaterals, do the diagonals

a) have the same length?

b) bisect each other?

c) intersect at 90°?

d) bisect each other at 90°?

7. a) Write a conjecture about the type of pentagon formed when the intersection points of the diagonals of a regular pentagon are joined.

b) Construct a regular pentagon.

c) Construct all the diagonals.

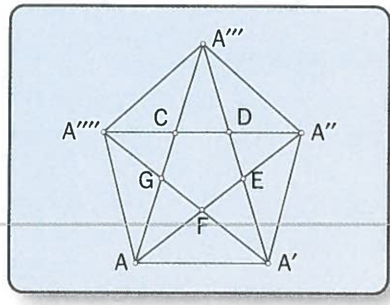
d) Construct all the points at the intersections of the diagonals.

e) Measure the distance between each pair of adjacent points of intersection to find the side lengths of the inner pentagon. How do the side lengths compare?

f) Measure the interior angles of the inner pentagon. How do the angle measures compare?

g) Drag one of the vertices to change the larger pentagon to a variety of sizes. For the inner pentagon in each case, how do the side lengths compare and how do the angle measures compare?

h) Is your conjecture from part a) valid? Explain.

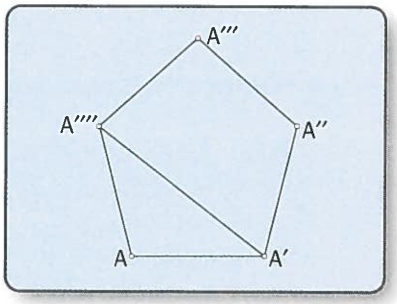


8. Golden ratio a) Construct a regular pentagon and one diagonal.

b) Measure the length of one side of the pentagon and the length of the diagonal.

c) Calculate the ratio of the length of the diagonal to the length of the side.

d) The number you found in part c) is an approximation of the **golden ratio**, a number found in nature, art, and architecture. Use your research skills to find examples of the golden ratio.



Exploring Sides and Diagonals of Polygons

Complete the following explorations using grid paper and a ruler. Use a protractor when necessary.

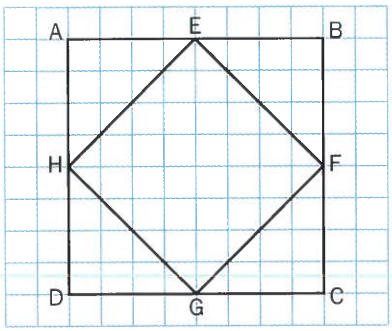
The explorations involve some special types of quadrilaterals, as follows.

- A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel and equal in length.
- A **rectangle** is a quadrilateral with both pairs of opposite sides parallel and equal in length, and all angles 90°.
- A **rhombus** is a quadrilateral with both pairs of opposite sides parallel and all sides the same length.
- A **square** is a quadrilateral with both pairs of opposite sides parallel, all sides the same length, and all angles 90°.
- A **kite** is a quadrilateral with two pairs of adjacent sides equal.

1 Sides

1. a) Make a conjecture about the type of quadrilateral that is formed when the midpoints of the sides of a square are joined.

b) Construct a square ABCD. Mark the midpoints of the four sides and label them E, F, G, and H, as shown. Join the midpoints to make quadrilateral EFGH.



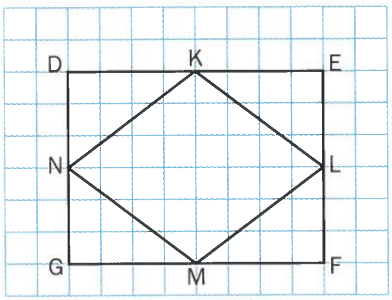
c) Find the lengths of EF, FG, GH, and HE. How do the lengths compare?

d) Find the measures of ∠HEF, ∠EFG, ∠FGH, and ∠GHE. How do the angle measures compare?

e) Is your conjecture from part a) valid? Explain.

2. a) Make a conjecture about the type of quadrilateral that is formed when the midpoints of the sides of a rectangle are joined.

b) Construct a rectangle DEFG. Mark the midpoints of the four sides and label them K, L, M, and N, as shown. Join the midpoints to make quadrilateral KLMN.

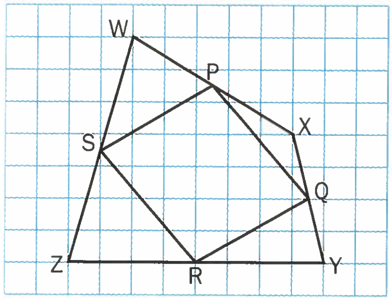


c) Find the lengths of KL, LM, MN, and NK. How do the lengths compare?

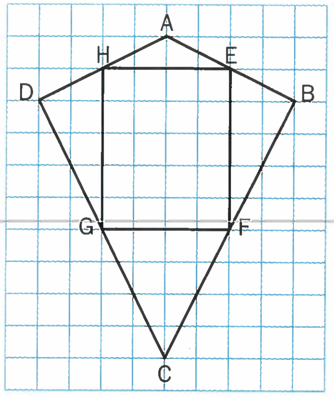
d) Find the measures of ∠KLM, ∠LMN, ∠MNK, and ∠NKL. How do the angle measures compare?

e) Is your conjecture from part a) valid? Explain.

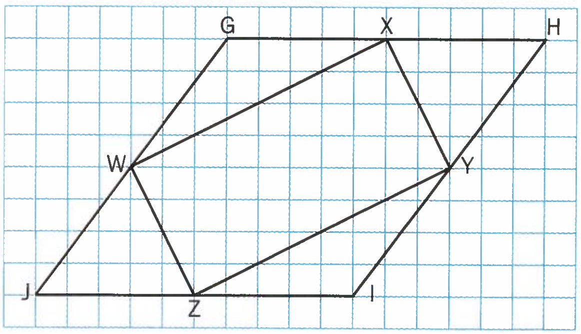
3. a) Make a conjecture about the type of quadrilateral that is formed when the midpoints of the sides of a quadrilateral are joined.
b) Construct any quadrilateral WXYZ. Find the midpoints of the four sides and label them P, Q, R, and S, as shown. Join the midpoints to make quadrilateral PQRS.
c) Find the lengths of SP and RQ. How do the lengths compare?
d) Find the lengths of PQ and SR. How do the lengths compare?
e) Find the measures of ∠SPQ, ∠PQR, ∠QRS, and ∠RSP. How do the angle measures compare?
f) Is your conjecture from part a) valid? Explain.



4. a) Make a conjecture about the type of quadrilateral that is formed when the midpoints of the sides of a kite are joined.
b) Construct a kite ABCD. Mark the midpoints of the four sides and label them E, F, G, and H, as shown. Join the midpoints to make quadrilateral EFGH.
c) Test your conjecture by measuring the sides and angles of quadrilateral EFGH.
d) Is your conjecture from part a) valid? Explain.

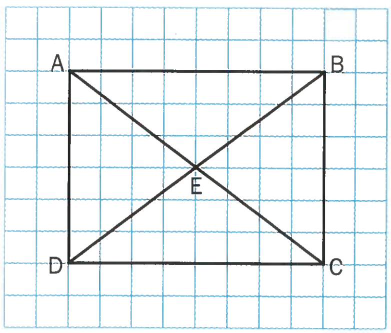


5. a) Make a conjecture about the type of quadrilateral that is formed when the midpoints of the sides of a rhombus are joined.
b) Construct a rhombus GHIJ. Mark the midpoints of the four sides and label them X, Y, Z, and W, as shown. Join the midpoints to make quadrilateral XYZW.
c) Test your conjecture by measuring the sides and angles of quadrilateral XYZW.
d) Is your conjecture from part a) valid? Explain.



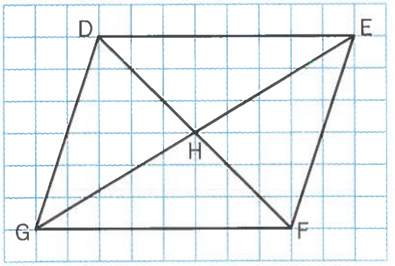
2 Diagonals

1. a) Construct a rectangle ABCD. Draw the diagonals AC and BD. Label the point of intersection E.
b) Find the lengths of AC and BD. How do the lengths compare?
c) Find the lengths of AE and CE. How do the lengths compare?
d) Find the lengths of BE and DE. How do the lengths compare?

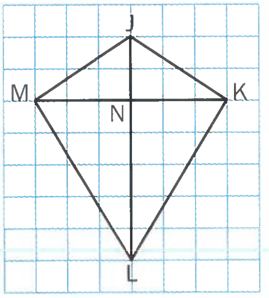


e) Find the measures of ∠AED, ∠DEC, ∠CEB, and ∠BEA. How do the angle measures compare?
f) Write a statement describing the relationship between the lengths of the diagonals of a rectangle, whether they bisect each other, and whether they intersect at 90°.

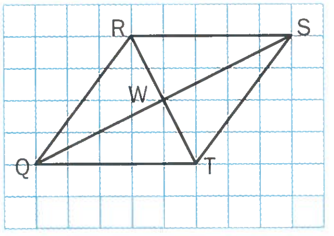
2. a) Construct a parallelogram DEFG. Draw the diagonals DF and EG. Label the point of intersection H.
b) Find the lengths of DF and EG. How do the lengths compare?
c) Find the lengths of DH and FH. How do the lengths compare?
d) Find the lengths of EH and GH. How do the lengths compare?
e) Find the measures of ∠DHG, ∠GHF, ∠FHE, and ∠EHD. How do the angle measures compare?
f) Write a statement describing the relationship between the lengths of the diagonals of a parallelogram, whether they bisect each other, and whether they intersect at 90°.



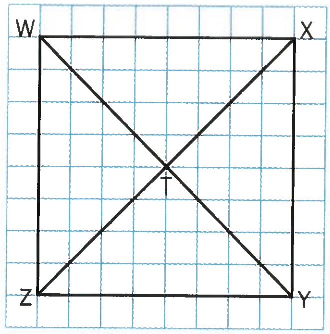
3. a) Construct a kite JKLM. Draw the diagonals JL and KM. Label the point of intersection N.
b) Find the lengths of JL and MK. How do the lengths compare?
c) Find the lengths of JN and LN. How do the lengths compare?
d) Find the lengths of KN and MN. How do the lengths compare?
e) Find the measures of ∠JNM, ∠MNL, ∠LNK, and ∠KNJ. How do the angle measures compare?
f) Write a statement describing the relationship between the lengths of the diagonals of a kite, whether they bisect each other, and whether they intersect at 90°.



4. a) Construct a rhombus RSTQ. Draw the diagonals RT and SQ. Label the point of intersection W.
b) Find the lengths of RT and QS. How do the lengths compare?
c) Find the lengths of RW and TW. How do the lengths compare?
d) Find the lengths of SW and QW. How do the lengths compare?
e) Find the measures of ∠RWQ, ∠QWT, ∠TWS, and ∠SWR. How do the angle measures compare?
f) Write a statement describing the relationship between the lengths of the diagonals of a rhombus, whether they bisect each other, and whether they intersect at 90°.



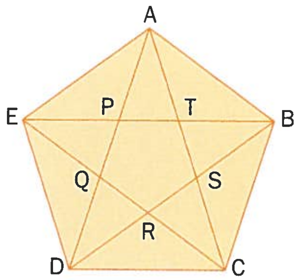
5. a) Construct a square WXYZ. Draw the diagonals WY and XZ. Label the point of intersection T.



b) Find the lengths of WY and XZ. How do the lengths compare?
c) Find the lengths of WT and YT. How do the lengths compare?
d) Find the lengths of ZT and XT. How do the lengths compare?
e) Find the measures of ∠WTZ, ∠ZTY, ∠YTX, and ∠XTW. How do the angle measures compare?
f) Write a statement describing the relationship between the lengths of the diagonals of a square, whether they bisect each other, and whether they intersect at 90°.

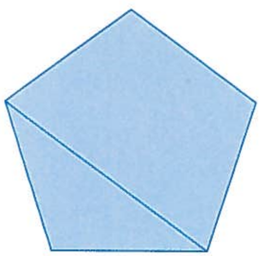
6. For which of the five quadrilaterals you have investigated in questions 1–5, do the diagonals
a) have the same length?
b) bisect each other?
c) intersect at 90°?
d) bisect each other at 90°?

7. a) Construct a regular pentagon ABCDE. Draw all the diagonals. Label the points of intersection of the diagonals P, Q, R, S, and T.



b) Find the lengths of PQ, QR, RS, ST, and TP. How do the lengths compare?
c) Find the measures of ∠PQR, ∠QRS, ∠RST, ∠STP, and ∠TPQ. How do the angle measures compare?
d) Is the pentagon PQRST a regular pentagon?

8. Golden ratio a) Construct a regular pentagon and draw one diagonal.



b) Find the length of one side of the pentagon and the length of the diagonal.
c) Find the ratio of the length of the diagonal to the length of the side. Round your answer to the nearest tenth.
d) The number you found in part c) is an approximation of the **golden ratio**, a number found in nature, art, and architecture. Use your research skills to find a more accurate value of the golden ratio and examples of where it is found.

CAREER CONNECTION

Design

In 1997, Canada Post issued a 45¢ stamp to acknowledge Canadian achievements in industrial design. The stamp was launched at Toronto's Design Exchange, where some of the items shown on the stamp are on display.

Stamp reproduced courtesy of Canada Post Corporation.

Designers work in many fields, including fashion, interior design, urban design, landscape architecture, and Website design. We constantly see the work of designers, which ranges from logos, packaging, and stage sets to the magazines we read, the cars we drive, and the streets we drive along.

1 Designing With Triangles and Quadrilaterals

1. Design 1 was created by connecting the midpoints of the sides of the equilateral triangles.

a) What is the measure of each angle in the largest triangle?

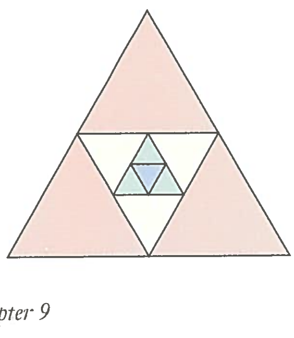
b) How many triangles in the design have these angle measures?

2. The triangles in Design 2 are congruent isosceles triangles.

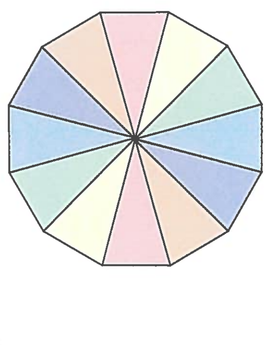
a) What are the angle measures in each triangle in the design?

b) What are the angle measures in each quadrilateral in the design?

Design 1

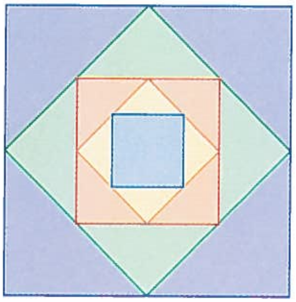


Design 2



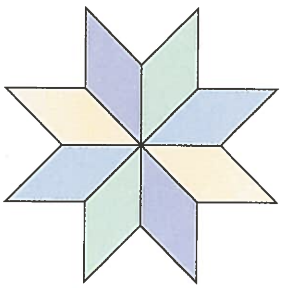
3. Design 3 was created by connecting the midpoints of the sides of the squares.
a) What are the angle measures in a triangle in the design?
b) How many triangles in the design have these angle measures?

Design 3

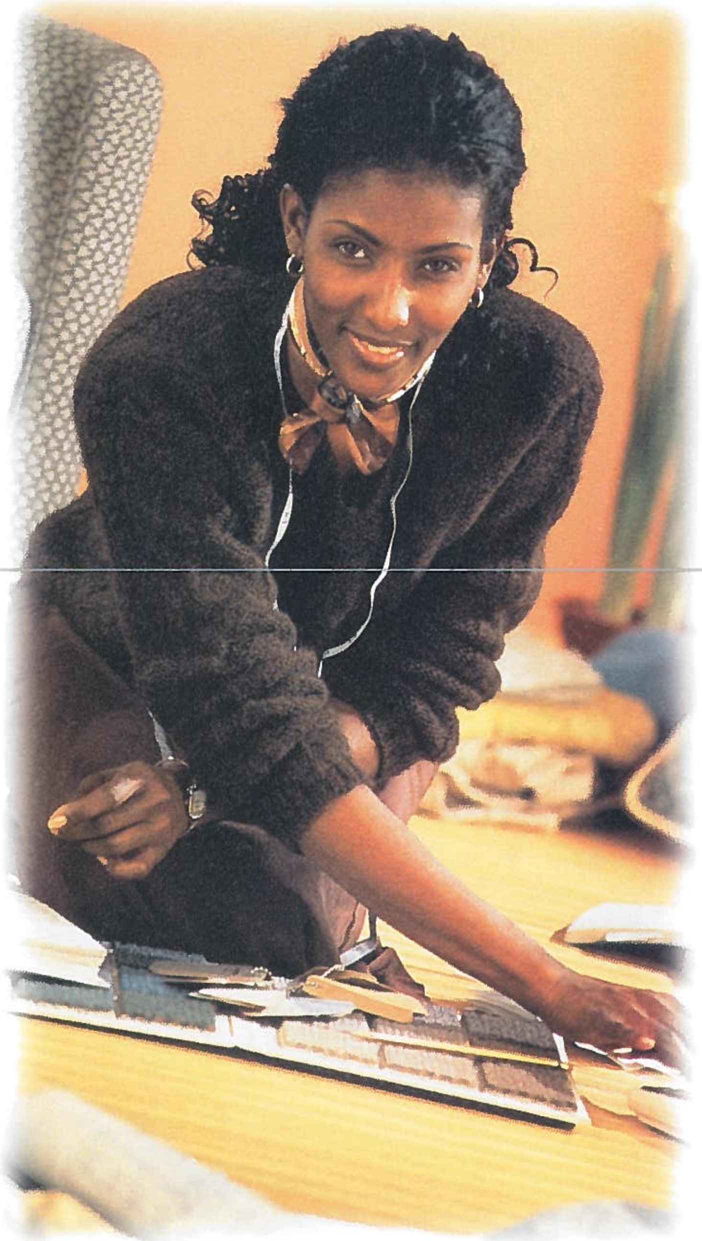


4. Design 4 consists of eight congruent rhombuses.
a) What are the angle measures in each rhombus?
b) How many right angles are formed by sides of rhombuses?

Design 4



5. a) Create your own design from triangles and/or quadrilaterals.
b) Write a problem about angle measures in your design. Have a classmate solve your problem.



2 Locating Information

1. Choose a design field that interests you. Use your research skills to find out how people train for this field and who employs them.

2. Describe a Canadian achievement in the field that you chose.

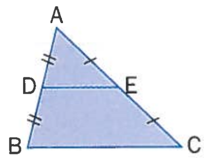
Posing Questions and Testing Conjectures

In the following explorations, you will
a) pose a question about a geometric relationship
b) make a conjecture, which is a possible answer to the question
c) test your conjecture
d) communicate your findings

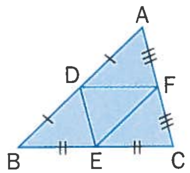
1 Posing Questions Related to Triangles

1. Pose a question about how the sum of the lengths of any two sides of a triangle compares with the length of the third side. Make a conjecture, test it, and communicate your findings.

2. △ABC is any triangle. Line segment DE joins the midpoints of AB and AC. Pose a question about the relationship between the line segments DE and BC. Make a conjecture, test it, and communicate your findings.



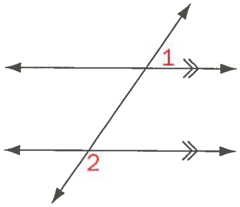
3. △ABC is any triangle. DE, DF, and EF join the midpoints of the sides of △ABC. Pose a question about the relationship between the lengths of the sides of △ABC and the lengths of the sides of △DEF, or a question about the measures of the angles of △ABC and the measures of the angles of △DEF. Make a conjecture, test it, and communicate your findings.



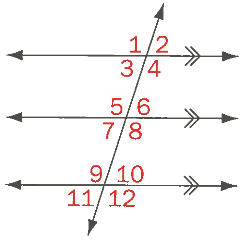
4. a) You have 3 straws that are 6 cm long, 3 straws that are 7 cm long, and 3 straws that are 8 cm long. Pose a question about the number of different triangles that can be made using 3 straws as the sides. Make a conjecture, test it, and communicate your findings.
b) Repeat part a) but use 3 straws that are 4 cm long, 3 straws that are 8 cm long, and 3 straws that are 9 cm long.
c) Write a similar question using straws of lengths that you choose. Make a conjecture, test it, and communicate your findings.

2 Posing Questions Related to Parallel Lines

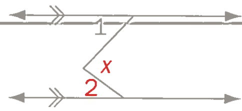
1. In the diagram, ∠1 and ∠2 are exterior angles on the same side of the transversal intersecting two parallel lines. Pose a question about the relationship between ∠1 and ∠2. Make a conjecture, test it, and communicate your findings.



2. a) Pose a question about the relationship between ∠12 and ∠1 in the diagram. Make a conjecture, test it, and communicate your findings.
b) Pose a question about the relationship between ∠11 and ∠4 in the diagram. Make a conjecture, test it, and communicate your findings.

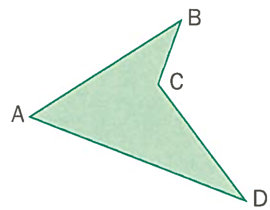


3. Pose a question about the relationship between ∠1 and ∠2 in the diagram. Make a conjecture, test it, and communicate your findings.

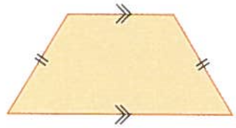


3 Posing Questions Related to Quadrilaterals

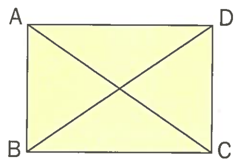
1. Quadrilateral ABCD is a concave quadrilateral. Pose a question about the sum of the interior angles of a concave quadrilateral. Make a conjecture, test it, and communicate your findings.



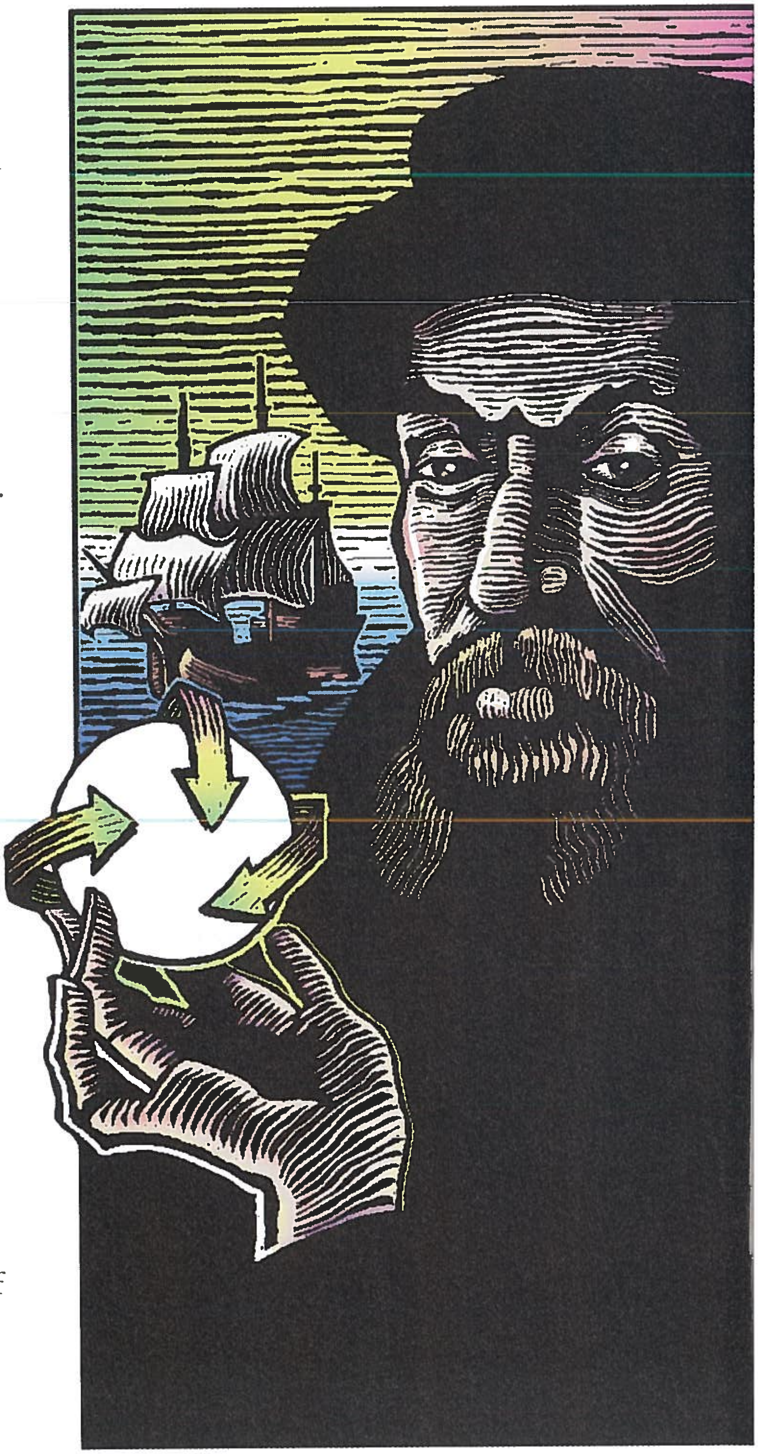
2. An **isosceles trapezoid** is a trapezoid whose two non-parallel sides are equal in length. Pose a question about the relationship between the interior angles of an isosceles trapezoid. Make a conjecture, test it, and communicate your findings.



3. ABCD is any rectangle. Pose a question about the relationship between the lengths of AC and BD. Make a conjecture, show how to test it without measuring, and communicate your findings.



10.3 Analyzing Conjectures Using Examples and Counterexamples

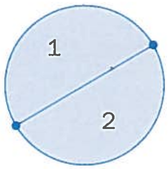


It is important to be cautious when making conjectures. They may or may not be true. To demonstrate that a conjecture is false, you need describe only one **counterexample**, which is an example for which the conjecture is false.

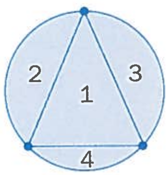
Many years ago, people thought the Earth was flat. Whenever a ship left on a voyage of exploration and never returned, it was believed that the ship had fallen off the edge of the Earth. The flat-Earth conjecture was believed by many until Ferdinand Magellan's ship arrived back in Portugal on September 6, 1522, after taking three years to sail around the world. This trip was seen as a counterexample to the conjecture that the Earth was flat.

Explore: Look for a Pattern

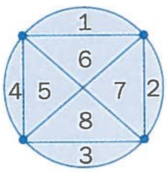
When 2 points are placed on the circumference of a circle and joined, 2 regions are formed.



When 3 points are placed on the circle, and each point is joined to every other point, 4 regions are formed.



When 4 points are used, 8 regions are formed.



Make a conjecture by describing the pattern in the number of regions formed.

Inquire

1. a) Use the pattern to predict the number of regions formed when 5 points are used.
b) Use a diagram to check your prediction. Is the pattern valid for 5 points?

2. a) Use the pattern to predict the number of regions formed when 6 points are used.

b) Use a diagram to check your answer. Is the pattern valid for 6 points?

3. What conclusion can you make about your conjecture for describing the pattern in the number of regions formed?

4. Give a counterexample to show that each of the following conjectures is false.

a) If you live in a country bordering the United States, then you live in Canada.

b) If a quadrilateral has four right angles, then it is a square.

c) A heavier-than-air mechanically driven vehicle that flies is an airplane.

Example Isosceles Triangles

Consider the conjecture:

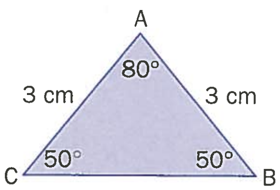
In any isosceles triangle, all three angles are acute.

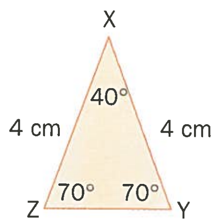
a) Draw two examples of triangles for which the conjecture holds.

b) Draw one counterexample that shows the conjecture is not true.

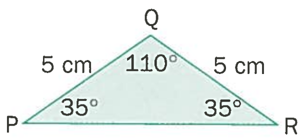
Solution

a)





b)



Since $\angle Q$ is obtuse, this counterexample shows the conjecture is not true.

Practice

A

Give one example that supports each conjecture, and then give one counterexample that shows the conjecture is false.

1. Provinces in Canada share a land border with the United States.

2. If the name of a province in Canada contains the letter *s*, then the province has a coastline.

3. Canadian provincial capitals are south of the 53°N parallel of latitude.

4. The square root of a number is smaller than the number.

5. If the *x*-coordinate of a point in the Cartesian plane is positive, then the point is in the first quadrant.

6. The square of a number is greater than the number.

7. All multiples of 4 are divisible by 8.

8. The square root of a number is a rational number.

Draw a diagram to illustrate each of the following conjectures. Then, draw a counterexample diagram showing the conjecture is not true.

9. If a quadrilateral has two equal diagonals, then it is a rectangle.

10. If two opposite angles in a quadrilateral are equal, then the other two opposite angles are supplementary.

11. If a quadrilateral has four equal sides, then it is a square.

12. If a triangle has exactly two obtuse exterior angles, then it is a right triangle.

13. An altitude of a triangle lies within the triangle.

14. If the measures of two sides of a right triangle are 3 cm and 4 cm, then the third side measures 5 cm.

Applications and Problem Solving

B

Use examples or counterexamples to confirm or deny each conjecture in questions 15–24.

15. If a scalene triangle has a 60° angle, then this angle is opposite the shortest side of the triangle.

16. From a point not on a line, it is possible to draw two perpendiculars to the line.

17. If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

18. If a quadrilateral has perpendicular diagonals, then it is a square.

19. A quadrilateral cannot have exactly three interior angles that are right angles.

20. If two interior angles of a quadrilateral are right angles, then the quadrilateral is a rectangle.

21. If a quadrilateral has four equal sides and four equal angles, then it is a square.

22. If two opposite angles of a quadrilateral both measure 90°, then the quadrilateral is a rectangle.

23. If an exterior angle of a triangle is acute, then the triangle is obtuse.

24. A pentagon with five equal sides is a regular pentagon.

25. Danielle conjectured that, if point P is the vertex of two angles, ∠APB and ∠BPC, then ∠APC is obtuse. Draw a diagram for which the conjecture is true and one counterexample diagram to show that the conjecture is false.

26. Pascal conjectured that, if line segments AB and BC have the same length, then B is the midpoint of line segment AC. Draw a diagram for which his conjecture is true and one counterexample diagram to show that his conjecture is false.

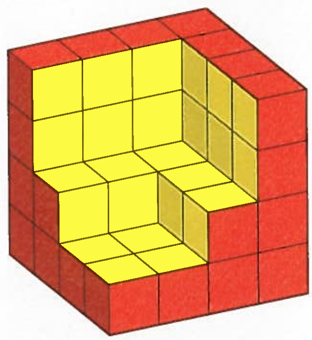
C

27. Sarita conjectured that, if the diagonals of a quadrilateral are perpendicular, then the quadrilateral cannot have all sides with different lengths. Draw diagrams to show whether her conjecture is true or false.

28. Ray conjectured that, if one diagonal of a quadrilateral bisects two interior angles, then the quadrilateral is a parallelogram. Draw diagrams to show whether his conjecture is true or false.

LOGIC POWER

Assume that no small cubes are missing from the back of the stack.



1. How many small cubes are needed to complete the large cube?

2. The faces on the outside of the large cube must all be red. The faces hidden inside the large cube must all be yellow. Some of the small cubes you found in question 1 need to have 2 red faces and 4 yellow faces.

a) What other combinations of red and yellow faces are needed?

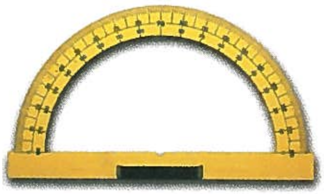
b) Find how many small cubes are needed with each combination of red and yellow faces.

10.4 How Can We Model Your Field of Vision?

Angles of peripheral vision were introduced on page 509 at the beginning of this chapter. Complete the following experiment to determine your angles of peripheral vision. You will need to work in a group of at least three.

1 Modelling Horizontal Peripheral Vision

To determine your angle of horizontal peripheral vision, you will need a chalkboard protractor. If one is not available, make one from cardboard and mark the scale in 5° intervals.



1. Decide who is to be tested (the subject), who is to do the testing (the tester), and who is to perform the angle measurements (the recorder). Then, use the following steps.

- The subject and the tester sit in chairs and face each other with their knees almost touching.
- To test the left eye, the subject covers the right eye and stares at the tester's nose.
- The tester holds a pen vertically just behind and about 25 cm from the subject's left ear.
- The subject keeps staring at the tester's nose while the tester slowly moves the pen in a horizontal arc toward the centre of the subject's field of vision.
- The subject says "stop" when the pen first comes into view.
- The recorder places the protractor under the subject's chin and measures the angle formed by two lines — one that connects the subject's nose and the tester's nose, and the other that connects the subject's nose and the pen.

2. To test the subject's right eye, repeat the above steps but with the subject's left eye covered and the pen behind the subject's right ear.

3. Add the two angle measures found above to find the subject's angle of horizontal peripheral vision.

4. Change roles until all students in the class have been tested. Record all the results.

5. How does your own angle of horizontal peripheral vision compare with your estimate from page 509?

6. Calculate the mean angle of horizontal peripheral vision for the students in your class.

2 Modelling Vertical Peripheral Vision

Recall from page 509 that there are two angles of vertical peripheral vision. They are upward and downward from the horizontal. The procedure for finding these angles is similar to the procedure for finding the angle of horizontal peripheral vision.

1. Find the upward angle of peripheral vision as follows.
• With the subject and the tester positioned as before, the subject stares at the tester's nose with both eyes.
• The tester holds a pen horizontally about 25 cm above the subject's head.
• The subject keeps staring at the tester's nose while the tester moves the pen in a vertical arc down toward the centre of the subject's field of vision.
• The subject says "stop" when the pen first comes into view.
• The recorder measures the angle formed by two lines — one that connects the subject's nose and the tester's nose, and the other that connects the subject's nose and the pen.

2. Find the subject's downward angle of peripheral vision in a similar way, except that the tester holds the pen horizontally below the subject's chin and moves it up toward the centre of the subject's field of vision.

3. Change roles until all students in the class have been tested. Record all the results.

4. How do your own upward and downward angles of peripheral vision compare with your estimates from page 509?

5. Calculate the mean upward and downward angles of peripheral vision for the students in your class.

6. Which is greater — the upward or the downward angle of peripheral vision? Explain why it is greater.

3 Analyzing and Displaying Data

1. As well as calculating the mean of each set of values, how could you analyze the data?

2. Prepare a report using the class results. Organize and analyze the data in ways that you choose.

4 Applying the Model

1. Helmets One standard for bicycle helmets recommends an angle of vertical peripheral vision of 40° upward and an angle of horizontal peripheral vision of 240°, or 120° from the centre line on each side. For go-kart racing helmets, the recommended angles of vertical peripheral vision are 5° upward and 20° downward, and the angle of horizontal peripheral vision is 180°, or 90° from the centre line on each side.

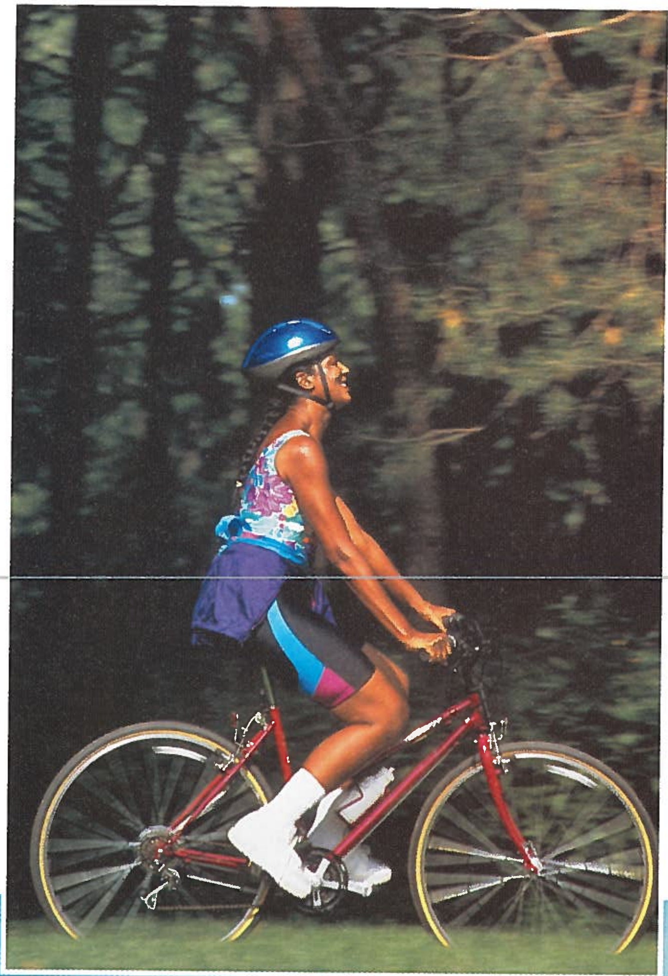
a) Why are the angles of horizontal peripheral vision different for the two helmets?

b) Why is there a recommended angle of downward peripheral vision for a go-kart helmet, but not for a bicycle helmet?

c) For what other types of headgear do designers need to consider the horizontal and vertical angles of peripheral vision?

2. Tunnel vision An eye condition called glaucoma can result in "tunnel vision." What is tunnel vision and why is it given this name?

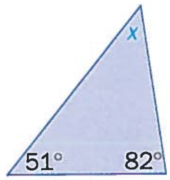
3. Eye tests Describe other methods that ophthalmologists use to determine angles of peripheral vision.



Review

10.1 *Find the missing angle measures.*

1.



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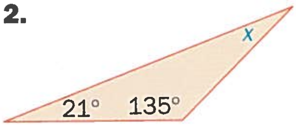

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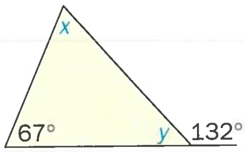

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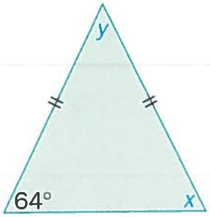

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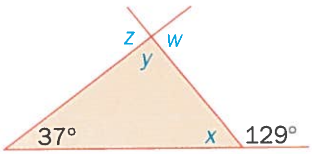

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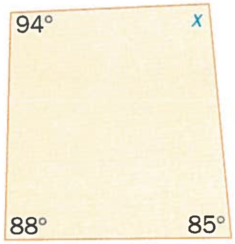

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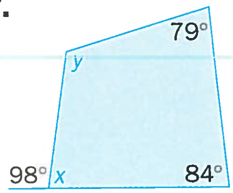

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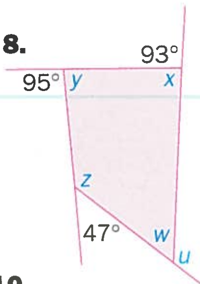

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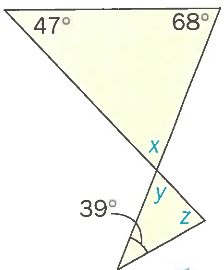

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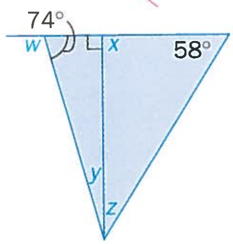

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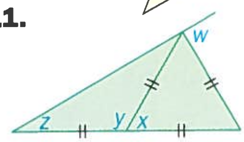

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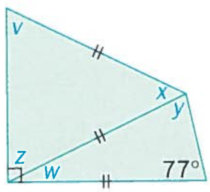

Find the value of x. Then, find the measures of all the indicated angles.

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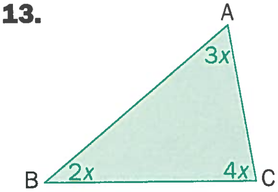

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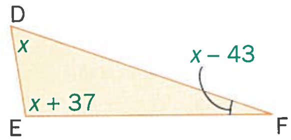

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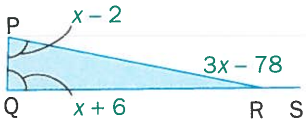

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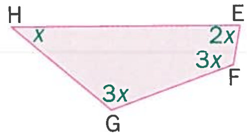

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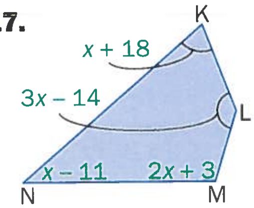

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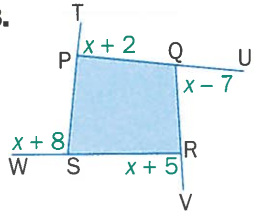

10.2 *Find the measure of each indicated angle.*

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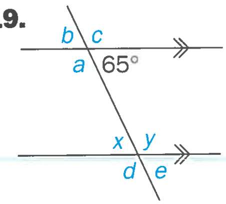

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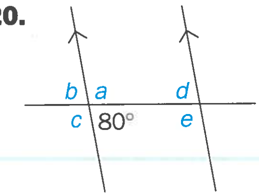

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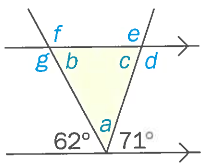

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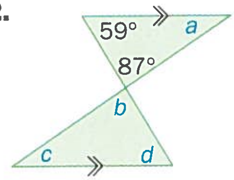

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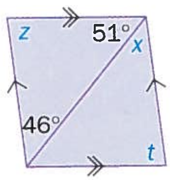

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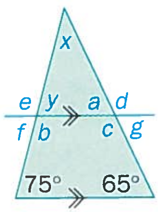

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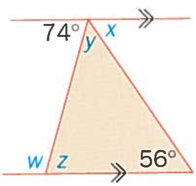

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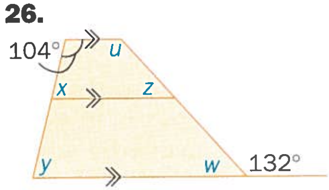
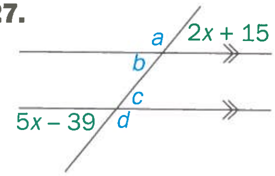


Find the value of x. Then, find the measures of the indicated angles.

27.



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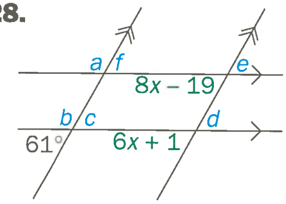

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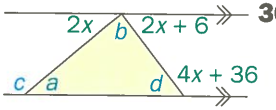
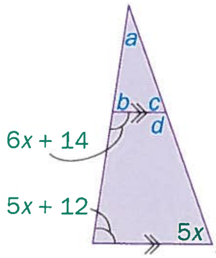
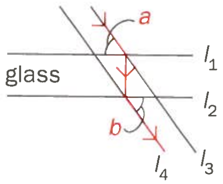


30.



31. Refraction of light In the diagram, light enters the pane of glass along l_3 and leaves along l_4. If $l_3 \parallel l_4$, $l_1 \parallel l_2$, and $\angle a = 55°$, find the measure of $\angle b$. Give reasons for your answer.



10.3 *Draw a diagram to illustrate each of the following conjectures. Then, draw a counterexample diagram showing the conjecture is not true.*

32. The three altitudes of a triangle intersect at a point inside the triangle.

33. If a quadrilateral has two equal diagonals, then it is a square.

Confirm or deny each statement using examples or counterexamples.

34. If a quadrilateral has four equal angles, then it is a square.

35. If a quadrilateral contains no right angles and has two pairs of equal angles, then it is a parallelogram.

36. It is not possible for the sum of the lengths of two sides of a triangle to equal the length of the third side.

37. If one interior angle of a quadrilateral is a reflex angle, then the quadrilateral is concave.

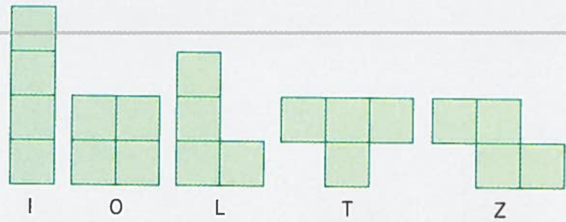
Exploring Math

Tetrominoes

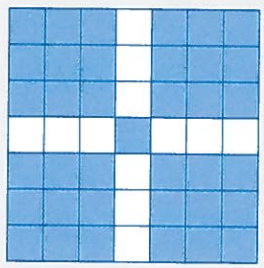
Polyominoes are shapes formed by joining identical squares along their edges. The domino, which is made up of two squares, can have only one shape.

A triomino, which is made up of three squares, can have two shapes.

Tetrominoes are made up of four squares and can have the following five shapes.



On the 7-by-7 grid shown, squares have been shaded so that no group of shaded squares forms the I shape, either vertically or horizontally. The maximum number of squares that can be shaded without shading the I shape is 37, as shown.



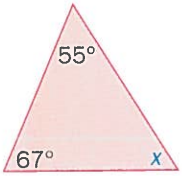
Determine the maximum number of squares that can be shaded on a 7-by-7 grid without shading the following shapes. Allow for all possible orientations; for example, an upside down T shape is still considered a T shape.

1. the O shape
2. the L shape
3. the I, O, and Z shapes
4. the I, O, T, Z, and L shapes

Chapter Check

Find the missing angle measures.

1.



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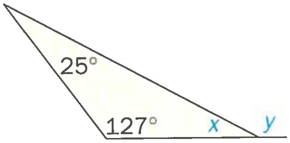

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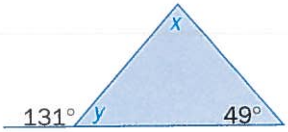

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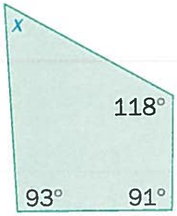

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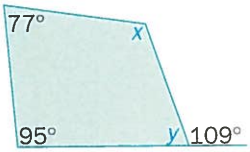

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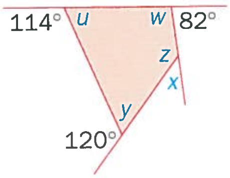

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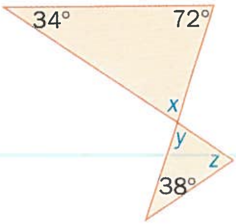

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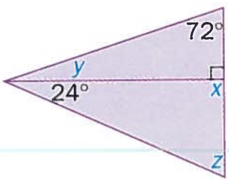
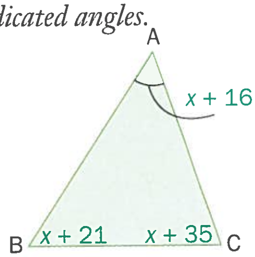


Find the value of x. Then, find the measures of all the indicated angles.

9.



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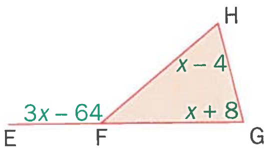

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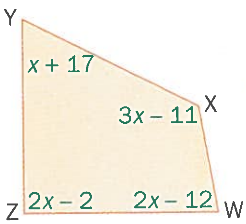

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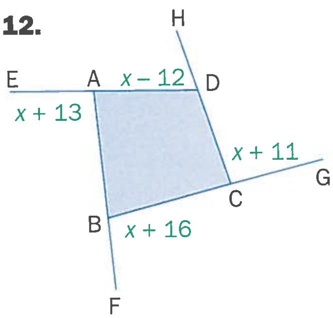
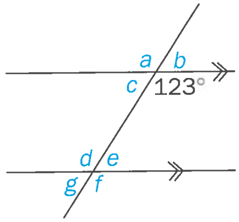


Find the measure of each indicated angle.

13.



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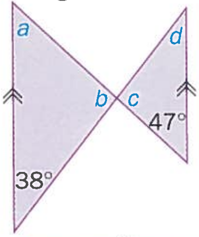

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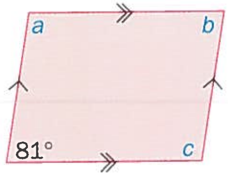

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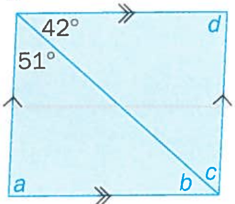

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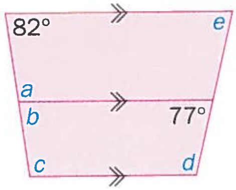

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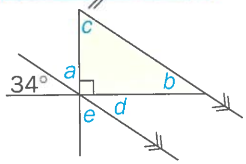
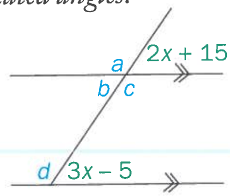


Find the value of x. Then, find the measures of the indicated angles.

19.



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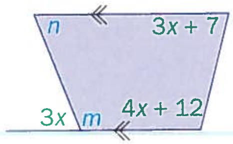


Draw a diagram to illustrate each of the following conjectures. Then, draw a counterexample diagram that shows the conjecture is not true.

21. If the measure of one angle in a triangle is 85°, the triangle is not scalene.

22. A quadrilateral with four equal sides is regular.

23. A trapezoid has two pairs of equal angles.

24. Two co-interior angles on the same side of a transversal that intersects two parallel lines are equal.

Confirm or deny each statement using examples or counterexamples.

25. A quadrilateral cannot have three interior angles that are obtuse angles.

26. A kite has two pairs of equal angles.

27. The exterior angles of an acute triangle are obtuse.

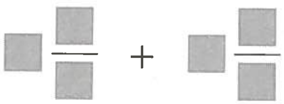
Using the Strategies

1. **Latin Squares** The diagram shows a Latin Square. The numbers 1, 2, and 3 have been placed so that each number appears only once in each row and column.

1	2	3
2	3	1
3	1	2

There are 11 other different Latin Squares that use the numbers 1, 2, and 3. Draw them.

2. **Smallest sum** Use each of the digits 1, 3, 4, 6, 7, and 8 only once. Make the smallest possible sum by arranging the numbers as indicated in the diagram. What is the sum?

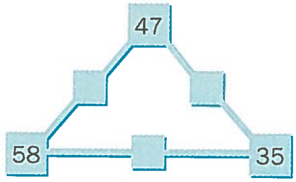


3. **Counterfeit coin** What is the minimum number of mass comparisons an inspector needs to make to find 1 counterfeit coin in a collection of 40 coins? Assume that the counterfeit coin is lighter than the others.

4. **Phone calls** About how many hours do all the high school students in Ontario spend on the telephone in a year?

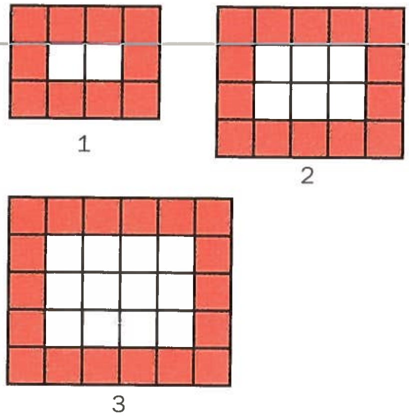
5. **Shopping** You spend $2.75 in a store and receive $7.25 change from $10.00. Notice that the arrangement of the digits in the amount you spent is a rearrangement of the digits in your change. Find 4 other pairs of amounts spent and change from $10.00 that share this property.

6. The numbers in the large squares are found by adding the numbers in the small squares. What are the numbers in the small squares?



7. **GCF** The greatest common factor of two numbers, m and n, is 14. If $m = 2 \times 5 \times 7^2$, name three numbers that could be n.

8. **Design** Each rectangular design is made with red border squares and white interior squares. Each square has a side length of 1 unit.



a) How many red squares are in the fourth diagram? the fifth diagram?
b) Write an expression for the number of red squares in terms of the length of the design, l.
c) How many red squares are there in the design with a length of 14? 40?

9. **Running** Lauryn and Yolanda had a 100-m race. Lauryn beat Yolanda by 10 m. For the second race, Yolanda suggested that Lauryn start 10 m behind the starting line. Yolanda thought this way of starting would give her a fair chance.
a) Who won the race and by how much?
b) What assumptions did you make?

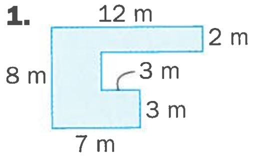
10. **Rivers** What percent of Canada's 20 longest rivers flow into Hudson Bay?

CUMULATIVE REVIEW, CHAPTERS 9–10

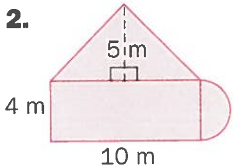
Chapter 9

Calculate the area. Round to the nearest square unit, if necessary.

1.

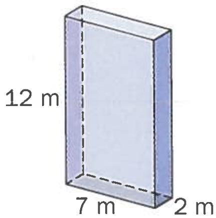


2.



Calculate the surface area. Round to the nearest square metre, if necessary.

3.



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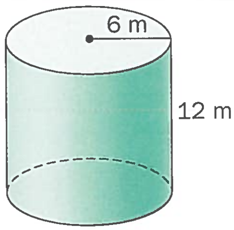
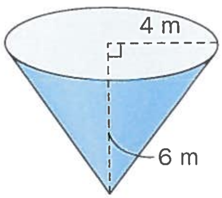


5.



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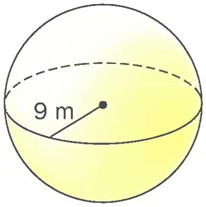
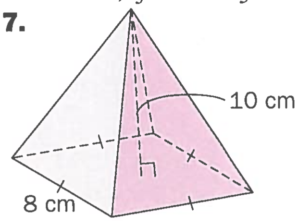


Calculate the volume. Round to the nearest cubic centimetre, if necessary.

7.



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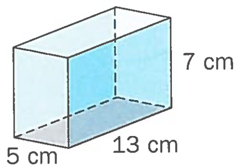

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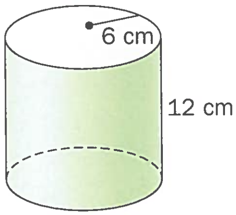
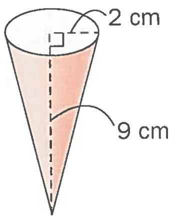


10.



11. Soup can A soup can is a cylinder with a diameter of 6.5 cm and a height of 9.5 cm.

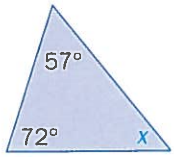
a) Calculate the volume, to the nearest cubic centimetre.

b) How many millilitres of soup will the soup can hold?

Chapter 10

Find the missing angle measures.

1.



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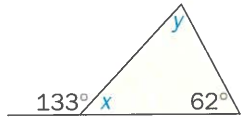
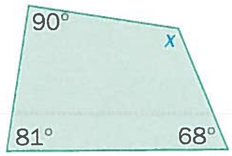


3.



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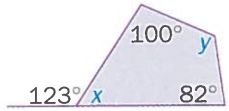
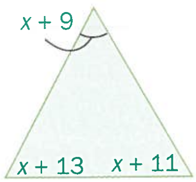


Find the measures of all the indicated angles.

5.



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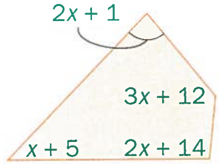
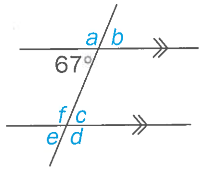


Find the measure of each indicated angle.

7.



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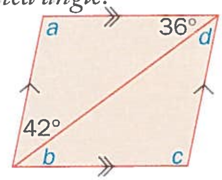
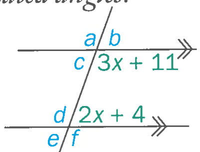
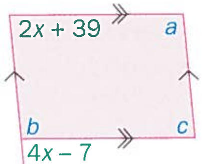


Find the value of x. Then, find the measures of all the indicated angles.

9.



10.



Draw a diagram to illustrate each of the following conjectures. Then, draw a counterexample diagram showing the conjecture is not true.

11. If the diagonals of a quadrilateral bisect each other at 90°, then it is a square.

12. If two opposite angles in a quadrilateral are equal, then the other two opposite angles are equal.

Confirm or deny each statement using examples or counterexamples.

13. If a triangle has one exterior angle that is acute, then the triangle is scalene.

14. If two exterior angles of a quadrilateral measure 90°, then it is a rectangle.

Estimate, and then calculate.

1. $34.6 + 121.9 + 55.2$ **2.** $11.2 - 5.9 + 0.8$
3. 5.2×115 **4.** $45.36 \div 7.2$

5. Evaluate $4x - 7$ when x equals
a) 9.4 **b)** 7.3 **c)** 10.6

6. Evaluate for $x = 2$ and $y = 3$.
a) $x^4 - y^2$ **b)** $(y - x)^3$ **c)** $7x^2 - y^2$

Simplify. Express each answer in exponential form.

7. $2^5 \times 2^3$ **8.** $5^6 \div 5^4$ **9.** $(7^3)^4$
10. $x^7 \times x^4$ **11.** $x^8 \div x^3$ **12.** $(x^5)^2$

Evaluate.

13. $3 - (-5) - 4 - 2$ **14.** $5 - 8 - 9 - 2$
15. -7×8 **16.** $-30 \div (-5)$
17. $(-4)^2$ **18.** -5^3

Evaluate.

19. $14 \div (-7) - 2^3$ **20.** $(3 - 6)^2 - 5 \times 2$
21. $3 \times (-1)^3 + 4 \times (-3)$ **22.** $-5 \times 3 - 8$

23. Evaluate for $x = -4$ and $y = -3$.
a) $5x - 6y$ **b)** $3xy - 2$
c) $2x^2 - xy$ **d)** $x^2 - y^3$

Write each ratio in simplest form.

24. 15 to 20 **25.** 8:10 **26.** $\frac{18}{27}$

Solve for x.

27. $\frac{x}{1.5} = \frac{10}{3}$ **28.** $\frac{3}{2} = \frac{7}{x}$

29. Running speed A jackrabbit can run at a speed of 72 km/h. How far can it run in 1 s?

30. Comparison shopping Decide which is the better value.
\$4.60 for 4 L of milk or \$2.45 for 2 L of milk

Write as percents.

31. 0.55 **32.** 0.09 **33.** 1.34

Write each percent as a decimal.

34. 24% **35.** 0.8% **36.** 143%

37. Markup The cost price for a sweater for a retailer was \$110. The retailer marked it up by 30%. What was the selling price before taxes?

Write in scientific notation.

38. 12 300 000 **39.** 0.000 042

Write in standard form.

40. 3.2×10^8 **41.** 5.8×10^{-6}

Simplify.

42. $4^{-3} \times 4^4$ **43.** $3^{-3} \div 3^{-4}$ **44.** $(5^{-2})^{-5}$

Evaluate.

45. $3^0 \times 3^2$ **46.** $4^3 \div 4^5$ **47.** $(3^{-2})^{-2}$

Calculate.

48. $-3.4 \times (-6.5)$ **49.** -5.2^2
50. $10.8 \div (-0.2)$ **51.** $-3.9 - 6.8$
52. $-\frac{7}{2} \div \frac{7}{6}$ **53.** $-4 - 5\frac{5}{8}$
54. $-\frac{5}{4} \times \frac{8}{7}$ **55.** $11\frac{3}{5} - 17$

Evaluate.

56. $2(4.6 - 5.1) \div (-0.5)$
57. $(-3.2)^2 + 4$
58. $8 - \frac{3}{4} \times \frac{8}{5}$
59. $(8 - 12)^2 \div \frac{1}{4}$

Estimate. Then, calculate to the nearest tenth.

60. $\sqrt{531}$ **61.** $\sqrt{4562}$ **62.** $\sqrt{0.85}$

63. Measurement A square has an area of $60\ m^2$. Calculate the perimeter, to the nearest tenth of a metre.

64. Music lessons Identify the population and suggest a sampling procedure that could be used to answer the following question.

What percent of secondary school students in your city or town take music lessons?

65. Test scores **a)** Draw a box-and-whisker plot to represent the following geography test scores.
71, 78, 79, 68, 70, 72, 72, 81, 84, 61,
63, 63, 86, 86, 79, 77, 74, 87, 83
b) About what percent of the values lie in the box? in each whisker?

66. Education spending The table gives the amount, in billions of dollars, spent on elementary and secondary education in Canada in several years.

Year	1971	1976	1981	1986	1991	1996
Spending ($Billions)	5	10	17	23	33	36

a) Display the data on a scatter plot.
b) Draw the line of best fit.
c) Use the graph to estimate the amount that will be spent on elementary and secondary education in 2012.

67. Measurement **a)** Plot the points A(2, 2), B(5, 2) and C(2, 6) on a grid. Join them to form a triangle.
b) Find the perimeter and area of the triangle.

68. Giant tortoise A giant tortoise moves at a speed of 4 m/min.
a) Copy and complete the table of values for a moving giant tortoise.

Time (min)	Distance (m)
0	
1	
2	
3	
4	

b) Draw a graph of distance versus time.
c) Use the graph to estimate the distance a giant tortoise can cover in 1.7 min; in 3.3 min.

a) *Graph the following relations using paper and pencil or a graphing calculator.*
b) *State whether each relation is a direct variation or a partial variation.*

69. $y = -2x + 2$ **70.** $y = -0.5x$

71. Modelling motion Sketch a graph of the distance you are from your locker versus time on a typical school day, from when you enter the school until you leave.

Simplify.

72. $(x^2 - 3x + 8) + (3x^2 + 7x - 9)$
73. $(2y^2 + y - 11) - (5y^2 - y + 4)$

Expand and simplify.

74. $5(y - 3) + 7(y + 2)$
75. $3(3x - 5) - (4x - 9) + 7$
76. $-2(x^2 - 6x - 1) - 3(2x^2 - x + 4)$
77. $3x(x - 4) + 2x(3x + 1)$

Simplify.

78. $(-4x^2)(-3xy)$ **79.** $(2de)(-3d^2e^3)$
80. $(-3x^2y^3)^2$ **81.** $(-2wx^3y^3)^3$
82. $\dfrac{40x^4y^5}{-8x^2y^2}$ **83.** $\dfrac{-42r^2s^3t^4}{-6r^2s^3t}$

Factor.

84. $15x^2 - 10x$ **85.** $9x^2y^2 - 3x^2y + 6xy^2$

Solve and check.

86. $3x - 13 = 11$ **87.** $15 = 2y + 21$
88. $5x + 12 = x - 12$ **89.** $6y - 11 = 5 + 8y$
90. $8(m + 1) = 3(m - 4)$
91. $2(x + 4) = 5(x - 1) - 2$
92. $0.3x + 0.4 = 0.8x - 0.1$
93. $\dfrac{x}{6} - \dfrac{2}{3} = \dfrac{x}{2}$ **94.** $\dfrac{x+1}{2} = \dfrac{x+2}{3}$

95. National parks The total area of Point Pelee National Park and Georgian Bay Islands National Park is 40 km^2. The area of Point Pelee National Park is 10 km^2 less than the area of Georgian Bay Islands National Park. Find the area of each park. Explain how you used a mathematical model to solve the problem.

Find the slope of the line passing through each pair of points, if possible.

96. A(2, 5) and B(4, 11)
97. C(−3, 4) and D(−2, 2)
98. E(−1, 7) and F(5, 7)
99. G(3, −4) and H(3, 6)

Write an equation in standard form for the line passing through the given point and having the given slope.

100. A(3, 5); $m = 4$ **101.** B(−1, 2); $m = -\dfrac{3}{4}$

Write an equation in standard form for the line passing through the given points.

102. A(3, 8) and B(1, −6)
103. C(2, −3) and D(−4, −5)

Find the slope and y-intercept of each of the following lines.

104. $3x + y - 4 = 0$ **105.** $5x - 2y + 10 = 0$

Graph each equation using a method of your choice.

106. $2x + y - 5 = 0$ **107.** $5x - 3y + 15 = 0$

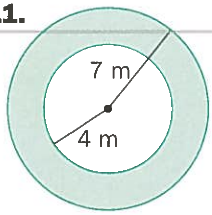
108. Write an equation of a line perpendicular to $3x + y - 4 = 0$ and passing through the point $(-2, 5)$.

109. Use paper and pencil or a graphing calculator to find the coordinates of the point of intersection of the lines $y = 4x + 1$ and $y = 2x + 3$.

110. a) Graph the ordered pairs and draw the line of best fit.
$(5, 6), (3, 4), (3, 2), (0, 0), (0, -2), (-2, -4), (-2, -6)$
b) Find an equation of the line of best fit.

Calculate the area of each shaded region. Round answers to the nearest tenth of a square unit, if necessary.

111.



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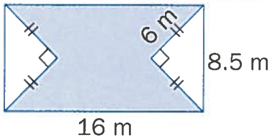
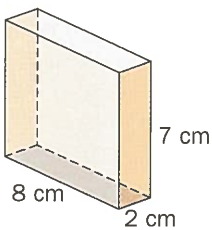


Calculate the surface area and volume. Round answers to the nearest whole number of square or cubic units, if necessary.

113.



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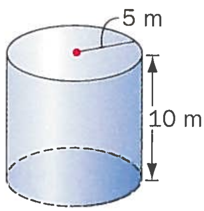

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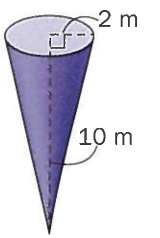

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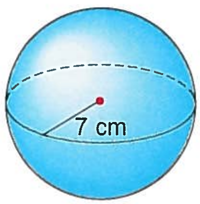

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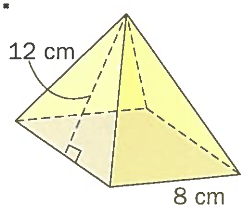

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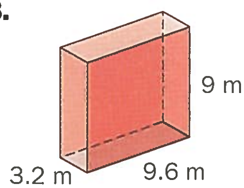
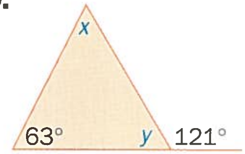


119. Tomato paste can A tomato paste can has the shape of a cylinder with a radius of 2.5 cm and a height of 8 cm.
a) Calculate the volume of the can, to the nearest cubic centimetre.
b) How many millilitres of paste will it hold?

Find the missing angle measures.

120.



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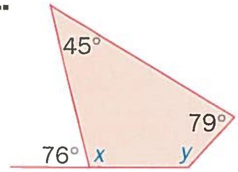

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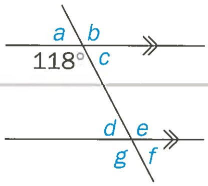

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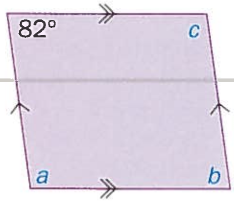
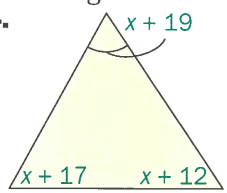
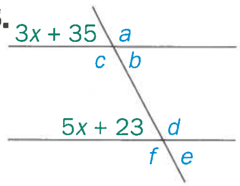


Find the value of x. Then, find the measures of all the indicated angles.

124.



125.



126. Draw a diagram to illustrate the following conjecture. Then, draw a counterexample diagram showing the conjecture is not true.
If the exterior angles of a triangle are obtuse, then the triangle is equilateral.

127. Confirm or deny the statement using examples or a counterexample.
If a quadrilateral has exactly two exterior angles that measure 90°, then it is a kite.