CHAPTER 9

Measurement

How Can We Model the Earth in Two Dimensions?

The position of a place on the Earth can be described using its latitude and longitude.

1. Parallels of latitude are imaginary circles around the Earth parallel to the equator, which has been assigned a latitude of 0°.
a) What is located at a latitude of 90°N? 90°S?
b) Which parallel of latitude has a radius equal to the radius of the Earth?
c) What is the radius of the 90°N or the 90°S parallel of latitude? Explain.

2. Meridians of longitude are imaginary half-circles that connect both poles. In 1884, the meridian through Greenwich, near London, England, was selected as the prime meridian and assigned a longitude of 0°.
a) How does the length of a meridian of longitude compare with the length of the equator? Explain.
b) Longitudes are described as being east (E) or west (W) of Greenwich. How are the meridians at 180°E and 180°W related? Explain.
c) Use an atlas or other source to find the longitude of where you live, to the nearest degree. If the prime meridian were through where you live, what would the longitude of Greenwich be? Explain.

In Modelling Math — Cartography on pages 501 to 503, you will learn more about how we can model the Earth in two dimensions.

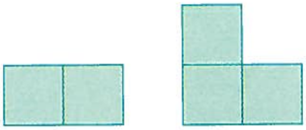
GETTING STARTED

Perimeter and Area

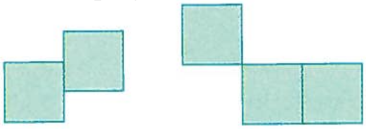
1 Polyominoes

A polyomino is a polygon formed by joining identical squares along whole sides.

These are polyominoes:

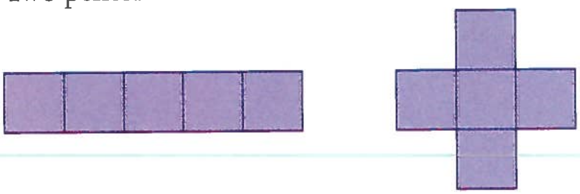


These are not polyominoes:



1. A pentomino is made up of 5 identical squares joined at their sides. A number of different pentominoes can be formed by rearranging the squares.

Two pentominoes are shown:



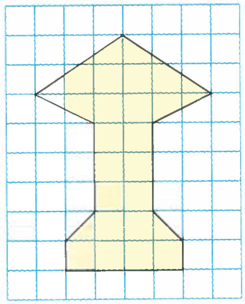
a) Draw as many different pentominoes as possible on grid paper.
b) Which shape has the smallest perimeter?

2. A tetromino is made up of 4 identical squares joined at their sides.
a) Draw as many different tetrominoes as possible on grid paper.
b) Which shape has the smallest perimeter?

3. A hexomino is made up of 6 identical squares joined at their sides.
a) Predict which shape of hexomino has the smallest perimeter.
b) Explain your reasoning.

2 Areas of Irregular Figures

The figure has been drawn on a grid, with each square equal to 1 square unit.



The area of the figure can be estimated by counting the number of whole and part squares that it covers.

The number of whole squares is 16. The number of part squares is 14.

$$\begin{aligned}\text{Total Area} &= (\text{Number of whole squares}) \\ &\quad + \frac{1}{2}(\text{Number of part squares}) \\ &= 16 + 0.5(14) \\ &= 16 + 7 \\ &= 23\end{aligned}$$

The total area of the figure is about 23 square units.

1. Why do you think you calculate one half of the part squares?

2. Trace your hand on a sheet of 1-cm grid paper. Estimate the area of your hand, in square centimetres, by counting whole squares and part squares.

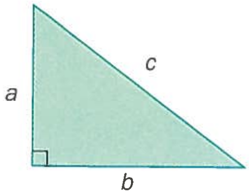
3. Measure your arm length, in centimetres.

4. Collect a class set of arm lengths and hand areas. Graph hand area versus arm length.

5. Can you make a general conclusion about arm length and hand area from the data? If you can, what is it? Explain and justify your reasoning.

3 The Pythagorean Theorem

In a right triangle, the longest side is opposite the right angle and is called the **hypotenuse**. The other two sides are called **legs**. The Pythagorean Theorem states that, in any right triangle, if c is the length of the hypotenuse, and a and b are the lengths of the legs, then $a^2 + b^2 = c^2$.



Find the unknown side length in each right triangle. If necessary, round answers to the nearest tenth of a unit.

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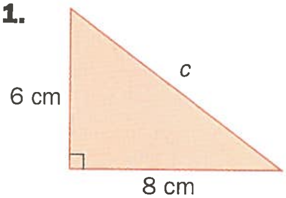

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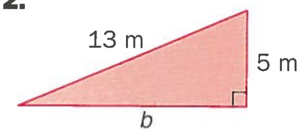

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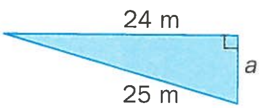

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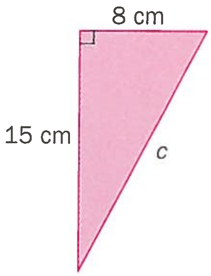

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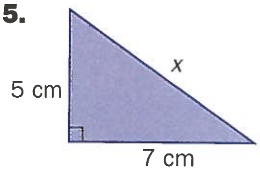

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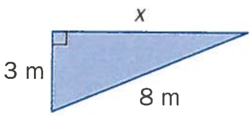

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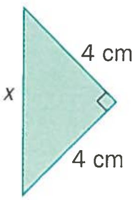

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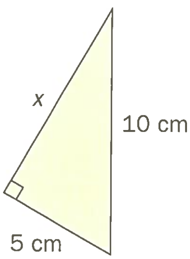


Mental Math

Operations With Decimals

Calculate.

1. 3.14×1000 **2.** $3.14 \div 100$
3. 2.5×6 **4.** $15.5 \div 5$
5. $0.5(3.9 + 8.1)$ **6.** $0.5(6.2 + 4.8)$

Estimate.

7. $(3.14)(2.2)^2$ **8.** $(3.14)(4.9)^2$
9. $(3.14)(9.8)^2$ **10.** $(3.14)(2.1)(3.2)$
11. $(3.14)(5.2)(2.4)$ **12.** $(3.14)(7.2)(10.4)$

Subtracting Using Compatible Numbers

Suppose that you are subtracting one number from another and that the number you are subtracting is close to a multiple of 10. The mental subtraction is easier if you adjust both numbers so that you subtract a multiple of 10.

For 83 – 49, think

$$(83 + 1) - (49 + 1) = 84 - 50 = 34$$

So, $83 - 49 = 34$

For 91 – 42, think

$$(91 - 2) - (42 - 2) = 89 - 40 = 49$$

So, $91 - 42 = 49$

Calculate.

1. $55 - 19$ **2.** $73 - 28$ **3.** $80 - 39$
4. $81 - 22$ **5.** $70 - 31$ **6.** $61 - 23$

Adapt the method to calculate the following. Describe how you adapted the method.

7. $6.4 - 2.9$ **8.** $10 - 3.8$ **9.** $7.2 - 4.3$
10. $650 - 280$ **11.** $1800 - 990$ **12.** $710 - 320$

13. a) Complete the following, where x, y, n, m, b, and c represent whole numbers.
$(10x + y) - (10n + m)$
$(10x + y + b) - (10n + m + b)$
$(10x + y - c) - (10n + m - c)$

b) Explain how the subtractions in part a) are related to the method shown above.

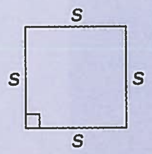
c) For the method to work as shown, what is the value of $m + b$? the value of $m - c$?

INVESTIGATING MATH

Reviewing Formulas

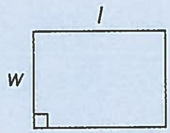
Recall the formulas for the perimeter, P, and the area, A, of the following plane figures.

Square



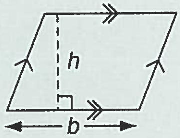
$P = 4s$
$A = s^2$

Rectangle



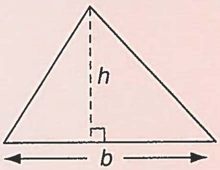
$P = 2(l + w)$
$A = l \times w$

Parallelogram



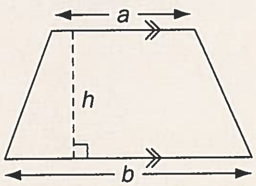
$P =$ sum of side lengths
$A = b \times h$

Triangle



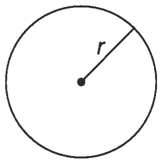
$P =$ sum of side lengths
$A = \frac{1}{2} \times b \times h$

Trapezoid



$P =$ sum of side lengths
$A = \frac{1}{2} \times (a + b) \times h$

Circle

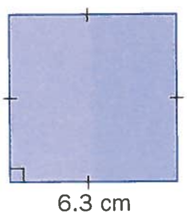


$C = \pi \times d$ or $2 \times \pi \times r$
$A = \pi \times r^2$

1 Finding the Perimeter

Calculate the perimeter of each of the following figures.

1.



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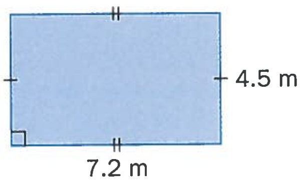

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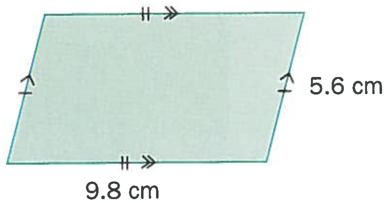

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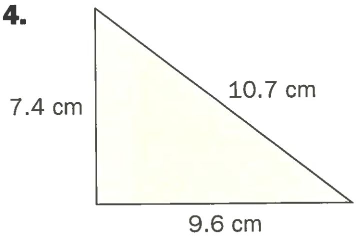

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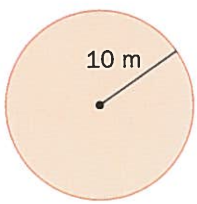

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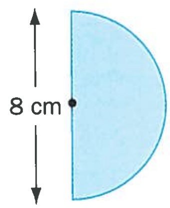

2 Finding the Area

Calculate the area of each of the following figures. If necessary, round answers to the nearest tenth of a square unit.

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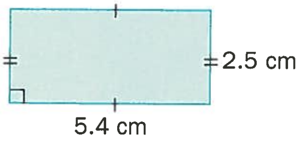

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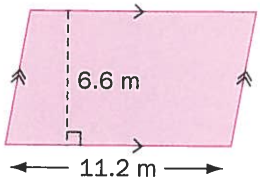

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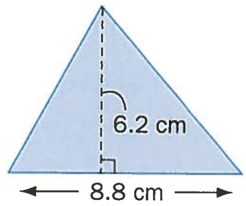

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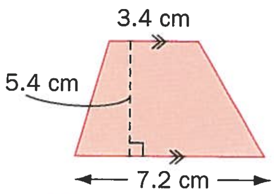

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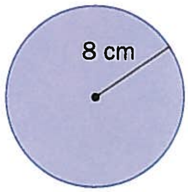

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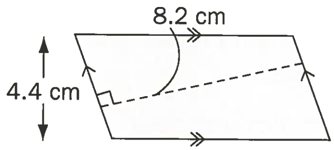

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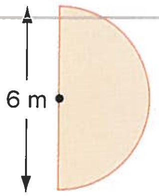

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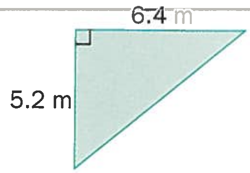

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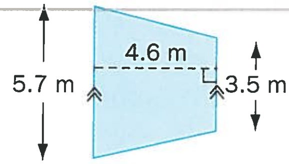

3 Finding the Perimeter and the Area

Calculate the perimeter and the area of each figure.

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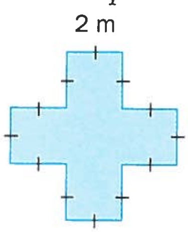

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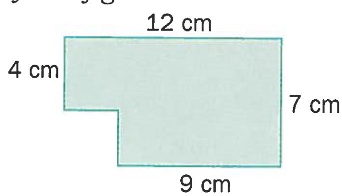

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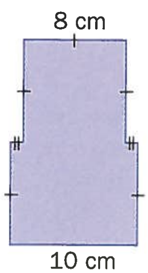

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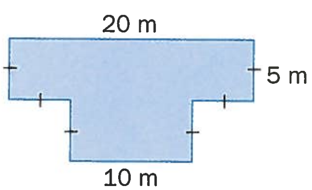

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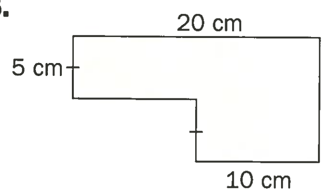

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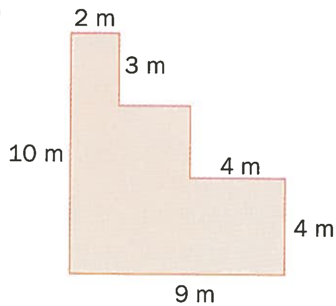


9.1 Areas of Composite Figures

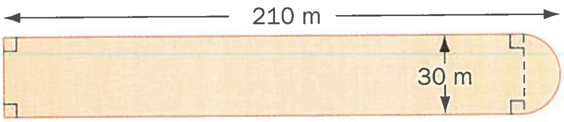
A **composite figure** is a figure made up of two or more distinct figures, such as a rectangle and a semicircle. To find the area of a composite figure, add the areas of the individual parts.

Explore: Use a Diagram

The first recorded Olympic games were held in 776 B.C., near the small city of Olympia, in Greece. Other Greek cities and towns based the plans for their Olympic sites on the plan used at Olympia.

The games were held in a *stadion*, from which we get the modern word stadium. The name *stadion* came about because the first race was 1 *stade*, or about 192 m, in length. A stadion was usually set into a valley to provide seating for the spectators, who, in some cases, numbered about 50 000.

A stadion was a rectangle with a semicircle at one end. The dimensions of a typical Greek stadion are shown in the diagram.



a) What is the diameter of the semicircle?
b) What is the radius of the semicircle?
c) What is the length of the rectangle?

Inquire

1. What is the area of the rectangular part of the stadion?

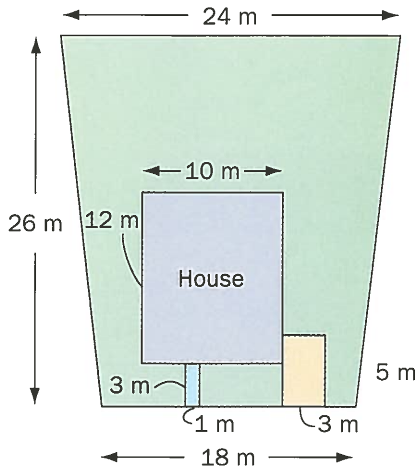
2. What is the area of the semicircle, to the nearest square metre?

3. What is the total area of the stadion, to the nearest square metre?

4. a) Which modern-day Olympic race is most like the first race in the ancient Olympics?
b) Name the Canadians who have won the gold medal in this modern-day race and state the years in which they won.

Example Landscaping

The diagram shows a house on a lot that is in the shape of a trapezoid. The parallel sides of the trapezoid measure 18 m and 24 m. The height of the trapezoid is 26 m. The dimensions of the house, the path to the front door, and the driveway are shown in the diagram. The remaining area of the lot is to be covered with sod. What is the area that needs to be covered with sod?



Solution

Calculate the total area of the house, path, and driveway. Then, subtract this area from the area of the lot.

Area of the house: $A_h = 12 \times 10$
$= 120$

Area of the path: $A_p = 3 \times 1$
$= 3$

Area of the driveway: $A_d = 5 \times 3$
$= 15$

Total Area: $120 + 3 + 15 = 138$

Area of the lot:
$$A_l = \frac{1}{2} \times (a+b) \times h$$
$$= \frac{1}{2} \times (18 + 24) \times 26$$
$$= \frac{1}{2} \times 42 \times 26$$
$$= 21 \times 26$$
$$= 546$$

Estimate
$20 \times 30 = 600$

Area to be covered with sod: $546 - 138 = 408$

Estimate
$550 - 150 = 400$

The area of the lot that needs to be covered is 408 m^2.

Practice

A

Calculate the area of each figure. Round answers to the nearest tenth of a square unit, if necessary.

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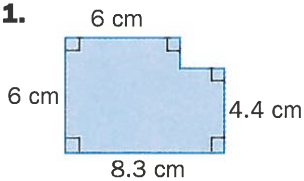

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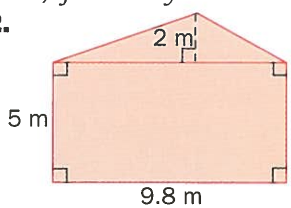

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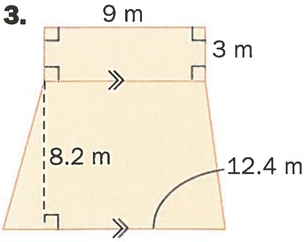

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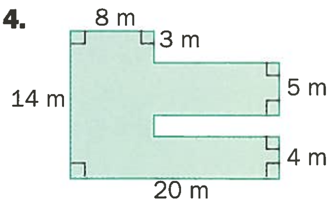

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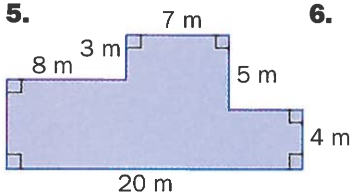

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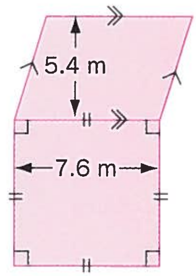

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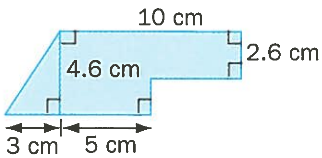

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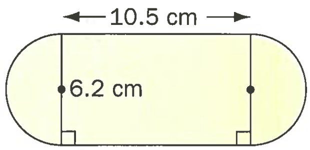

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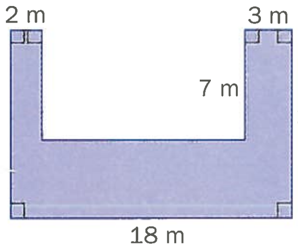

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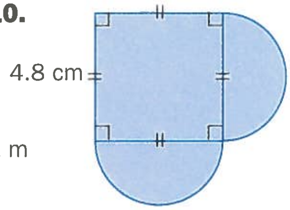

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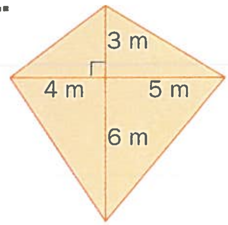

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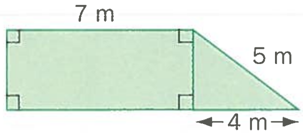
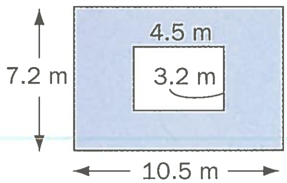


Applications and Problem Solving

Calculate the area of each shaded region. Round answers to the nearest tenth of a square unit, if necessary.

13.



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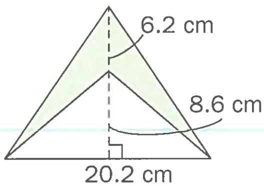

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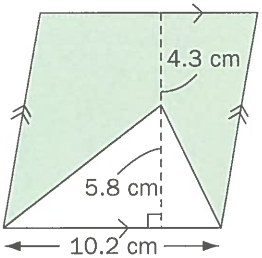

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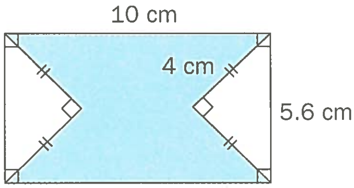

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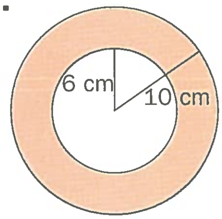

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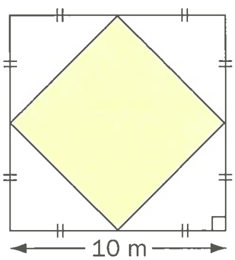
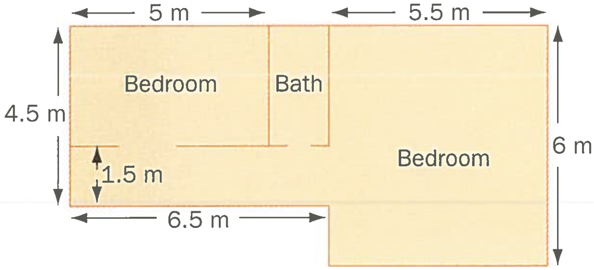


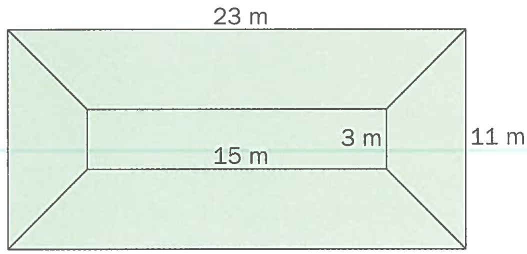
B

19. Carpeting The diagram shows the plan of the second floor of a new house. The hall and the two bedrooms are to be carpeted.



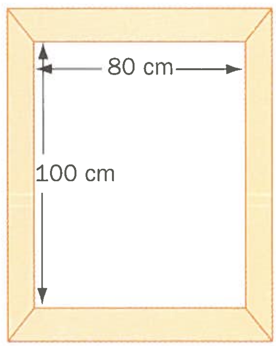
Calculate the area of carpet needed, in square metres.

20. Roofing Sandrine needs to replace the roofing material on her home. The roof has a rectangle on top and sloping trapezoids on each side. The dimensions of the roof are shown. Each trapezoid has a height of 4.5 m.

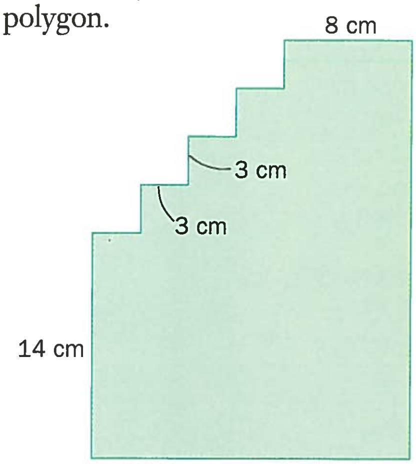


a) Calculate the area of the roof.
b) The trapezoids are to be covered with shingles. If a package of shingles covers 10 m^2, how many packages will be needed?

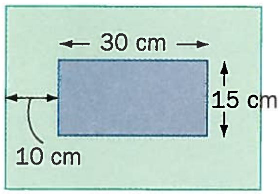
21. Carpentry Lamar works as a carpenter. He is framing a window that measures 100 cm by 80 cm. The frame is made up of four trapezoids. The frame is 10 cm wide. Find the total area of the four trapezoids.



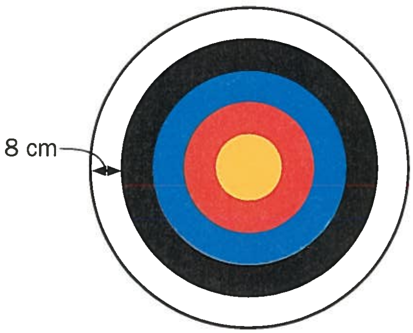
22. Each "step" of the polygon has a width of 3 cm and a height of 3 cm. Calculate the area of the polygon.



23. **Picture framing** The picture measures 30 cm by 15 cm. The mat around the picture is 10 cm wide. Calculate the area of the mat.



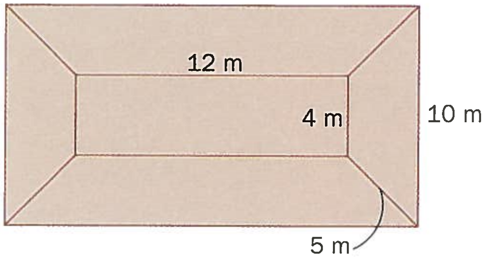
24. **Archery** One of the targets used in archery competitions has a diameter of 80 cm. The circle in the centre is gold and has a radius of 8 cm. The gold circle is surrounded by 4 rings, which are coloured red, blue, black, and white. Each of these 4 rings is 8 cm wide.



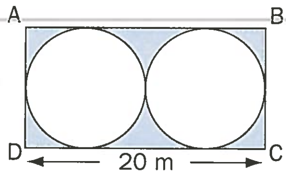
a) Calculate the area of the blue ring, to the nearest square centimetre.
b) What percent of the target is blue?

C

25. **House roof** A diagram of the roof of a house is shown. The top of the roof is a rectangle. The four trapezoids have the same height. Calculate the area of the roof.

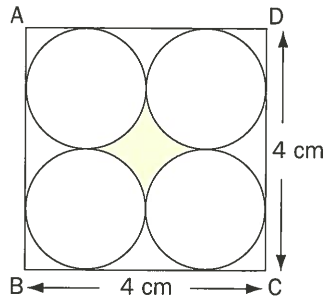


26. The two circles are identical, and figure ABCD is a rectangle.



a) Determine the total area of the shaded regions. Write your answer in terms of π.
b) Calculate the total area of the shaded regions, to the nearest square metre.

27. The four circles are identical, and figure ABCD is a square.



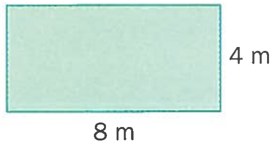
a) Determine the area of the shaded region. Write your answer in terms of π.
b) Calculate the area of the shaded region, to the nearest tenth of a square centimetre. Explain how you can judge whether your answer is reasonable.

INVESTIGATING MATH

Optimizing Perimeter and Area

1 Fencing on Four Sides

1. Fencing a garden a) A landscaper has 24 sections of fence, each 1 m long, to enclose a rectangular garden. The diagram models one of the possible rectangular shapes with a perimeter of 24 m. Sketch all the other rectangles that have whole-number dimensions and a perimeter of 24 m.
b) Copy and complete the table, or use a spreadsheet like the one shown.



Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1			24	
2			24	
3			24	
⋮				

	A	B	C	D	E
1	Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
2	1	1	=12–B2	24	=B2*C2
3	2	=B2+1	=12–B3	24	=B3*C3

c) What dimensions give the maximum area?
d) What is the maximum area?

2. Repeat question 1 for 32 sections of fence and a garden with a perimeter of 32 m.

3. Without restricting the dimensions to whole numbers of metres, use a table or spreadsheet to determine the maximum area of a garden with a perimeter of 30 m.

4. What shape of rectangle gives the maximum area for a given perimeter?

5. a) Describe a method for calculating the maximum area of a rectangular garden from its perimeter.
b) Use your method to calculate the maximum area of a rectangular garden with a perimeter of 34 m; 27.6 m; 36.8 m.

6. Fencing a pen a) A farmer wants to fence a rectangular animal pen with the minimum amount of fencing, so that the pen has an area of 36 m^2. The rectangle shown has an area of 36 m^2. Sketch the other rectangles that have whole-number dimensions and an area of 36 m^2.

b) Copy and complete the table, or use a spreadsheet like the one shown.

Rectangle	Width (m)	Length (m)	Area (m^2)	Perimeter (m)
1			36	
2			36	
3			36	
⋮				

	A	B	C	D	E
1	Rectangle	Width (m)	Length (m)	Area (m^2)	Perimeter (m)
2	1	1	=36/B2	36	=2*B2+2*C2
3	2	2	=36/B3	36	=2*B3+2*C3

c) What dimensions use the minimum amount of fencing?
d) What is the minimum perimeter?

7. Repeat question 6 for a pen with an area of 16 m^2.

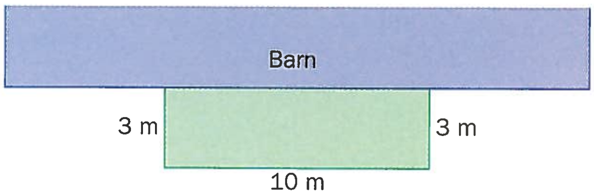
8. Without restricting the dimensions to whole numbers of metres, use a table or spreadsheet to determine the minimum perimeter of a pen with an area of 30.25 m^2.

9. What shape of rectangle gives the minimum perimeter for a given area?

10. a) Describe a method for calculating the minimum perimeter of a rectangular pen from its area.
b) Use your method to calculate the minimum perimeter of a rectangular pen with an area of 20.25 m^2; 27 m^2; 35 m^2. Round your answers to the nearest tenth of a metre, if necessary.

2 Fencing on Three Sides

1. Fencing a corral a) A rancher is adding a rectangular corral to the side of a barn. The barn will form one side of the rectangle, as shown in the diagram. The rancher has 16 m of fence. The rectangle shown uses 16 m of fence. Sketch all the other rectangular corrals that have whole-number dimensions and use 16 m of fence.



b) Copy and complete the table, or use a spreadsheet like the one shown on the following page.

Rectangle	Width (m)	Length (m)	Fence Used (m)	Area (m^2)
1			16	
2			16	
3			16	
⋮				

	A	B	C	D	E
1	Rectangle	Width (m)	Length (m)	Fence Used (m)	Area (m²)
2	1	1	=16–2*B2	16	=B2*C2
3	2	=B2+1	=16–2*B3	16	=B3*C3

c) What dimensions give the maximum area?
d) What is the maximum area?

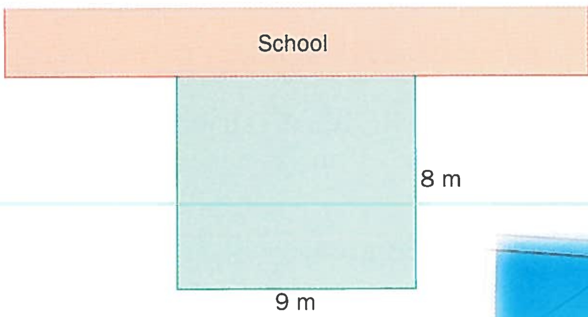
2. Repeat question 1 for a corral that uses 28 m of fence.

3. Without restricting the dimensions to whole numbers of metres, use a table or spreadsheet to determine the maximum area of a corral that uses 26 m of fence.

4. Describe the shape of a corral that gives the maximum area for a given length of fence.

5. a) Describe a method for calculating the maximum area of a rectangular corral from the length of fence used.
b) Use your method to calculate the maximum area of a rectangular corral, if the length of fence used is 30 m; 21.8 m; 26.4 m. Round your answers to the nearest tenth of a metre, if necessary.

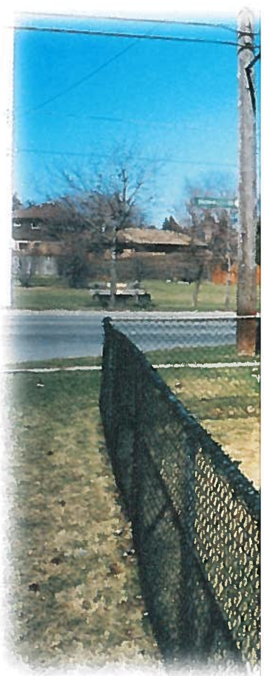
6. Fencing a playground a) An architect is adding a rectangular kindergarten playground to the side of a school. The school will form one side of the rectangle, as shown in the diagram. The area of the playground is to be 72 m^2. The rectangle shown has an area of 72 m^2. Sketch all the other rectangles that have whole-number dimensions and an area of 72 m^2.



b) Copy and complete the table, or use a spreadsheet like the one shown.

Rectangle	Width (m)	Length (m)	Area (m²)	Fence Used (m)
1			72	
2			72	
3			72	
⋮				

	A	B	C	D	E
1	Rectangle	Width (m)	Length (m)	Area (m²)	Fence Used (m)
2	1	1	=72/B2	72	=C2+2*B2
3	2	2	=72/B3	72	=C3+2*B3



c) What dimensions use the minimum length of fence?
d) What is the minimum length of fence?

7. Repeat question 6 for a playground with an area of 128 m^2.

8. Without restricting the dimensions to whole numbers of metres, use a table or spreadsheet to determine the minimum length of fence for a playground with an area of 60.5 m^2.

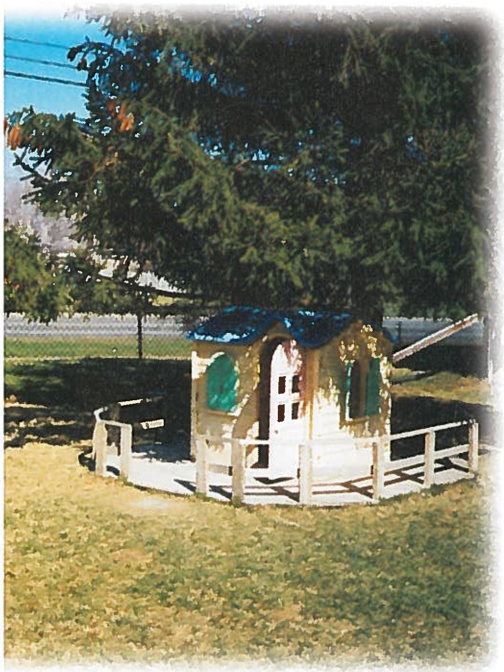
9. How are the side lengths related in a rectangular playground that uses the minimum length of fence for a given area?

10. a) Describe a method for calculating the minimum length of fence needed for a playground from its area.
b) Use your method to calculate the minimum length of fence for a playground with an area of 50 m^2; 112.5 m^2; 70 m^2. Round your answers to the nearest tenth of a metre, if necessary.

3 Applications and Problem Solving

1. **Corkboards** a) If you were designing a rectangular corkboard with an area of 1 m^2 and with a wooden frame, what shape would you make it so that the manufacturing cost was as small as possible? Explain.
b) If you were designing a corkboard with an area of 9 m^2 for use in a classroom, why might you not make the corkboard the same shape as in part a)?

2. **Animal pen** If an animal pen does not have to be rectangular, predict the shape that would require the minimum length of fence for a given area. Test your prediction and share your results with your classmates.

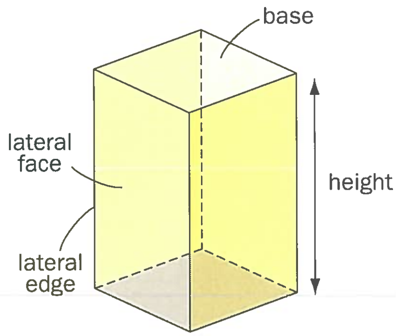


3. **Garden** Pose a problem involving the relationship between the perimeter and the area of a garden of a shape that you choose. Check that you can find the solution, and then have a classmate solve your problem.

4. Describe other applications in which it is important to know
a) the minimum perimeter of a rectangle for a given area
b) the maximum area of a rectangle for a given perimeter

INVESTIGATING MATH

Surface Areas and Volumes of Right Prisms



A **polyhedron** is a 3-dimensional object in which the faces are polygons. A **prism** is a polyhedron with two parallel congruent bases that are in the shape of a polygon. The lateral faces are parallelograms.

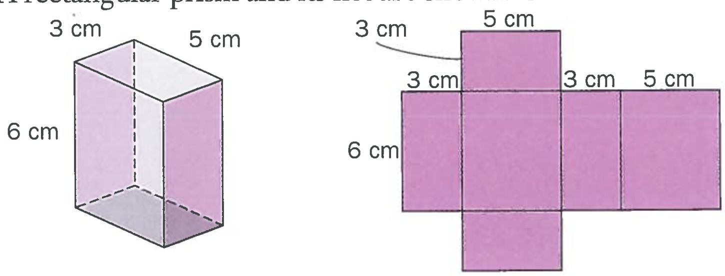
In a **right prism**, the lateral edges are perpendicular to the bases. So, in a right prism, the lateral faces are rectangles. The height of a right prism is the length of a lateral edge. Assume that all the prisms referred to in this book are right prisms.

Prisms are named according to the shape of the bases. For example, a prism with square bases is a square-based prism. A prism with triangular bases is a triangular prism.

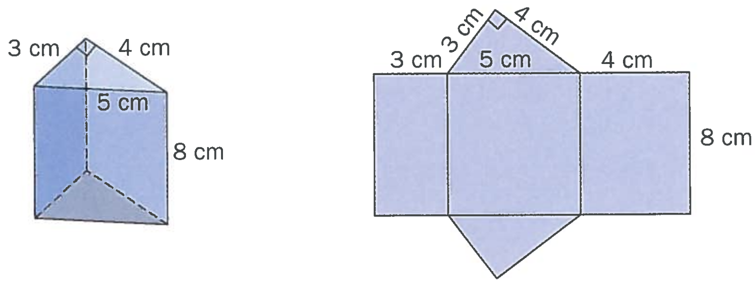
1 Modelling the Surface Area

The surface area of a prism is found by adding the areas of all the faces.

1. A rectangular prism and its net are shown. Calculate the surface area of the prism.

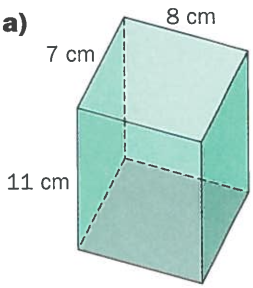


2. A triangular prism and its net are shown. Calculate the surface area of the prism.

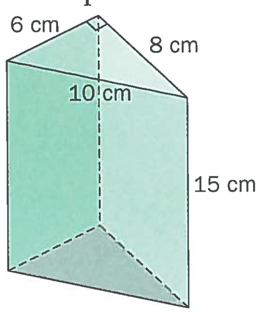


3. Calculate the surface area of each prism.

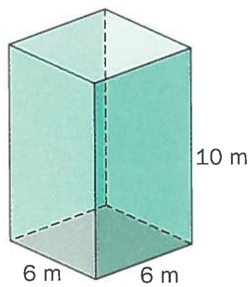
a)



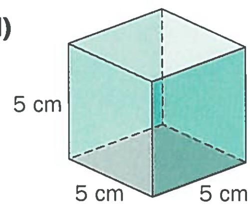
b)



c)



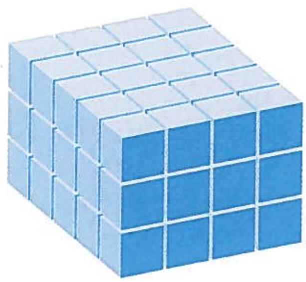
d)



2 Modelling the Volume

Volume is the amount of space that a figure occupies. Volume is measured in cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3).

1. The rectangular prism is made up of 1-cm cubes.



a) How many cubes make up the prism?

b) What is the volume of the prism?

c) What is the area of the base of the prism?

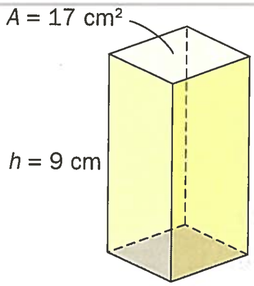
d) What is the height of the prism?

e) How does the product of the area of the base and the height compare with the volume of the prism?

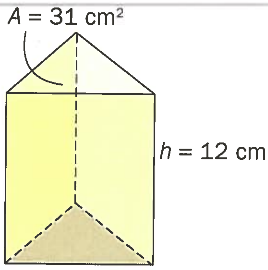
f) Write an equation that expresses the volume of a prism, V, in terms of the area of the base, B, and the height, h.

2. Given the area of the base and the height of each prism, calculate the volume.

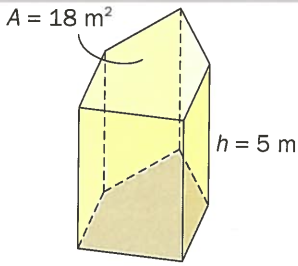
a)



b)

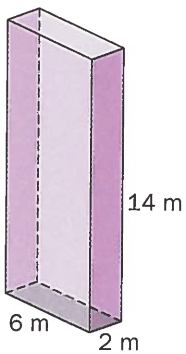


c)

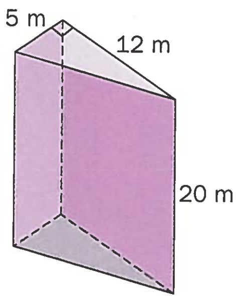


3. Calculate the volume of each prism.

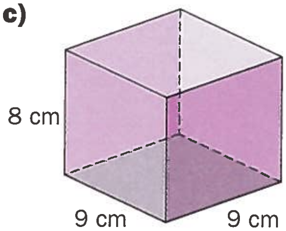
a)



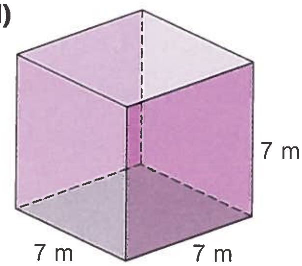
b)



c)

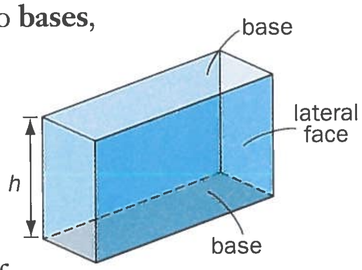


d)



9.2 Surface Area and Volume of a Prism

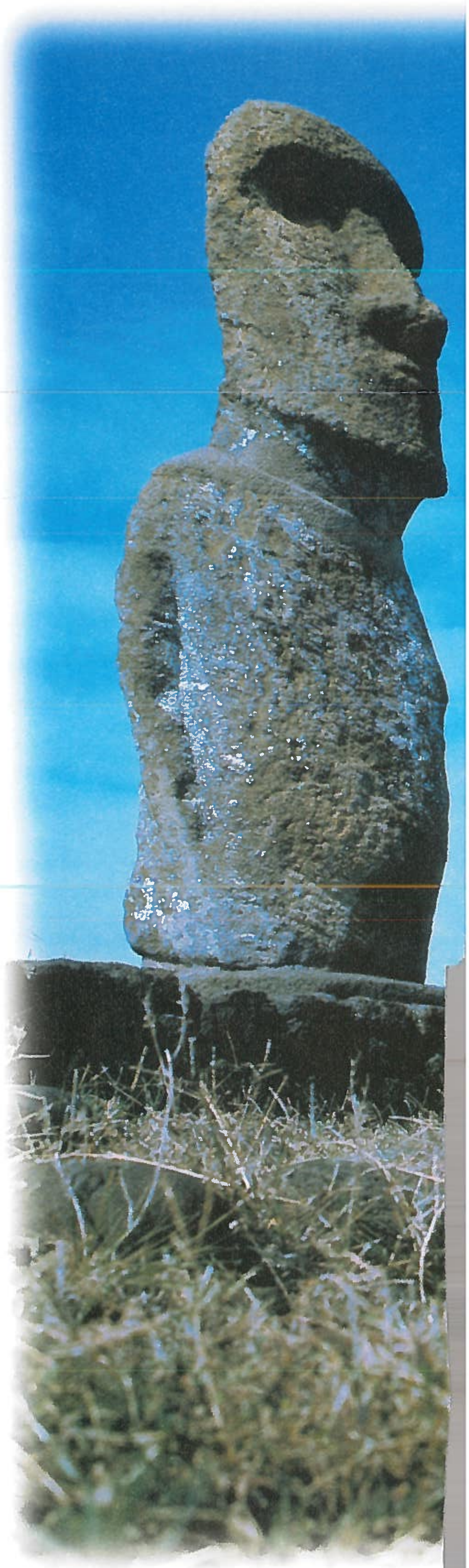
The right rectangular prism has two **bases**, which are parallel, congruent polygons. The **height** of the prism, h, is the perpendicular distance between the two bases.



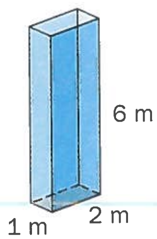
The **lateral faces**, the faces that are not bases, are rectangles. The **lateral area** of a prism is the sum of the areas of the lateral faces. The **surface area** is the sum of the areas of the lateral faces and the two bases.

Explore: Develop a Formula

Easter Island, also known as Rapa Nui, belongs to Chile. The island is located in the Pacific Ocean, about 3700 km west of Chile. Easter Island is important to archaeologists because of the gigantic statues found there. The statues consist of large heads with elongated noses and ears.



The construction of the statues started about 1800 years ago. They were carved from prisms made of volcanic rock. Many of the statues were carved from rectangular prisms with bases measuring 1 m by 2 m and with a height of 6 m. For one of the rectangular prisms, calculate the area of



a) each base **b)** each lateral face

Inquire

1. What is the total area of the bases?

2. What is the lateral area?

3. What is the surface area of the prism?

4. What is the perimeter of a base?

5. Multiply the perimeter of a base by the height.

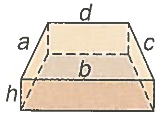
6. How do the answers to questions 2 and 5 compare? Explain why they compare in this way.

7. Write a formula for the lateral area, $L.A.$, in terms of the perimeter of a base, P, and the height, h.

8. Write a formula for the surface area, $S.A.$, in terms of
a) $L.A.$ and the area of each base, B
b) P, h, and B

You can find the surface area of a prism by adding the areas of all the faces. However, a formula can be developed to make the process much faster. To find the formula for the surface area of a prism, first find a formula for the lateral area.

In the prism shown, a, b, c, and d are the measures of the sides of the base, and h is the height. The perimeter of the base, P, is the sum of its side lengths, $a + b + c + d$.

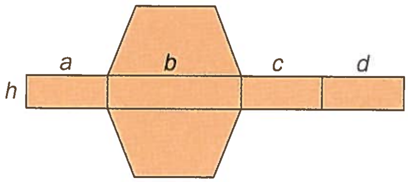


Use the net to write the formula for the lateral area, $L.A.$

Add the areas of the lateral faces: $L.A. = ah + bh + ch + dh$

Remove the common factor: $L.A. = h(a + b + c + d)$

Substitute P for $a + b + c + d$: $L.A. = hP$ or Ph



The bases of a prism are congruent polygons, so they have the same area, B.

The surface area, $S.A.$, of a prism is found by adding the lateral area and the total area of the two bases, $2B$.

So, $S.A. = Ph + 2B$

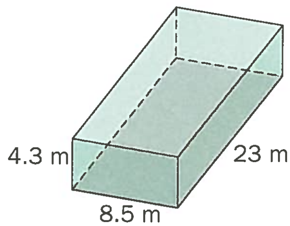
The formula $S.A. = Ph + 2B$ can be used to find the surface area of any right prism.

Example 1 *Anik E-2* Satellite

Canada's *Anik E-2* is a communications satellite that provides services to television networks and telephone systems. The satellite approximates a rectangular prism with a length of 23 m, a width of 8.5 m, and a height of 4.3 m. Calculate the surface area of the satellite.

Solution

Draw a diagram.



Calculate the perimeter of a base, P:

$$\begin{aligned} P &= 2(l + w) \\ &= 2(23 + 8.5) \\ &= 2(31.5) \\ &= 63 \end{aligned}$$

Calculate the area of a base, B:

$$\begin{aligned} B &= (23)(8.5) \\ &= 195.5 \end{aligned}$$

Estimate

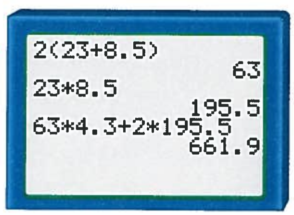
$20 \times 10 = 200$

Calculate the surface area, $S.A.$:

$$\begin{aligned} S.A. &= Ph + 2B \\ &= (63)(4.3) + 2(195.5) \\ &= 661.9 \end{aligned}$$

Estimate

$60 \times 4 + 2 \times 200$
$= 240 + 400$
$= 640$

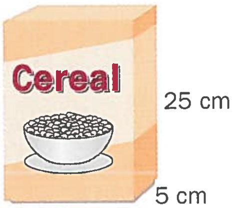


The surface area of the satellite is 661.9 m^2.

Example 2 Volumes of Packages

Calculate each volume.

a)



b)

Solution

The volume of a prism is the area of its base times its height.

a) The cereal box is a rectangular prism.

$V = B \times h$
$= [(19)(5)](25)$
$= 2375$

Estimate

$(20)(5)(25) = 2500$

The volume of the box is 2375 cm^3.

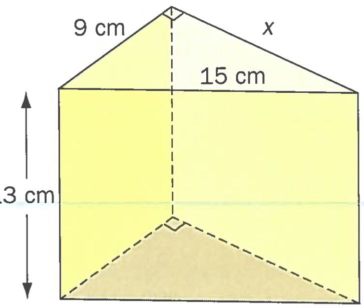
b) The cheese box is a triangular prism. The base and height of the triangle are 10 cm each.

$V = B \times h$
$= [\frac{1}{2}(10)(10)](2)$
$= (50)(2)$
$= 100$

The volume of the box is 100 cm^3.

Example 3 Triangular Prism

The triangular prism has a height of 13 cm and a right triangular base, with one leg measuring 9 cm and a hypotenuse of 15 cm. Calculate the volume and surface area of the prism.



Solution

Use the Pythagorean Theorem to find the length, x, of the third side of the triangular base.

$$x^2 + 9^2 = 15^2$$
$$x^2 + 81 = 225$$
$$x^2 = 225 - 81$$
$$x^2 = 144$$
$$x = \sqrt{144}$$
$$x = 12$$

Find the area of a base.

$$B = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2}(12)(9)$$
$$= 54$$

Now, find the volume.

$V = B \times h$
$= 54 \times 13$
$= 702$

Estimate

$50 \times 13 = 650$

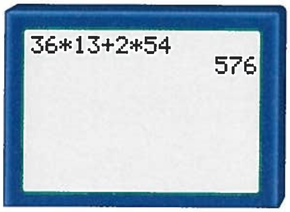
$S.A. = Ph + 2B$

The perimeter of the base is $9 + 12 + 15$ or 36.

So, $S.A. = (36)(13) + 2(54)$
$= 576$

Estimate

$40 \times 10 + 100 = 500$

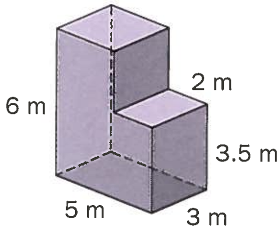


The volume of the prism is 702 cm^3, and the surface area is 576 cm^2.

A composite solid is made up of 2 or more prisms joined together.

Example 4 Composite Solid

Find the volume of the composite solid.



Solution

The solid is composed of a smaller prism, where $l = 3$ m, $w = 2$ m, and $h = 3.5$ m, and a larger prism, where $l = 3$ m, $w = 3$ m, and $h = 6$ m.

Volume of smaller prism $= (3)(2)(3.5)$
$= 21$

Volume of larger prism $= (3)(3)(6)$
$= 54$

Total volume $= 21 + 54$
$= 75$

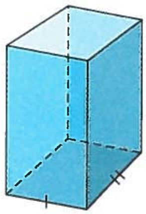
The volume of the composite solid is 75 m^3.

Practice

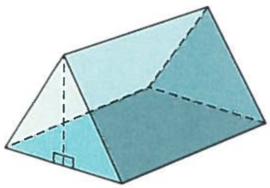
A

Name each prism.

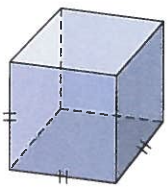
1.



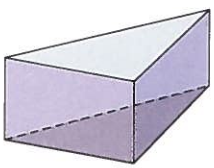
2.



3.

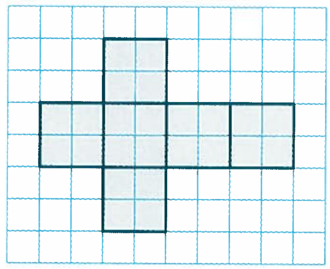


4.

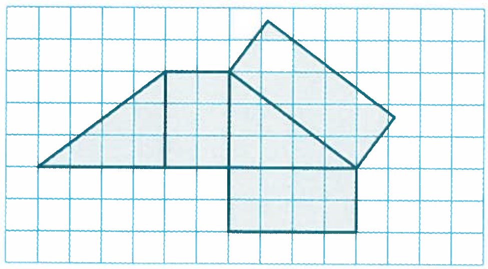


Which prisms in questions 1–4 can be formed from each of the following nets?

5.

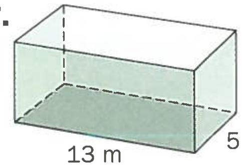


6.



Estimate the surface area of each prism. Then, calculate it, to the nearest square centimetre or square metre.

7.



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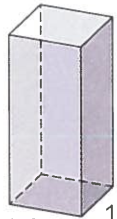
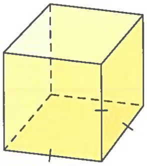
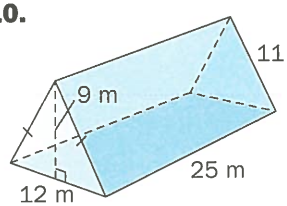


Calculate each surface area.

9.

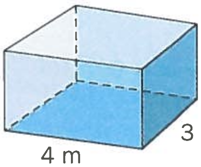


10.

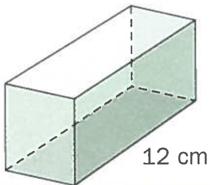


Estimate, then calculate the volume of each rectangular prism.

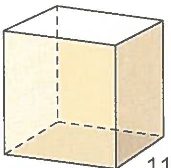
11.



12.



13.



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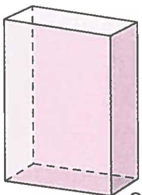
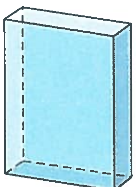
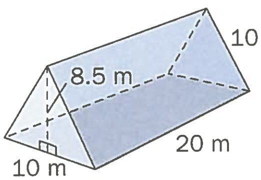


Calculate the surface area and volume of each prism, to the nearest square or cubic unit.

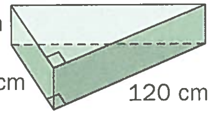
15.



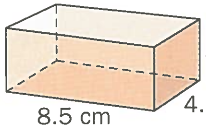
16.



17.

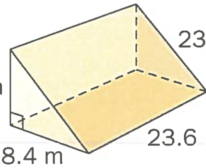


18.



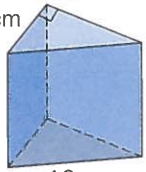
19.

20.



Calculate the surface area and volume of each prism.

21.



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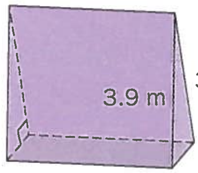
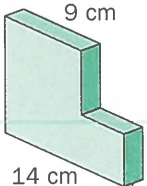
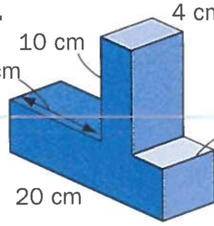


Calculate the surface area and volume of each composite solid.

23.



24.



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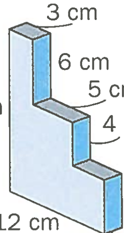

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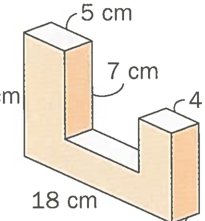
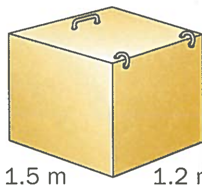


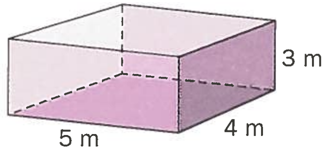
Applications and Problem Solving

27. Garbage bin A covered garbage bin is to be built so that it measures 1.5 m by 1.2 m by 1.0 m. How much plywood will it take to build the garbage bin?



B

28. Painting **a)** Calculate the surface area of this room.

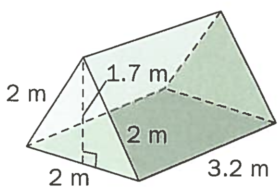


b) One 4-L can of paint will cover 36 m^2. If you want to give the ceiling and walls of the room two coats of paint, how many 4-L cans will you need? What assumptions have you made? Explain how you can judge whether your answer is reasonable.

29. Composter **a)** The dimensions of the square base of a composter are 1 m by 1 m. Its height is 0.65 m. It is a prism with a top, a bottom, and 4 sides. Calculate its surface area.
b) The cost of material to build this composter is \$9.98/m^2. What is the total cost of the material?
c) If a town has set aside \$1 250 000 for the materials to build these composters, how many composters can be built?

30. **a)** A prism has a height of 10 cm. Find its surface area if the dimensions of the base are 8 cm by 2 cm.
b) Draw and label a diagram of the prism on dot paper or centimetre grid paper.
c) What is the name of the prism?

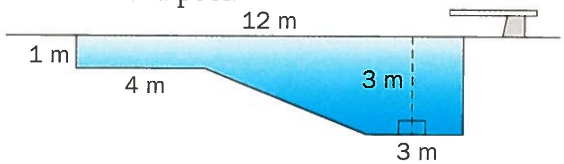
31. Tent How many cubic metres of air does the tent contain?



32. Office tower The Toronto-Dominion Bank Tower is one of Canada's tallest buildings. It approximates a rectangular prism with a height of 168 m and a base of 60 m by 36 m. Calculate the total area of the outside walls and the roof.

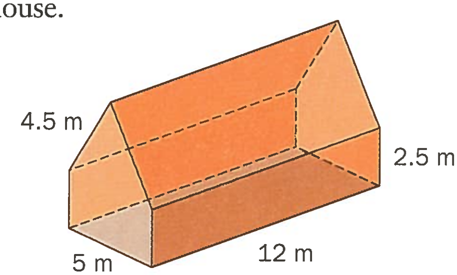
33. The surface area of a cube is 216 cm^2. What are the dimensions of this cube?

34. Swimming pool The diagram shows the side view of a pool.



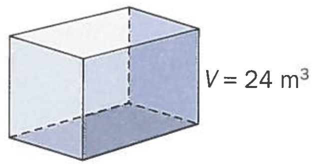
a) The pool is 5 m wide. Calculate its volume.
b) A pump can drain water from the pool at 0.3 m^3/min. How long does it take to drain the pool?

35. Greenhouse The top and sides of a greenhouse are made of plastic. Calculate the amount of plastic needed to construct the greenhouse.



C

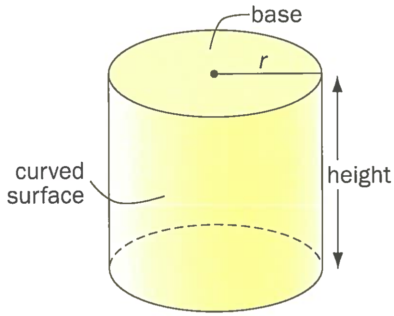
36. Garden shed A garden storage shed is to be built in the shape of a rectangular prism before the roof is added. The volume of the shed before the roof is put on is 24 m^3. What are the most appropriate dimensions for the rectangular prism? Explain and justify your reasoning.



37. Calculate the surface area and volume of the interior of your classroom.

38. Write a problem that requires the calculation of the surface area and volume of a prism. Have a classmate solve your problem.

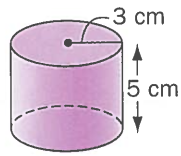
Surface Area and Volume of a Cylinder



A **cylinder** is a three-dimensional object with two bases, which are congruent circles, and a curved surface connecting the two bases. The **height** of a cylinder, h, is the perpendicular distance between the two bases.

The surface area of a cylinder is found by adding the areas of the bases and the area of the curved surface.

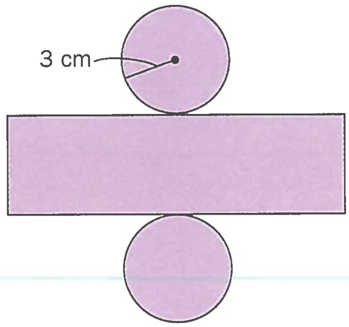
1 Modelling the Surface Area



A cylinder and its net are shown. The area of a base is $\pi \times r^2$. In terms of π, the area of a base is $\pi \times 3^2$, or 9π cm^2.

1. What is the sum of the areas of the two bases, in terms of π?

2. In the net, the curved surface of the cylinder becomes a rectangle.
a) How is the height of the rectangle related to the height of the cylinder?
b) What is the height of the rectangle?



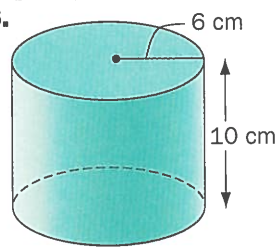
3. a) How is the length of the rectangle related to the circumference of the circular base of the cylinder?
b) What is the circumference of the circular base, in terms of π?
c) What is the length of the rectangle, in terms of π?

4. a) How is the area of the rectangle related to the area of the curved surface of the cylinder? Explain.
b) What is the area of the rectangle, in terms of π?
c) What is the area of the curved surface of the cylinder, in terms of π?

5. What is the surface area of the cylinder
a) in terms of π? **b)** to the nearest square centimetre?

Draw the nets and determine the surface area of each cylinder. Express your answers in terms of π.

6.



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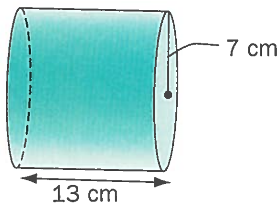

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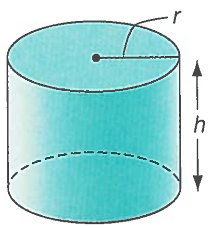
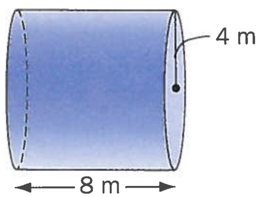


9. Use your formula from question 8 to find the surface area of the given cylinder, to the nearest square metre.



2 Modelling the Volume

For this investigation, you will need an empty cylindrical tin can. Make sure that there are no sharp edges. You will also need a graduated cylinder.

1. Use a graduated cylinder to fill the can with water. Record the volume of water in the can.

2. What is the volume of the can, in cubic centimetres? Explain.

3. a) Measure the inside diameter of the can, in centimetres.

b) What is the inside radius of the can?

4. Calculate the area of the inside base of the can, in square centimetres.

5. Measure the inside height of the can, in centimetres.

6. Find the product of the area of the inside base of the can and the height of the can.

7. How do your answers to questions 2 and 6 compare? Share your findings with your classmates.

8. Use your results to write a formula for the volume of a cylinder, V, in terms of the area of the base, B, and the height, h.

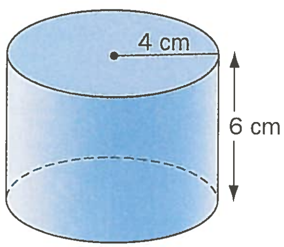
9. Write a formula for the volume of a cylinder, V, in terms of the radius of a base, r, and the height of the cylinder, h.

Use your formula from question 9 to determine the volume of each of the following cylinders

a) *in terms of* π

b) *to the nearest cubic centimetre*

10.



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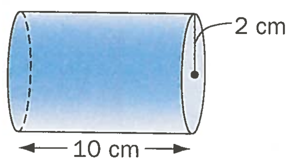
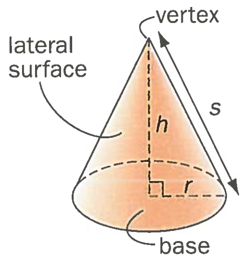


INVESTIGATING MATH

Surface Area and Volume of a Cone

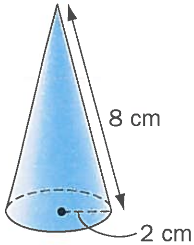
A **cone** is a three-dimensional object with a circular base and a curved lateral surface, which extends from the base to a point called the **vertex**. The **height**, or **altitude**, of a cone is the perpendicular distance from the vertex to the base. The **slant height**, s, of a cone is the distance from the vertex to a point on the edge of the base.

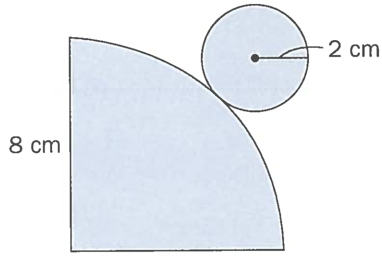


The surface area of a cone can be found by adding the area of the base and the area of the lateral surface.

1 Modelling the Surface Area

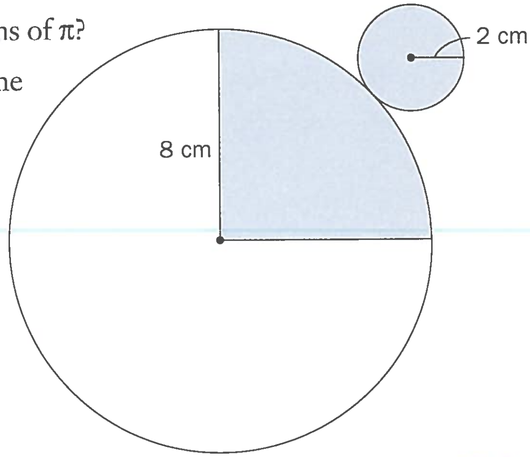
A cone and its net are shown. The cone has a base of radius 2 cm and a slant height of 8 cm.





In the net, the lateral surface of the cone is called a **sector**. It is a part of the large circle whose radius is the slant height of the cone, which is 8 cm. In terms of π, the circumference of the large circle is $2 \times \pi \times r$, which is $2 \times \pi \times 8$, or 16π cm.

1. What is the circumference of the base of the cone, in terms of π?



2. What fraction of the circumference of the large circle is the circumference of the base?

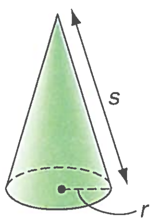
3. What is the area of the large circle, in terms of π?

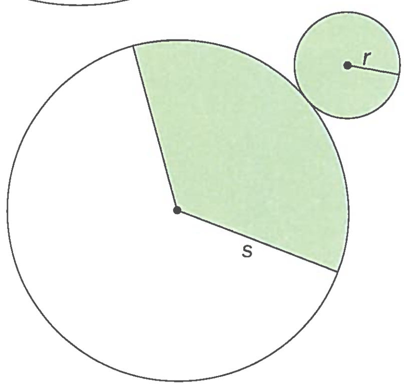
4. a) The area of the sector is the same fraction of the area of the large circle as you found in question 2. Explain why.
b) What is the area of the sector, in terms of π?

5. What is the area of the base of the cone, in terms of π?

6. What is the surface area of the cone,
a) in terms of π?
b) to the nearest square centimetre?

7. Use questions 1 to 6a) as a guide to complete the following statements for a general cone with a slant height, s, and a base of radius r. The circumference of the large circle is $2 \times \pi \times s$ or $2\pi s$.





a) The circumference of the base of the cone is ____.

b) The circumference of the base is $\frac{\_\_\_\_}{\_\_\_\_}$ of the circumference of the large circle.

c) The area of the large circle is ____.

d) The area of the sector is $\frac{\boxed{}}{\boxed{}}$ × ____ or ____.

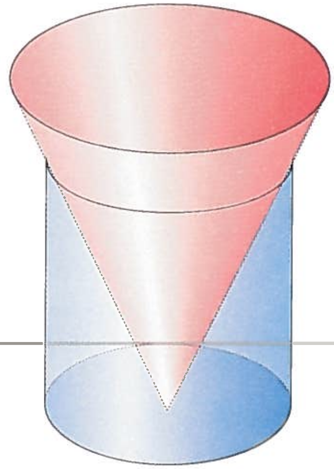
e) The area of the lateral surface of the cone is ____.

f) The area of the base of the cone is ____.

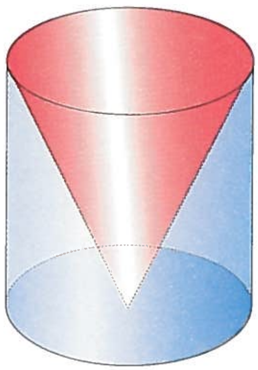
g) The surface area of the cone is ____.

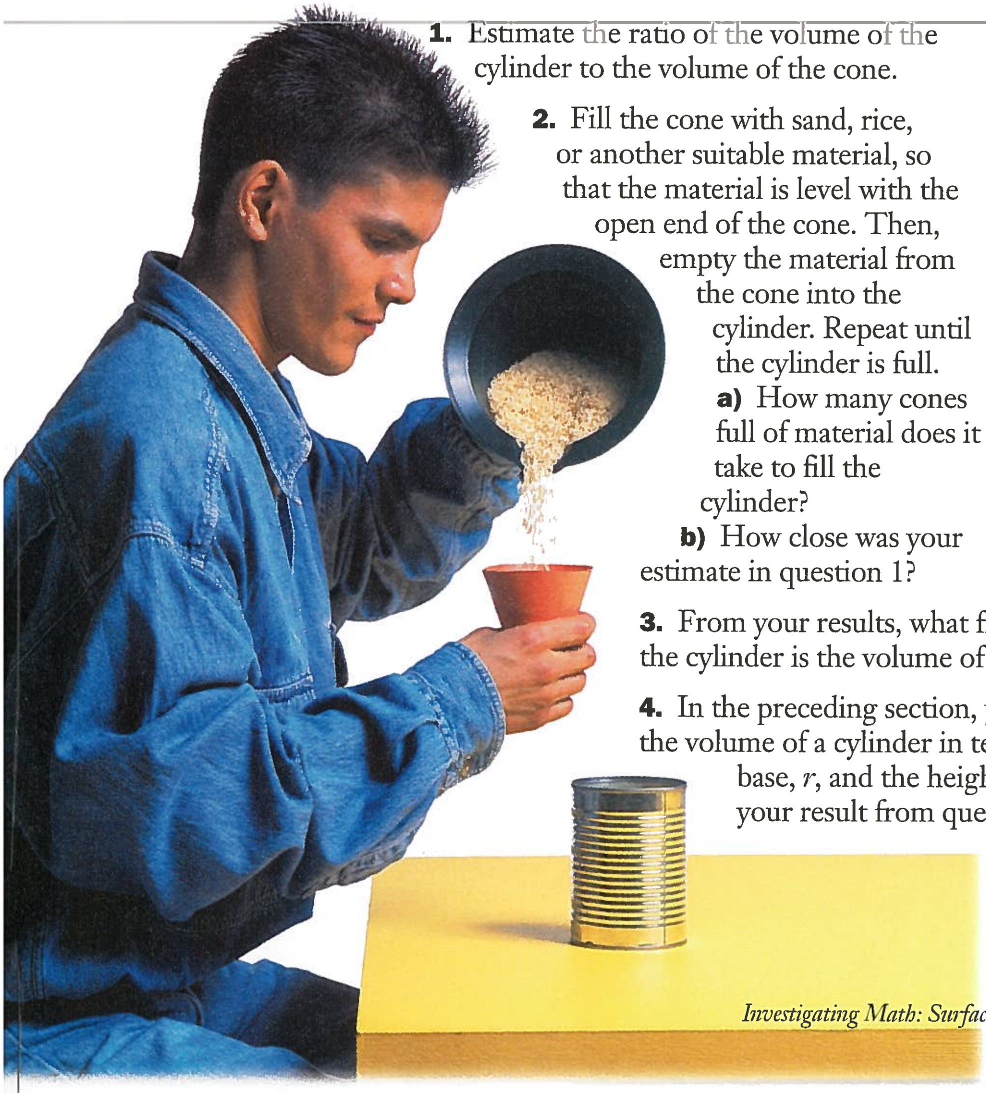
2 Modelling the Volume

For this investigation, you will need an empty cylindrical tin can. Make sure that there are no sharp edges. You will also need construction paper, scissors, tape, and sand, rice, or some other suitable material.



Use a piece of construction paper to form a cone whose tip just touches the bottom of the tin can. The curved surface of the cone should just touch the inside surface of the top of the can. Tape the cone and cut it so that its height is the same as the height of the cylinder.





1. Estimate the ratio of the volume of the cylinder to the volume of the cone.

2. Fill the cone with sand, rice, or another suitable material, so that the material is level with the open end of the cone. Then, empty the material from the cone into the cylinder. Repeat until the cylinder is full.

a) How many cones full of material does it take to fill the cylinder?

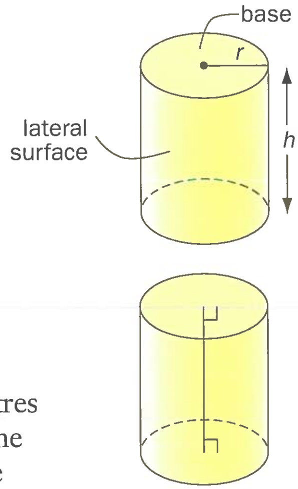
b) How close was your estimate in question 1?

3. From your results, what fraction of the volume of the cylinder is the volume of the cone?

4. In the preceding section, you wrote a formula for the volume of a cylinder in terms of the radius of the base, r, and the height, h. Use this formula and your result from question 3 to write a formula for the volume, V, of a cone in terms of the radius of the base, r, and the height, h.

9.3 Surface Area and Volume of a Cylinder and a Cone

Nearly perfect cylindrical shapes can be found in nature. An example is this part of the trunk of one of Canada's biggest trees. It is a Douglas fir that grows near Port Renfrew on Vancouver Island.

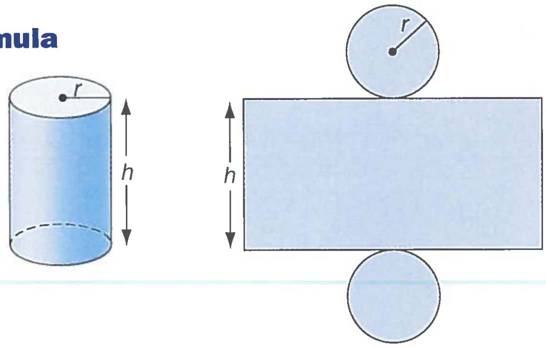


Recall that a cylinder is a three-dimensional figure, with two congruent circular bases connected by a curved lateral surface. The height of a cylinder, h, is the perpendicular distance between the two bases.

In a right cylinder, the segment joining the centres of the bases is perpendicular to the bases. Assume that all the cylinders referred to in this book are right cylinders.

Explore: Develop a Formula

In the diagram of the cylinder, each base has radius r, and the height is h. The net of the cylinder is also shown.



a) How does the surface area of the net compare with the surface area of the cylinder? Explain.
b) What shape in the net has the same area as the curved lateral surface of the cylinder?

Inquire

1. a) What name is given to the distance around the top of the cylinder?
b) Write a formula for this distance in terms of r.

2. a) How does the length of the rectangle compare with the distance around the top of the cylinder?
b) What is the length of the rectangle in terms of r?

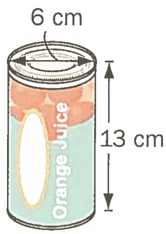
3. Write a formula for the area of the rectangle in terms of r and h.

4. Write a formula for the combined area of the two bases.

5. Use the results of questions 3 and 4 to write a formula for the surface area of the cylinder.

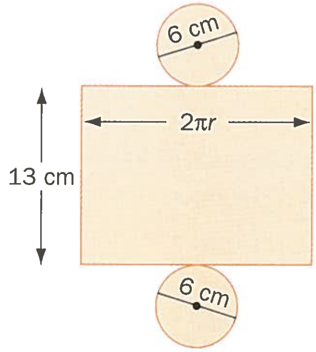
Example 1 Surface Area of a Juice Can

The cylindrical juice can has a diameter of 6 cm and a height of 13 cm. Find the surface area, to the nearest square centimetre.



Solution

The formula for the lateral area, *L.A.*, of a cylinder is $L.A. = 2\pi rh$.
The formula for the surface area, *S.A.*, of a cylinder is $S.A. = 2\pi r^2 + 2\pi rh$.



$$S.A. = 2\pi r^2 + 2\pi rh$$
$$= 2(\pi)(3)^2 + 2(\pi)(3)(13)$$
$$\doteq 302$$

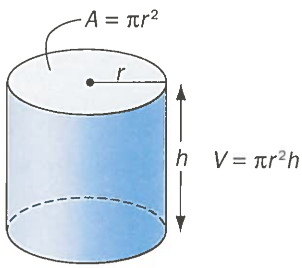
Estimate

$(2)(3)(10) + (2)(3)(40)$
$= 60 + 240$
$= 300$

2 [×] [π] [×] 3 [x^2] [+] 2 [×] [π] [×] 3 [×] 13 [=] 301.5928947

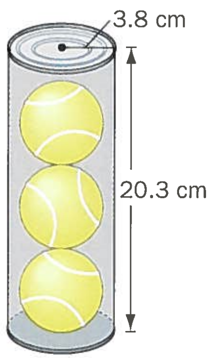
The surface area of the juice can is 302 cm^2, to the nearest square centimetre.

As with a prism, the volume of a cylinder can be found by multiplying the area of the base and the height.



Example 2 Volume of a Tennis-Ball Can

Find the volume of the tennis-ball can, to the nearest cubic centimetre.

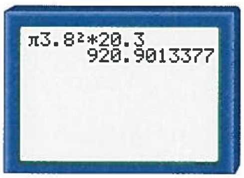


Solution

$$V = \pi r^2 h$$
$$= (\pi)(3.8)^2(20.3)$$
$$\doteq 921$$

Estimate

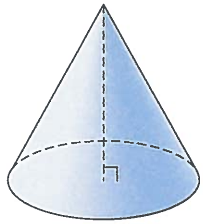
$(3)(4)^2(20) = 960$



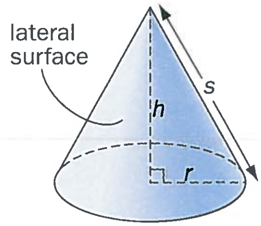
[π] [×] 3 [.] 8 [x^2] [×] 20 [.] 3 [=] 920.9013377

The volume of the tennis-ball can is 921 cm^3, to the nearest cubic centimetre.

Recall that a **cone** is a three-dimensional figure, with a circular base connected by a curved lateral surface to a point called the vertex.



In a **right cone**, the segment joining the centre of the base to the vertex is perpendicular to the base. Assume that all the cones referred to in this book are right cones.



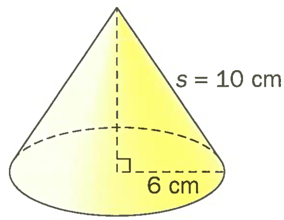
The height, h, of a cone is the perpendicular distance from the vertex to the base. The slant height, s, is the distance from the vertex to a point on the edge of the base.

The surface area, $S.A.$, of a cone is the sum of the lateral area, $L.A.$, and the area of the circular base.

The formula for the lateral area of a cone is $L.A. = \pi rs$.
The formula for the surface area of a cone is $S.A. = \pi r^2 + \pi rs$.

Example 3 Surface Area of a Cone

Calculate the surface area of the cone, to the nearest square centimetre.

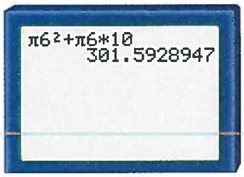


Solution

$S.A. = \pi r^2 + \pi rs$
$= (\pi)(6)^2 + (\pi)(6)(10)$
$\doteq 302$

Estimate

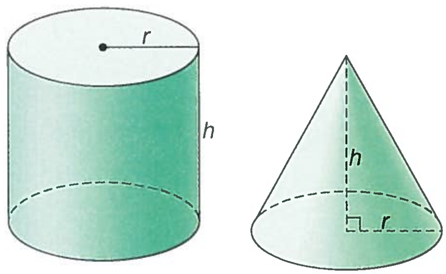
$(3)(40) + (3)(6)(10)$
$= 120 + 180$
$= 300$



π × 6 x² + π × 6 × 10 = 301.5928947

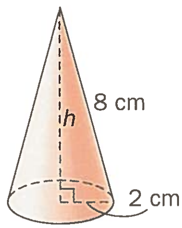
The surface area of the cone is 302 cm^2, to the nearest square centimetre.

The volume of a cone is one third of the volume of a cylinder with the same base and height.



Example 4 Volume of a Cone

Find the volume of the cone, to the nearest cubic centimetre.

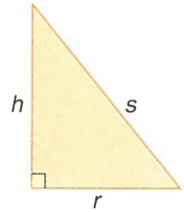


Solution

Use the Pythagorean Theorem to find the height, h.

$$h^2 + r^2 = s^2$$
$$h^2 + 2^2 = 8^2$$
$$h^2 + 4 = 64$$
$$h^2 = 60$$
$$h = \sqrt{60}$$

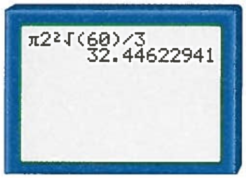
We could evaluate $\sqrt{60}$ as about 7.7, but $\sqrt{60}$ is the exact value.



$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{\pi(2)^2(\sqrt{60})}{3}$$
$$\doteq 32$$

Estimate

$$\frac{3 \times 4 \times 8}{3} = 32$$



π × 2 x² × 60 √x ÷ 3 =

32.4462294 1

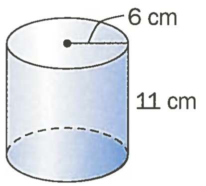
The volume of the cone is 32 cm^3, to the nearest cubic centimetre.

Practice

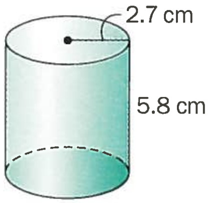
A

Calculate each surface area, to the nearest square centimetre.

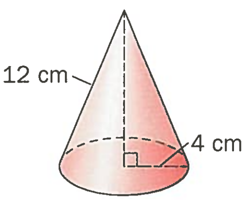
1.



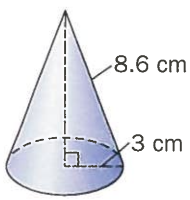
2.



3.

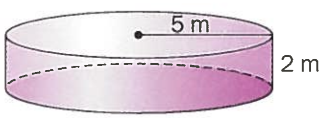


4.

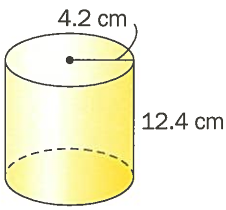


Estimate, and then calculate each volume, to the nearest cubic unit.

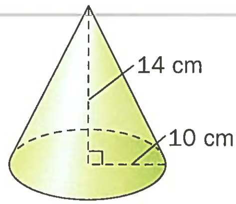
5.



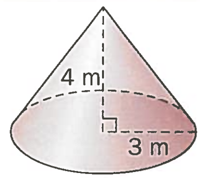
6.



7.

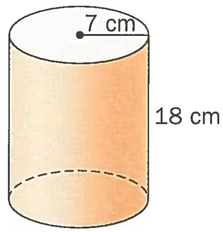


8.

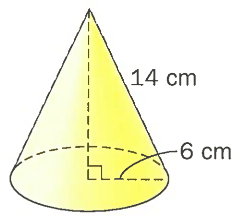


Calculate the lateral area, to the nearest square centimetre.

9.

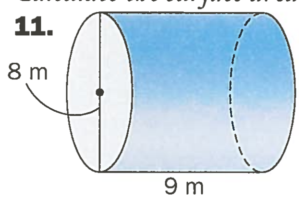


10.

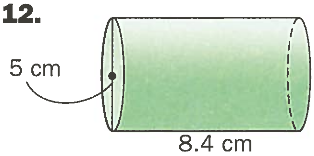


Calculate the surface area, to the nearest square unit.

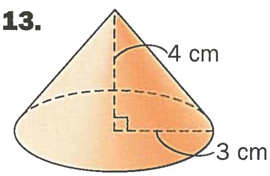
11.



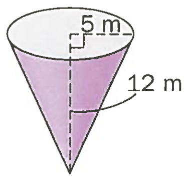
12.



13.

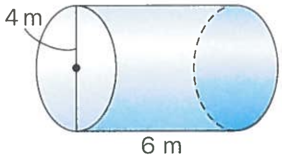


14.

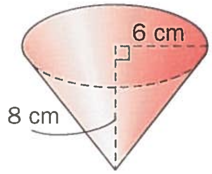


Calculate the surface area and volume of the following, to the nearest square or cubic unit.

15.



16.



Applications and Problem Solving

17. **Paper cup** A paper cup at a water dispenser has a conical shape. The radius of the cup is 3 cm, and its height is 6 cm.

a) Find the slant height of the cup, to the nearest tenth of a centimetre.

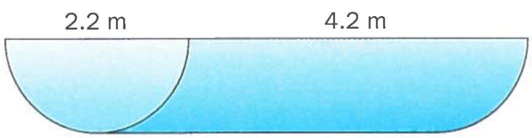
b) Calculate how much paper, to the nearest square centimetre, is required to make the cup.

B

18. **Canadarm** Canada's contribution to the International Space Station is a Space Station Remote Manipulator System. It is a larger version of the Canadarm that is used on the space shuttle. The arm is a cylinder that is 17 m long with a diameter of 40 cm. What is the lateral area, to the nearest square centimetre?

19. **Asphalt compactor** An asphalt compactor, used for paving highways, has two large cylindrical drums, one at the front and one at the rear. Each drum has a radius of 1 m and a width of 2.2 m. Calculate the area of asphalt that a single revolution of a drum compacts, to the nearest tenth of a square metre.

20. **Water tank** A hobby club runs remote-controlled boats in a tank that is shaped as shown.

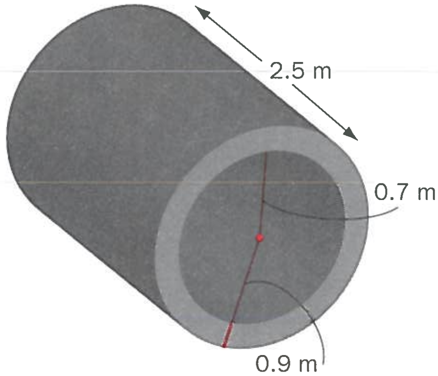


What volume of water can this tank hold, to the nearest cubic metre?

21. **Container** A cone-shaped container has a diameter of 20 cm and a height of 20 cm. How many litres of water will the cone hold, to the nearest litre?

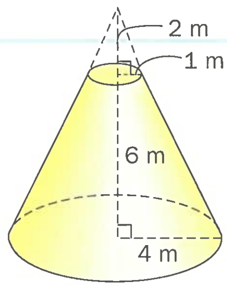
C

22. **Storm sewers** The dimensions for a section of concrete pipe used to make storm sewers is shown in the diagram.



Calculate the volume of concrete that is needed to make this section of pipe, to the nearest tenth of a cubic metre.

23. **Frustum of a cone** The frustum of a cone is the part that remains after the top portion has been cut off by a plane parallel to the base. A frustum of a cone is shown. Calculate the volume, to the nearest cubic metre.



24. **Shipping carton** Paper towels are sold in packages of 2 rolls. Each roll is a cylinder with a height of 30 cm and an outer diameter of 12 cm.

a) Design a shipping carton that will contain 12 packages of paper towels.

b) The inner diameter of each roll is 4 cm. How much wasted space is there in the carton?

25. Write a problem that requires the calculation of the surface area and volume of a cylinder or a cone. Have a classmate solve your problem.

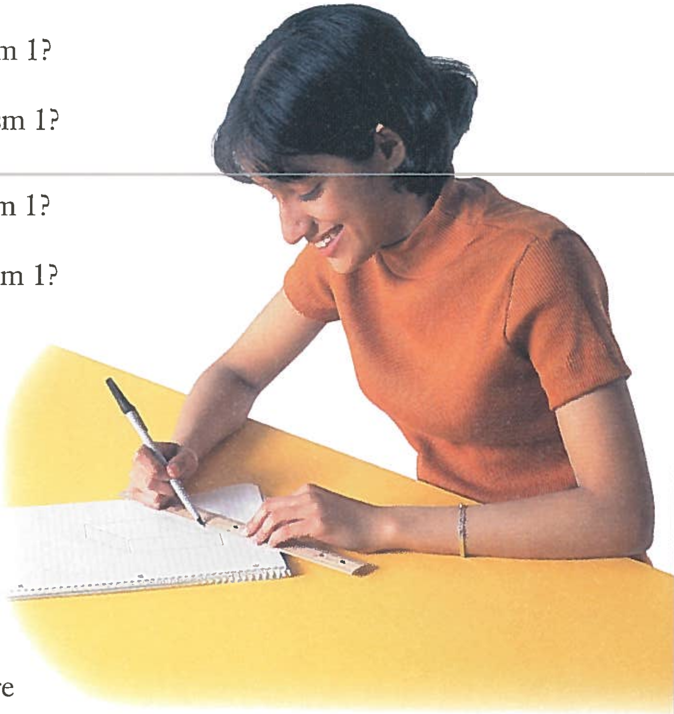
Varying the Dimensions of Prisms and Cylinders

1 Volumes and Surface Areas of Prisms

1. Sketch the three rectangular prisms with the dimensions shown, or build models using linking cubes. Then, copy and complete the table, or complete the calculations using a spreadsheet.

Prism	Width (cm)	Length (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	2	3		
2	2	4	6		
3	3	6	9		

2. How many times as great are
a) the dimensions of prism 2 as the dimensions of prism 1?
b) the volume of prism 2 as the volume of prism 1?
c) the surface area of prism 2 as the surface area of prism 1?



3. How many times as great are
a) the dimensions of prism 3 as the dimensions of prism 1?
b) the volume of prism 3 as the volume of prism 1?
c) the surface area of prism 3 as the surface area of prism 1?

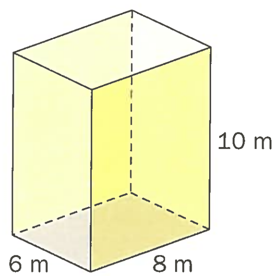
4. How does the volume of a prism change when all dimensions are increased by a factor of
a) 2? **b)** 3?

5. How does the surface area of a prism change when all dimensions are increased by a factor of
a) 2? **b)** 3?

6. Predict how the volume of a prism would change if all dimensions were increased by a factor of
a) 4 **b)** 10

7. Predict how the surface area of a prism would change if all dimensions were increased by a factor of
a) 4 **b)** 10

8. a) Calculate the surface area and volume of the prism shown.
b) If the dimensions were halved, what would the volume be?
c) What fraction would the smaller volume be of the larger volume?
d) If the dimensions were halved, what would the surface area be?
e) What fraction would the smaller surface area be of the larger surface area?



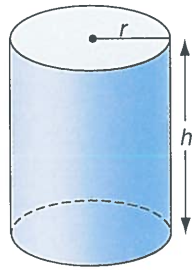
9. Write a statement about the change in the volume of a prism when all dimensions are changed by a factor of k.

10. Write a statement about the change in the surface area of a prism when all dimensions are changed by a factor of k.

2 Volumes and Surface Areas of Cylinders

1. Copy and complete the table for the three cylinders, or use a spreadsheet. Express each volume and surface area in terms of π. For example, the volume of the first cylinder is $\pi \times 1^2 \times 3$, or 3π cm^2.

Cylinder	Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	3		
2	2	6		
3	3	9		



2. How many times as great are
a) the dimensions of cylinder 2 as the dimensions of cylinder 1?
b) the volume of cylinder 2 as the volume of cylinder 1?
c) the surface area of cylinder 2 as the surface area of cylinder 1?

3. How many times as great are
a) the dimensions of cylinder 3 as the dimensions of cylinder 1?
b) the volume of cylinder 3 as the volume of cylinder 1?
c) the surface area of cylinder 3 as the surface area of cylinder 1?

4. How does the volume of a cylinder change when all dimensions are increased by a factor of
a) 2? **b)** 3?

5. How does the surface area of a cylinder change when all dimensions are increased by a factor of
a) 2? **b)** 3?

6. Predict how the volume of a cylinder would change if all dimensions were increased by a factor of
a) 4 **b)** 10

7. Predict how the surface area of a cylinder would change if all dimensions were increased by a factor of
a) 4 **b)** 10

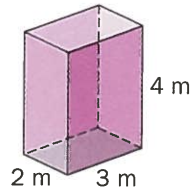
8. a) Calculate, in terms of π, the volume and surface area of a cylinder with a radius of 4 cm and a height of 6 cm.
b) If the dimensions were halved, what would the volume be, in terms of π?
c) What fraction would the smaller volume be of the larger volume?
d) If the dimensions were halved, what would the surface area be, in terms of π?
e) What fraction would the smaller surface area be of the larger surface area?

9. Write a statement about the change in the volume of a cylinder when all dimensions are changed by a factor of k.

10. Write a statement about the change in the surface area of a cylinder when all dimensions are changed by a factor of k.

11. a) Calculate the volumes of cylinders 1 and 2 to the nearest tenth of a cubic centimetre, instead of expressing them in terms of π.
b) Divide the larger volume from part a) by the smaller volume from part a). Is the result a whole number?
c) Explain why the volumes and surface areas compared in questions 2, 3, and 8 were expressed in terms of π.

3 Changing Selected Dimensions of Prisms



1. The prism has a length of 3 m, a width of 2 m, and a height of 4 m. Calculate
a) the volume
b) the surface area

2. Calculate the new volume when the width and height are not changed but the length of the original prism is increased by a factor of
a) 2 **b)** 3 **c)** 4

3. a) What happens to the volume when only the length of the prism is multiplied by 2? by 3? by 4?
b) If only the length of the prism were changed, would a graph of volume versus length be linear? If so, would the variation be direct or partial? Explain.

4. Calculate the new surface area when the width and height are not changed but the length of the original prism is increased by a factor of
a) 2 **b)** 3 **c)** 4

5. a) What happens to the surface area when only the length of the prism is multiplied by 2? by 3? by 4?
b) If only the length of the prism were changed, would a graph of surface area versus length be linear? If so, would the variation be direct or partial? Explain.

6. Calculate the new volume when the height is not changed but both the length and the width of the original prism are increased by a factor of
a) 2 **b)** 3 **c)** 4

7. What happens to the volume when both the length and the width of the prism are multiplied by 2? by 3? by 4?

8. Calculate the new surface area when the height is not changed but both the length and the width of the original prism are increased by a factor of
a) 2 **b)** 3 **c)** 4

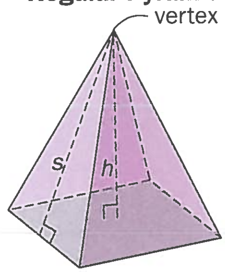
9. What happens to the surface area when both the length and the width of the prism are multiplied by 2? by 3? by 4?

INVESTIGATING MATH

Surface Area and Volume of a Regular Pyramid

A **pyramid** is a polyhedron with one base in the shape of a polygon and the same number of lateral triangular faces as there are sides in the base. The lateral triangular faces meet at a point called the **vertex** of the pyramid.

Regular Pyramid



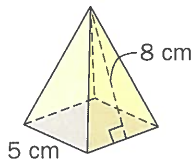
A **regular pyramid** has a regular polygon as its base. All the sides of the base are the same length, and all the lateral faces are congruent triangles. Assume that all the pyramids referred to in this book are regular pyramids.

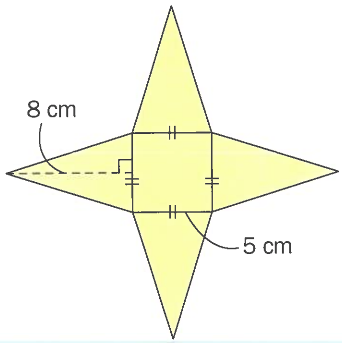
The **height**, *h*, of a pyramid is the perpendicular distance from the vertex to the base. The **slant height**, *s*, is the height of each triangular face.

1 Modelling the Surface Area

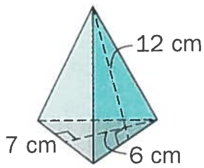
The surface area of a pyramid can be found by adding the areas of the lateral triangular faces and the area of the base.

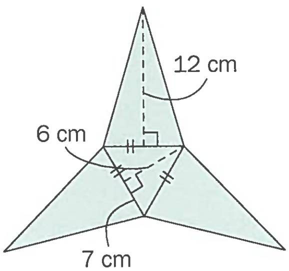
1. A square-based pyramid and its net are shown. Calculate the surface area of the pyramid.





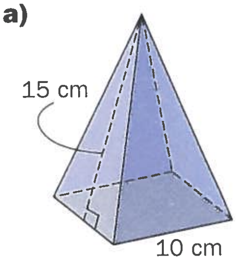
2. A regular triangular pyramid and its net are shown. Calculate the surface area of the pyramid.



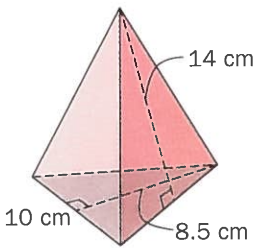


3. Calculate the surface area of each regular pyramid.

a)



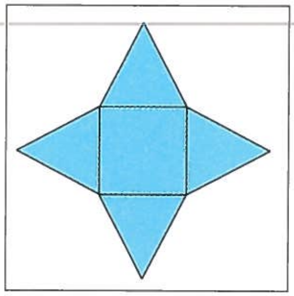
b)



2 Modelling the Volume

For this investigation, you will need an empty 250-mL milk carton, construction paper or cardboard, scissors, tape, and sand, rice, or another suitable material. The investigation involves finding the formula for the volume of a pyramid by comparing it to a prism with the same base and height as the pyramid.

Cut the top from an empty 250-mL milk carton to form a prism. On construction paper or cardboard, draw the net of a pyramid that has the same base dimensions and the same height as the prism. Cut out the net, tape it, and place the pyramid inside the prism.

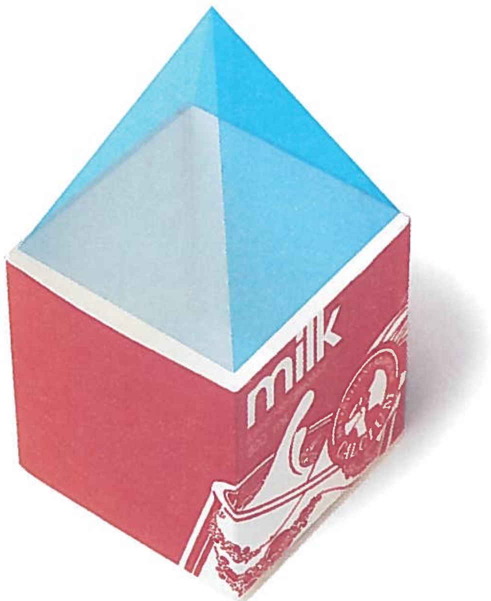


1. Estimate the ratio of the volume of the prism to the volume of the pyramid.

2. Cut along three sides of the base of the pyramid. Fill the pyramid with sand, rice, or another suitable material, so that the material is level with the open end of the pyramid. Then, empty the material from the pyramid into the prism. Repeat until the prism is full.
a) How many pyramids full of material does it take to fill the prism?
b) How close was your estimate in question 1?

3. From your results, what fraction of the volume of the prism is the volume of the pyramid?

4. The formula for the volume of a prism, V, with base area B and height h is $V = B \times h$. What is the formula for the volume of a pyramid, V, with the same base area, B, and the same height, h?



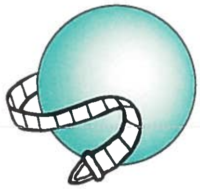
INVESTIGATING MATH

Surface Area and Volume of a Sphere

1 Modelling the Surface Area

For this investigation you will need a ball, such as a baseball or tennis ball, which approximates a sphere. You will also need a tape measure, scissors, compasses, paper, and tape.

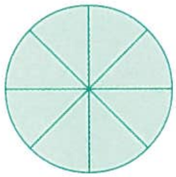
1. Use a tape measure to find the circumference of the ball, which is the greatest distance around its surface.



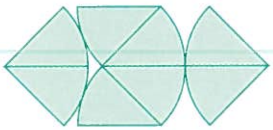
2. Substitute the circumference, C, into the formula $C = 2\pi r$, where r is the radius of the ball. Find r, to the nearest tenth of a centimetre.

3. Draw a circle whose radius is equal to the radius of the ball. Cut out the circle.

4. Fold the circle into eighths. Then, unfold it and cut the pieces apart.



5. Rearrange the pieces into the pattern shown, and tape the pattern together.



6. Tape the pattern to the ball.

7. Estimate the fraction of the surface of the ball that the pattern appears to cover.

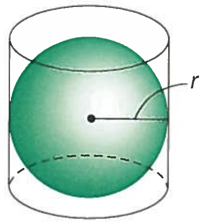
8. How many patterns would be needed to cover the sphere?

9. What is the area of the pattern in terms of r, the radius of the sphere?

10. Write a formula for the surface area of a sphere, in terms of r.

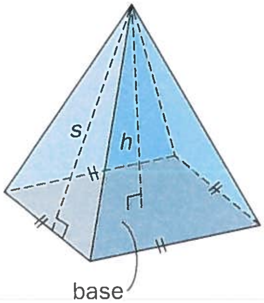
11. For each sphere with the given radius, use the formula from question 10 to calculate the surface area, to the nearest square centimetre.
a) 8 cm **b)** 10 cm **c)** 20 cm

2 Modelling the Volume



1. A sphere of radius r fits inside the cylinder, so that the sphere just touches the top, bottom, and sides of the cylinder.
   a) State the radius of the cylinder, in terms of r.
   b) State the height of the cylinder, in terms of r.
2. Write a formula for the volume of the cylinder, in terms of r.
3. The volume of the sphere is two thirds of the volume of the cylinder. Write a formula for the volume of the sphere, in terms of r.
4. For each sphere with the given radius, use the formula from question 3 to calculate the volume, to the nearest cubic metre.
   a) 4 m **b)** 5 m **c)** 7 m
5. Suppose you have a sphere and a cylinder, with sizes that compare as described in question 1. You also have a measuring cylinder and a supply of water.
   a) Design an experiment you could perform to show how the volume of the cylinder and the volume of the sphere are related.
   b) If possible, carry out the experiment. Do your results verify that the volume of the sphere is two thirds of the volume of the cylinder?

9.4 Surface Area and Volume of a Pyramid and a Sphere



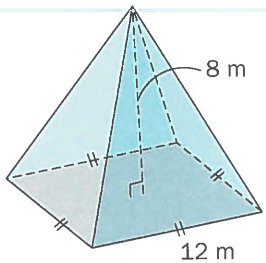
A **regular pyramid** is a polyhedron with a base that is a regular polygon and lateral faces that are congruent triangles. The **slant height**, s, is the length of the altitude of a triangular face. The **height**, h, of the pyramid is the perpendicular distance from the vertex to the base.

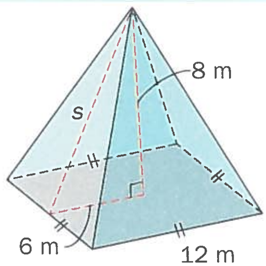
The **lateral area** of a pyramid is the sum of the areas of the lateral triangular faces. The **surface area** is the sum of the areas of all the faces, or the sum of the lateral area and the area of the base.

The Great Pyramid of Khufu is an example of a regular pyramid. It is the largest of the three pyramids built at Giza by an ancient Egyptian civilization around 2500 B.C. Each side of the square base of the pyramid measures about 230 m. The slant height of a lateral face is 187 m. It is believed that 4000 workers took 30 years to build the pyramid.

Explore: Use the Information

The diagram shows a square-based pyramid. The side length of the base is 12 m, and the height is 8 m.





In the red triangle, use the Pythagorean Theorem to calculate the slant height.

Inquire

1. What is the area of each triangular face?

2. What is the lateral area of the pyramid?

3. What is the area of the base?

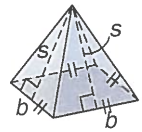
4. What is the surface area of the pyramid?

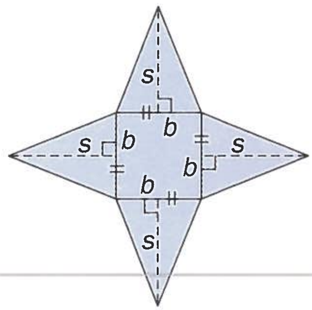
5. a) What is the lateral area of the Great Pyramid of Khufu?

b) How many football fields would it take to cover this area?

The surface area of a pyramid can be found by adding the areas of all the faces. However, a formula can be developed to find the surface area.

For the regular pyramid shown, b is the side length of the base, and s is the slant height of each triangular face.
The perimeter of the base, P, is the sum of the side lengths, $b+b+b+b$ or $4b$.
Use the net to write the formula for the lateral area, $L.A.$





Add the areas of the lateral faces: $L.A. = \frac{1}{2}bs + \frac{1}{2}bs + \frac{1}{2}bs + \frac{1}{2}bs$

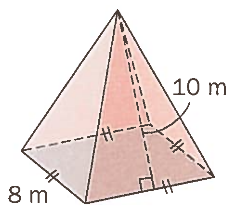
Use the distributive property: $L.A. = \frac{1}{2}s(b+b+b+b)$

$= \frac{1}{2}s(4b)$

Substitute P for $4b$: $L.A. = \frac{1}{2}sP$ or $\frac{1}{2}Ps$

The surface area, $S.A.$, is found by adding the lateral area and the area of the base, B.
So, $S.A. = \frac{1}{2}Ps + B$
The formula $S.A. = \frac{1}{2}Ps + B$ can be used to find the surface area of any regular pyramid.

Example 1 Surface Area of a Square-Based Pyramid
Calculate the surface area of the pyramid.
The side length of the base is 8 m and the slant height is 10 m.



Solution

Calculate the perimeter of the base, P: $P = 4 \times 8$

$= 32$

Calculate the area of the base, B: $B = 8 \times 8$

$= 64$

Calculate the surface area, $S.A.$: $S.A. = \frac{1}{2}Ps + B$

$= \frac{1}{2} \times 32 \times 10 + 64$

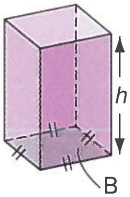
$= 160 + 64$

$= 224$

The surface area is 224 m^2.

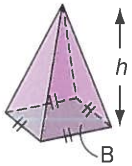
The volume of a pyramid is one third of the volume of a prism with the same base and height.

Prism



$V = B \times h$

Pyramid

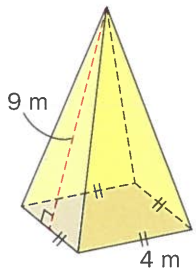


$V = \frac{1}{3} B \times h$

B is the area of the base and h is the height.

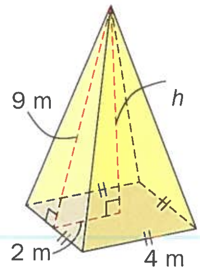
Example 2 Volume of a Pyramid

Find the volume of the pyramid, to the nearest cubic metre.



Solution

The slant height, s, is 9 m and the side length of the base is 4 m.
Use the Pythagorean Theorem to find the height, h.

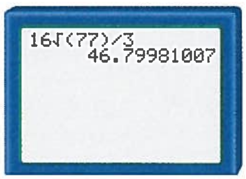


$$h^2 + 2^2 = 9^2$$
$$h^2 + 4 = 81$$
$$h^2 = 77$$
$$h = \sqrt{77}$$

We could evaluate $\sqrt{77} \doteq 8.8$, but there is no need to approximate at this stage when using a calculator.

Calculate the area of the base: $B = 4 \times 4$
$= 16$

Calculate the volume: $V = \frac{1}{3} \times B \times h$
$= \frac{1}{3} \times 16 \times \sqrt{77}$
$\doteq 47$



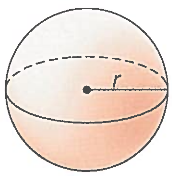
16 [×] 77 [√x] [÷] 3 [=] 46.7998 1007

Estimate

$\frac{1}{3} \times 16 \times 9 = 48$

The volume of the pyramid is 47 m^3, to the nearest cubic metre.

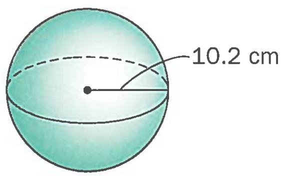
The formula for the surface area of a sphere with radius r is $S.A. = 4\pi r^2$.



$S.A. = 4\pi r^2$

Example 3 Surface Area of a Volleyball

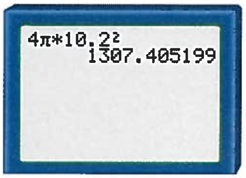
The radius of a volleyball is 10.2 cm. Calculate the surface area of the volleyball, to the nearest square centimetre.



Solution

The radius, r, is 10.2 cm.

$S.A. = 4\pi r^2$
$\quad = 4 \times \pi \times 10.2^2$
$\quad \doteq 1307$



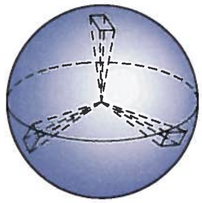
4 × π × 10 . 2 x^2 = 1307.405199

Estimate

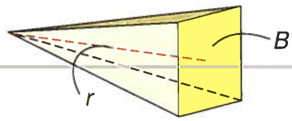
$4 \times 3 \times 100 = 1200$

The surface area of the volleyball is 1307 cm^2, to the nearest square centimetre.

To find the formula for the volume of a sphere, imagine a sphere being filled with a large number, n, of small identical pyramids, each with a base area B.



The vertex of each pyramid is the centre of the sphere. The height of each pyramid, h, is the radius of the sphere, r. The sum of the areas of the bases of all n pyramids, nB, approximates the surface area of the sphere, $4\pi r^2$.



The volume of each pyramid is $\frac{1}{3}Bh$ or $\frac{1}{3}Br$.

Express the total volume, V, of all n pyramids: $V = n \times \frac{1}{3}Br$

Rearrange the right side: $V = \frac{1}{3} \times nB \times r$

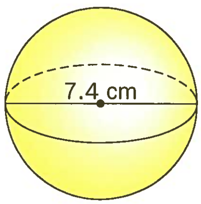
Substitute $4\pi r^2$ for nB: $V = \frac{1}{3} \times 4\pi r^2 \times r$

Simplify: $V = \frac{4}{3}\pi r^3$

So, the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Example 4 Volume of a Baseball

The diameter of a baseball is 7.4 cm. Calculate the volume of the baseball, to the nearest cubic centimetre.



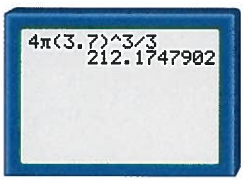
Solution

The diameter is 7.4 cm. The radius is 3.7 cm.

$V = \frac{4}{3}\pi r^3$
$\quad = \frac{4}{3}(\pi)(3.7)^3$
$\quad \doteq 212$

Estimate

$1 \times 3 \times 64 = 192$



4 × π × 3 . 7 y^x 3 ÷ 3 = 212.1747902

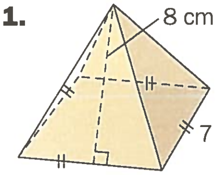
The volume of the baseball is 212 cm^3, to the nearest cubic centimetre.

Practice

A

Calculate the surface area. Round to the nearest square unit, if necessary.

1.



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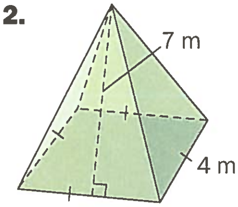

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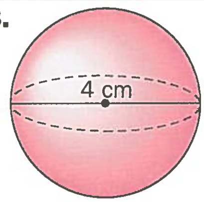

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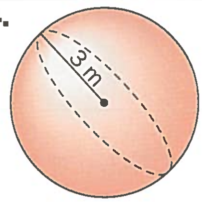
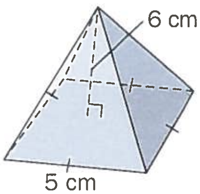
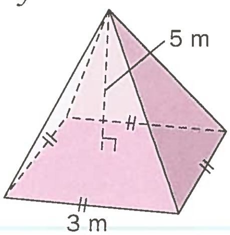


Estimate, and then calculate each volume. Round to the nearest cubic unit, if necessary.

5.



6.



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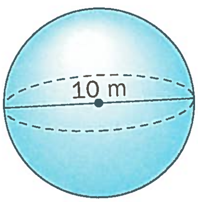

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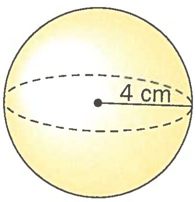
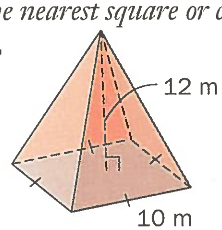
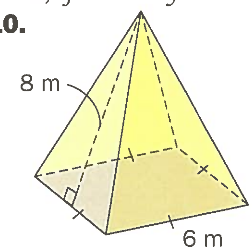


Calculate the surface area and the volume. Round to the nearest square or cubic metre, if necessary.

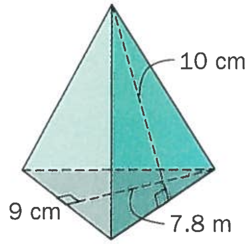
9.



10.



11. Calculate the surface area of the regular triangular pyramid.

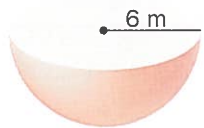


Applications and Problem Solving

12. Golf ball A golf ball is in the shape of a sphere with radius 20 mm. What is its surface area, to the nearest square millimetre?

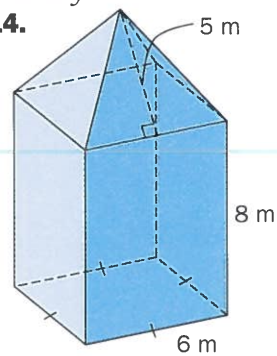
B

13. Calculate the surface area and volume of the hemisphere, to the nearest square or cubic metre.

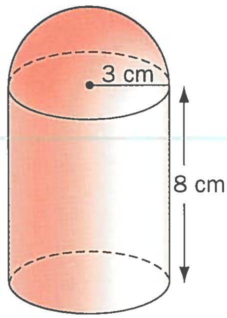


Find the surface area and volume of each composite figure. Round to the nearest square or cubic unit, if necessary.

14.

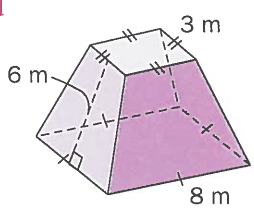


15.



16. A sphere has a surface area of 250 cm^2. What is its radius, to the nearest tenth of a centimetre?

17. Frustum of a pyramid The frustum of a pyramid is the part that remains after the top portion has been cut off by a plane parallel to the base. A frustum of a regular pyramid is shown. Find the surface area.



18. What is the volume, in terms of π, of a sphere with a surface area of 16π m^2? Explain how you used mathematical modelling to solve the problem.

19. Two spheres have diameters of 10 cm and 20 cm.
a) Calculate the surface area of each sphere, to the nearest square centimetre.
b) Divide the larger surface area by the smaller surface area from part a).
c) Can you be sure of the exact answer to part b)? Explain.
d) Express the surface area of each sphere in terms of π.
e) Divide the larger surface area by the smaller surface area from part d).
f) Can you be sure of the exact answer to part e)? Explain.
g) What is the advantage of expressing surface areas in terms of π before comparing them?

20. Use your findings in question 19 to help answer each of the following.
a) What happens to the surface area of a sphere when its diameter is doubled? tripled? halved?
b) What happens to the volume of a sphere when its diameter is doubled? tripled? halved?

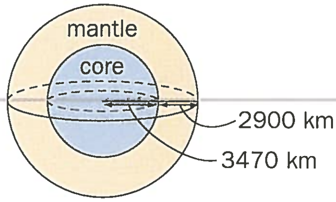
21. Planets The radius of Neptune is 10 times the radius of Mercury. State the ratio of
a) the surface area of Neptune to the surface area of Mercury
b) the volume of Neptune to the volume of Mercury

22. Canada's area The diameter of the Earth is 12 756 km.
a) Calculate the area of the Northern Hemisphere, to the nearest square kilometre.
b) What assumptions have you made?
c) The area of Canada is 9 970 610 km^2. Estimate the fraction of the Northern Hemisphere that Canada covers.

23. What happens to the volume of a square pyramid under each of the following conditions?
a) The base is unchanged, but the height is doubled.
b) The base is unchanged, but the height is tripled.
c) The height is unchanged, but the area of the base is doubled.

C

24. Earth's mantle The Earth's core has a radius of about 3470 km. The core is surrounded by the mantle, which is about 2900 km thick. What is the volume of the mantle, to the nearest billion cubic kilometres?



25. A pyramid sits inside a cube, so that the base of the pyramid is a face of the cube. What is the maximum possible volume of the pyramid, if the edge length of the cube is 24 cm?

26. A pyramid and a prism have congruent square bases and equal volumes. How do their heights compare? Explain and justify your reasoning.

27. Oranges If an orange of diameter 6 cm costs 25¢, and an orange of diameter 9 cm costs 50¢, which is the better value? Explain your answer, and state any assumptions that you made.

PATTERN POWER

Copy and complete the following.

$1^3 = 1$
$1^3 + 2^3 = 9$
$1^3 + 2^3 + 3^3 =$ ■
$1^3 + 2^3 + 3^3 + 4^3 =$ ■
$1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$ ■

Describe the pattern.

TECHNOLOGY

Design Problems in Three Dimensions

1 Designing Boxes

Many cardboard boxes are made in the shape of a rectangular prism. Suppose you are an industrial designer who must decide what dimensions to use for a box to hold 12 objects, each the size of a one-centimetre cube. The surface area of the box must be a minimum, so that the least amount of material is used to make the box.

1. Make a hypothesis in which you predict the dimensions of the box, if it must be

a) a square-based prism

b) a rectangular prism

Note that rectangular prisms include square-based prisms.

2. Using 12 one-centimetre cubes in each case, make the following prisms. Alternatively, sketch the prisms. Record their dimensions.

a) 2 different square-based prisms **b)** 4 different rectangular prisms

3. Set up a computer spreadsheet to determine the surface area of

a) each square-based prism **b)** each rectangular prism

	A	B	C	D	E	F
1	Prism	Width (cm)	Length (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
2	1				12	
3	2				12	
4	3				12	
5	4				12	

4. The most cost-effective, or cheapest, container of a given volume is the one that uses the least material. What dimensions of the prism make the most cost-effective box if it is

a) square based?

b) rectangular?

5. Compare the dimensions from question 4 with each hypothesis from question 1.

6. a) Add a final column to each spreadsheet to calculate the rate $\frac{\text{volume}}{\text{surface area}}$, to 2 decimal places, for each prism.

b) What happens to this rate as a box becomes more cost-effective?

7. Many boxes do not have whole-number dimensions. Suppose you had to design a box to hold 8 m^3 of product. Possible dimensions of the box would include 8 m × 1 m × 1 m, 2 m × 2 m × 2 m, 1.6 m × 2 m × 2.5 m, and 1 m × 1.25 m × 6.4 m. For dimensions that you choose, complete a spreadsheet to find the surface area of

a) each square-based prism

b) each rectangular prism

8. What are the dimensions and the shape of the 8-m^3 box that is most cost-effective, if it is

a) square based? **b)** rectangular?

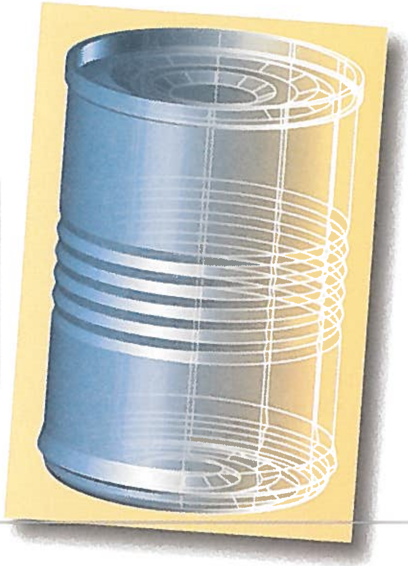
9. Are the following boxes made in the shape that is most cost-effective? If not, give possible reasons.

a) square-based cracker boxes

b) rectangular cereal boxes

2 Designing Cans

Industrial designers must also decide the height and radius to use for a cylindrical container. If the surface area is a minimum, the least amount of material is used to make the container.



1. The spreadsheet shows the radii and heights of four cylindrical cans.

	A	B	C	D	E
1	Can	Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
2	1	1	36		
3	2	2	9		
4	3	3	4		
5	4	6	1		

Make a hypothesis in which you predict which of the 4 cans is the most cost-effective. You may want to build models of the cans.

2. Use the spreadsheet to calculate each volume and surface area. Round each volume and surface area to the nearest whole number.

3. a) Which of the 4 cans is the most cost-effective?
b) Compare your answer from part a) with your hypothesis in question 1.

4. a) Sketch the front view of the most cost-effective can as it would appear to a customer in a grocery store.
b) Why do you think cylindrical cans are not all made in this shape?

5. Think about food products that come in cylindrical cans. Give possible reasons why each shape of cylinder is used for each can.

3 Heat Loss

1. Hot box You have been asked to design a rectangular container, called a hot box, for catering companies to use to transport hot food. The container must have a volume of 125 000 cm^3. Once the hot food is placed in the container, the container loses heat through its sides, top, and bottom. In order to keep the heat loss to a minimum, the total area of the faces must be a minimum.
a) What shape for the container has the minimum surface area?
b) What is the minimum surface area?

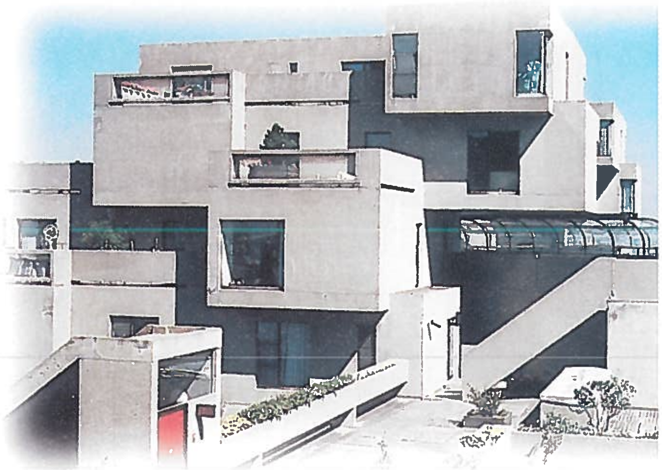
2. a) If you were not restricted to a rectangular shape, what shape would have the minimum surface area?
b) What are the disadvantages of using this shape for a hot box?

3. Use your answer to question 2a) to explain why cats curl up in a ball when they sleep.

CAREER CONNECTION

Architecture

Architecture is the science and art of planning, designing, and constructing buildings. A skilled architect designs buildings that are long-lasting, useful, comfortable, and pleasing to the eye.

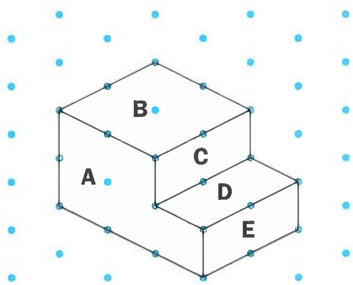


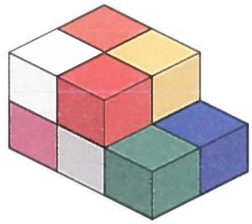
Habitat, designed by Canadian architect Moshe Safdie.

Moshe Safdie is one of Canada's most famous architects. His best-known buildings in Canada are Habitat, in Montréal, and the National Gallery, in Ottawa.

Habitat consists of about 150 private homes. Each is a prefabricated concrete unit. The units are arranged irregularly, so that there is as much privacy as possible.

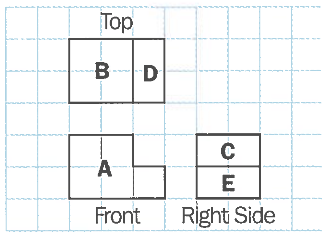
There are two common types of drawings that architects use to display an object.



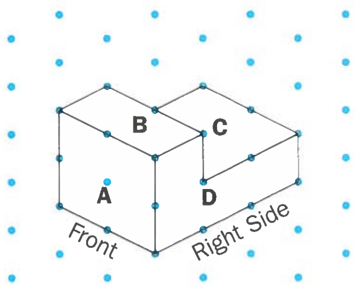


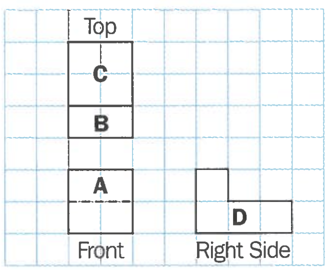
An **isometric view** is an attempt to represent a three-dimensional object in a two-dimensional drawing. An isometric view can also be called a corner view of the object. When drawing an isometric view, it is helpful to use isometric dot paper.

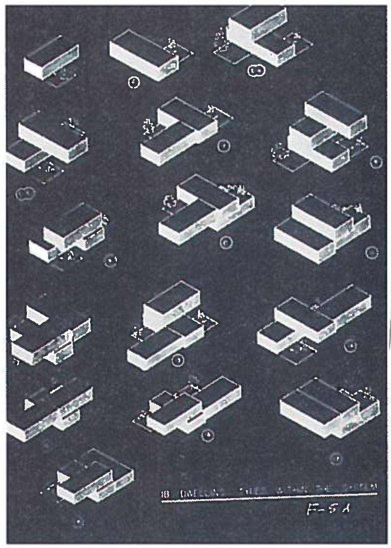
An **orthographic view** represents two dimensions of an object, as it appears from the top, front, and right side. When drawing orthographic views, it is helpful to use grid paper or square dot paper.



In an orthographic view, any hidden edges of an object are shown as dashed lines. In an isometric view, any hidden edges are not shown.





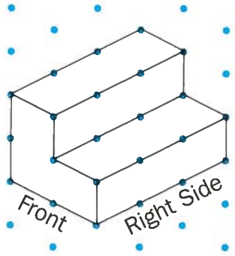


Moshe Safdie's isometric views of the homes in Habitat.

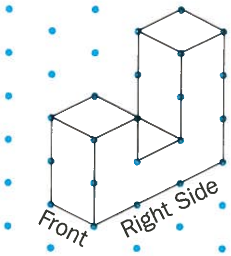
1 Isometric and Orthographic Views

1. Use the isometric view of each object to draw its orthographic views.

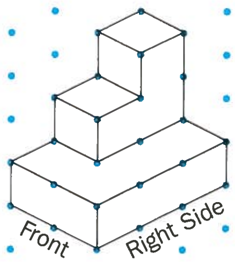
a)



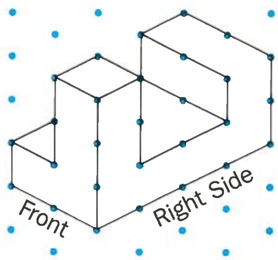
b)



c)

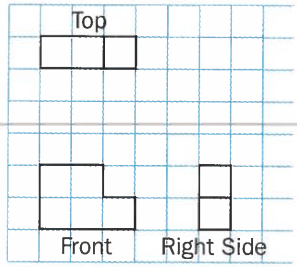


d)

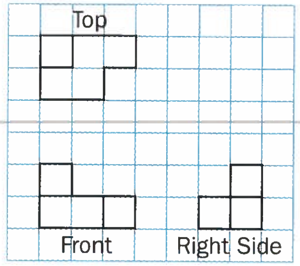


2. Use the orthographic views of each object to draw its isometric view.

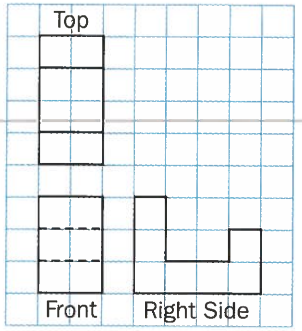
a)



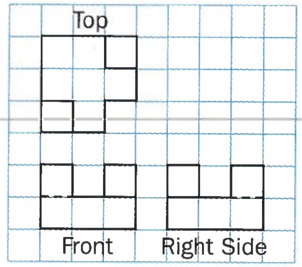
b)



c)



d)

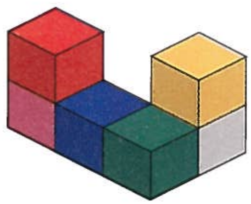


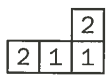
3. In the isometric views in question 1, assume that the shortest distance between two grid points represents 1 unit of length. For each object in question 1, determine the volume, the surface area, and the rate $\frac{\text{volume}}{\text{surface area}}$.

4. Why would an architect want to know the rate $\frac{\text{volume}}{\text{surface area}}$ for a building?

2 Base Designs

For an object built from linking cubes, the **base design** shows the top view of the object and the number of cubes in each column.



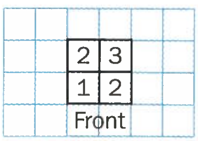


Base Design

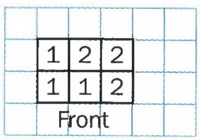
Use the following base designs to draw the orthographic and isometric views of each object.

1.

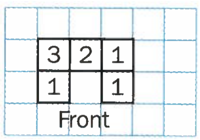
4.

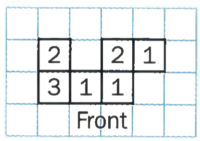


2.



3.





Graphing Calculator Programs for Surface Area and Volume

1 Using a Program

The graphing calculator program shown can be used to calculate the surface area of a cone.

This is the program.

This example used the program.

1. Describe what each line of the program does.

2. Explain why the symbol π is used in the program, rather than the value 3.14.

3. Input the program and use it to calculate the surface area of a cone with a base of radius 3.5 cm and a slant height of 4.3 cm. Round your answer to the nearest tenth of a square centimetre.

2 Writing Programs

1. Write a program that can be used to calculate the surface area of each of the following. In each case, describe what each line of the program does.
a) cylinder **b)** sphere **c)** square-based pyramid

2. Write a program that can be used to calculate the volume of each of the following. In each case, describe what each line of the program does.
a) cylinder **b)** sphere **c)** cone **d)** square-based pyramid

To check that the programs work, use them to calculate each of the following. Round each answer to the nearest tenth of a square or cubic unit.

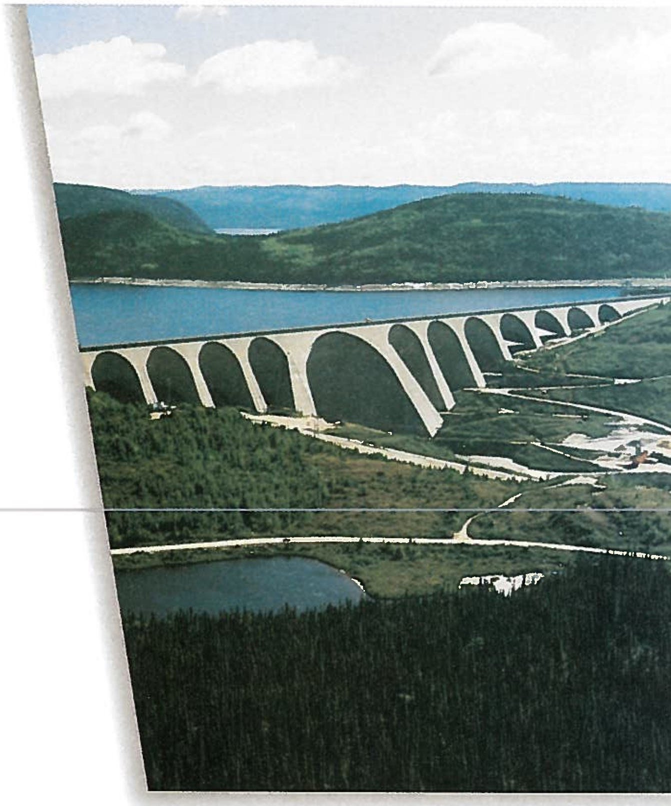
3. surface area and volume of a cylinder with a base of radius 4.5 cm and a height of 5.9 cm

4. surface area and volume of a sphere of radius 3.9 m

5. volume of a cone with a base of radius 2.1 m and a height of 3.8 m

6. surface area and volume of a square-based pyramid with base dimensions 2.4 cm by 2.4 cm, a height of 1.6 cm, and a slant height of 2 cm

9.5 Volume, Capacity, and Mass



The world's sixth largest reservoir is the Daniel Johnson Reservoir at Manicouagan, Québec. The volume of water in this reservoir when it is full is 141 852 000 000 m^3.

The greatest volume a container, such as a reservoir, can hold is known as the container's **capacity**. The capacity of a container is expressed in litres and millilitres. Large capacities may be expressed in kilolitres.

$$1000 \text{ mL} = 1 \text{L}$$
$$1000 \text{ L} = 1 \text{ kL}$$

The **mass** of an object is a measure of the quantity of matter in the object. Mass is measured in milligrams, grams, kilograms, and tonnes.

$$1000 \text{ mg} = 1 \text{ g}$$
$$1000 \text{ g} = 1 \text{ kg}$$
$$1000 \text{ kg} = 1 \text{ t}$$

Explore: Interpret the Diagrams

The diagrams of three different containers filled with water illustrate a special relationship between the volume and mass of water. Note that the diagrams are not drawn to scale.

	Container 1	Container 2	Container 3
Capacity of Container	1 mL	1000 mL or 1 L	1000 L or 1 kL
Volume of Water	1 cm^3	1000 cm^3 or 1 dm^3	1000 dm^3 or 1 m^3
Mass of Water	1 g	1000 g or 1 kg	1000 kg or 1 t

Inquire

1. a) Which container has a capacity of 1 kL?
b) What is the volume of water in this container?
c) What is the mass of water in this container?

2. a) Which container has a capacity of 1 L?
b) What is the volume of water in this container?
c) What is the mass of water in this container?

3. State the capacity of the Daniel Johnson Reservoir, in kilolitres.

4. State the mass of water in the Daniel Johnson Reservoir, in tonnes.

5. What mass of water could a 100-mL container hold?

6. State the capacity of a container that holds 5 kg of water.

Practice

A

Express each volume in cubic centimetres.

1. $5\ dm^3$ **2.** $0.032\ m^3$
3. $0.4\ dm^3$ **4.** $10\ m^3$

Express each volume in cubic metres.

5. $5000\ dm^3$ **6.** $45\ 000\ cm^3$
7. $1\ cm^3$ **8.** $25\ 200\ dm^3$

Express each capacity in litres.

9. 275 mL **10.** 15 mL
11. 1.5 kL **12.** 0.1 kL

Express each capacity in millilitres.

13. 7 L **14.** 0.25 L
15. 1 kL **16.** 0.045 L

Express each mass in grams.

17. 15 kg **18.** 4500 mg
19. 325 mg **20.** 0.35 kg

Express each mass in kilograms.

21. 28 000 g **22.** 15 t
23. 540 g **24.** 100 mg

State the volume, in cubic centimetres, of each mass of water.

25. 10 g **26.** 6.1 kg
27. 3.3 mg **28.** 0.1 g

Applications and Problem Solving

29. Food products Food products are sold in different units of measurement.
a) List 3 food products sold in units of mass.
b) List 3 food products sold in units of capacity.

30. Container State the capacity, in kilolitres, of the smallest container needed to hold 50 t of water.

31. Juice servings About how many 150-mL servings of grape juice can you pour from a 1-L container?

B

32. Orange juice A can of frozen orange juice holds 355 mL of concentrate. If you add 3 times this volume of water, how many litres of orange juice do you have? Explain how you can judge whether your answer is reasonable.

33. Trench A trench on a building site is 75% full of water. The trench measures 3 m long by 1 m wide by 2 m deep.
a) What is the volume of water in the trench?
b) What is the capacity of the trench?
c) How long does it take a pump that draws water at $0.6\ m^3/min$ to empty the trench?

C

34. Comparing containers Two containers have the same outside dimensions but different capacities. Explain how this situation is possible.

35. Gasoline can A can with a capacity of 4 L holds $4\ dm^3$ of gasoline. Can you state the mass of the gasoline? Explain.

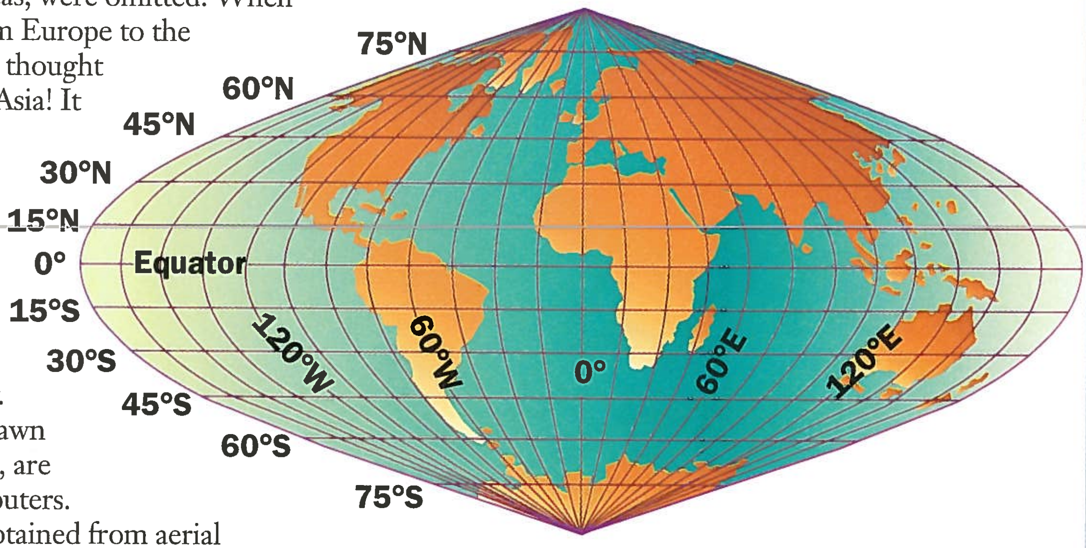
36. Milk carton **a)** What is the volume of milk in a 1-L carton?
b) Is the capacity of the carton exactly 1 L?
c) Would it be better to label the carton as 1 L of milk or $1\ dm^3$ of milk? Explain.

37. In your own words, distinguish the meanings of the terms "capacity" and "volume." Compare your answer with a classmate's.

9.6 How Can We Model the Earth in Two Dimensions?

Cartography, or the making of maps and charts, is a very old profession. For thousands of years, cartographers have tried to model the three-dimensional Earth using two-dimensional maps. The earliest surviving maps, dating from about 2300 B.C., are on clay tablets found in Iraq. Aboriginal people drew maps of parts of Canada long before Europeans arrived.

In the 2nd century A.D., an Egyptian named Ptolemy wrote a geography book that included maps of the known world. Vast parts of the world, including the Americas, were omitted. When Columbus sailed from Europe to the Americas in 1492, he thought that he had sailed to Asia! It was not until the 16th century that Gerardus Mercator's maps revolutionized navigation.



Cartography has changed dramatically. Maps, which were drawn by hand for centuries, are now created on computers. Information, often obtained from aerial photography or satellite imaging, is converted into digital form. Cartographers work with the digital information to decide colour, lines, lettering, and scales. A laser plotter draws maps directly onto photographic film for printing.

1 Modelling With Map Projections

A map projection models the three-dimensional Earth on a two-dimensional surface. It is impossible to do this without distortion. The projection a cartographer uses depends upon the purpose of the map, its scale, and the region of the Earth being mapped. Equal-area projections keep areas correct. Equidistant projections keep distances correct. Conformal projections keep shapes correct.

1. Using several different atlases, find

a) how many different projections are used

b) which projections are used most

2. To represent certain parts of the world, are some projections used more than others? If so, give possible reasons.

2 Modelling Antipodal Points

The city of Nanking, China, has a latitude of about 35°N and a longitude of about 120°E. If you travelled from Nanking through the centre of the Earth and out the other side, you would arrive close to the city of Buenos Aires, Argentina. Buenos Aires has a latitude of about 35°S and a longitude of about 60°W.

A location's antipodal point is the place on Earth directly opposite it. Nanking is the antipodal point to Buenos Aires. Two other cities related in this way are Lima, Peru, and Phnom Penh, Cambodia. Lima is located at about 10°S, 75°W. Phnom Penh is located at about 10°N, 105°E.

1. For each pair of cities, what is the relationship between the latitudes?

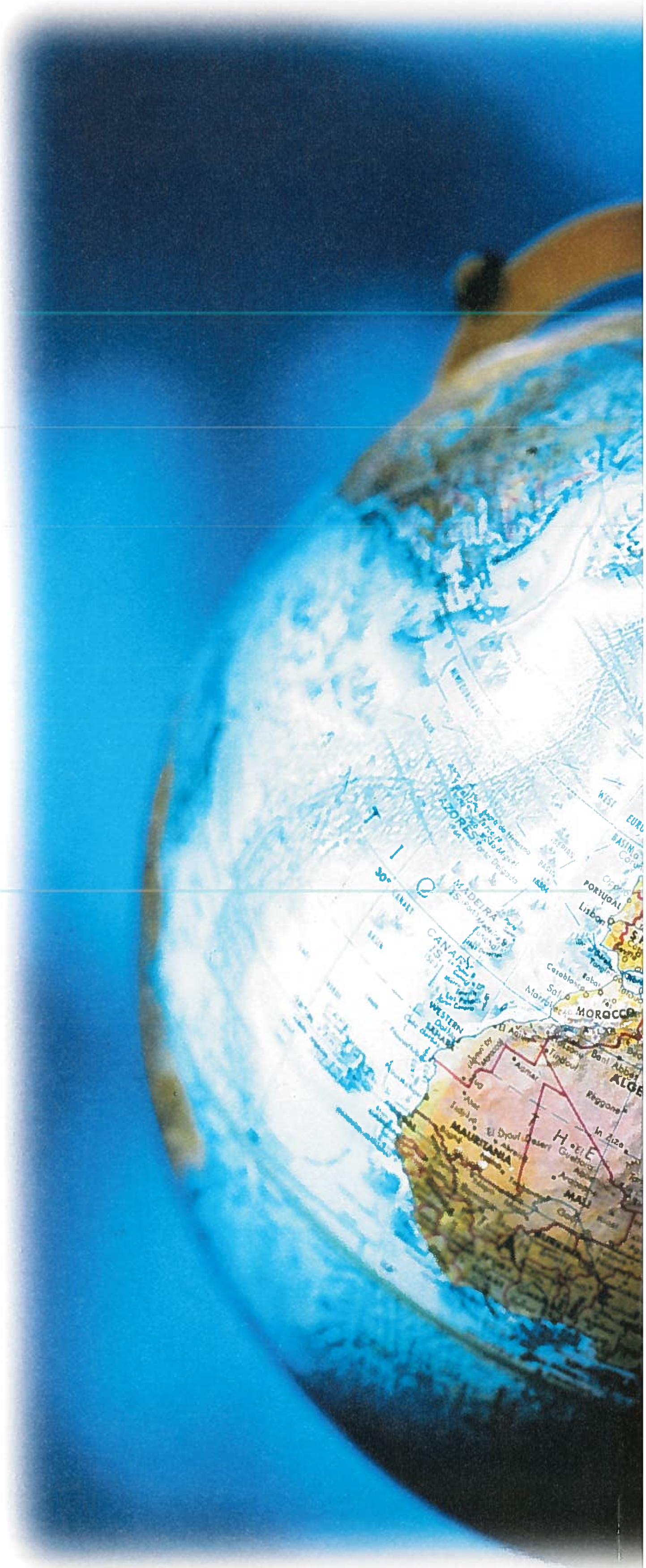
2. For each pair of cities, what is the relationship between the longitudes?

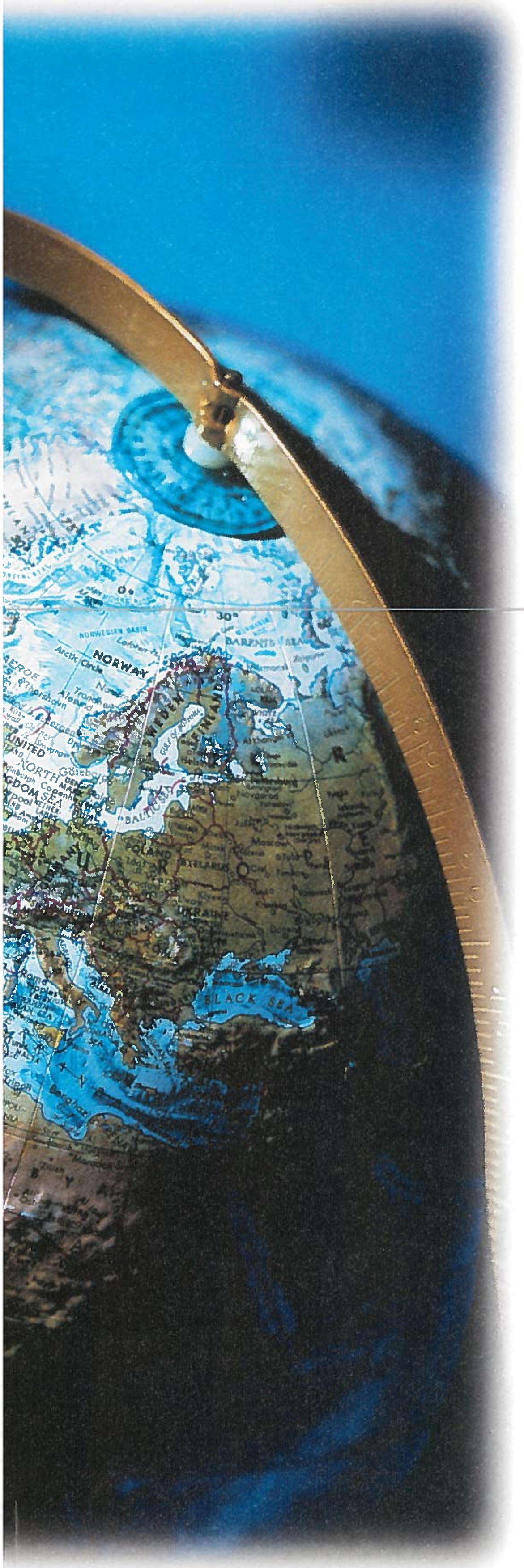
3. Without using latitudes and longitudes, use a map projection to estimate the location of the antipodal point to where you live.

4. Use a map to estimate the latitude and the longitude of the place where you live, to the nearest degree.

5. a) Find the latitude and the longitude of the antipodal point to where you live.
b) Does anyone live there?
c) About how far is this point from the location you estimated in question 3?

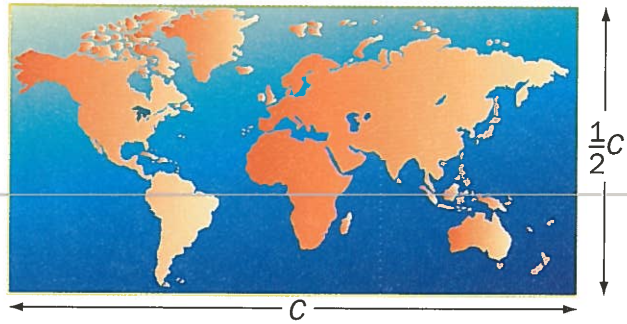
6. If two places are at antipodal points on the Earth, what is the straight-line distance between them?





3 Modelling the Area of the Earth

1. Assume that both the equatorial and polar radii of the Earth are 6400 km. Calculate the following, to the nearest whole number.
a) the surface area of the Earth
b) the equatorial circumference
c) the semi-circumference from pole to pole
d) the area that seems to be represented by the following Mercator projection, using dimensions from parts b) and c)



2. About how many times as great as the surface area of the Earth is the area that seems to be represented by this Mercator projection?

3. Which parts of the Earth are most distorted by a Mercator projection? Explain.

4 Extending the Model

Use your research skills to find more information about each of the following.
1. the Mercator projection, and why it revolutionized navigation

2. other projections and what they distort

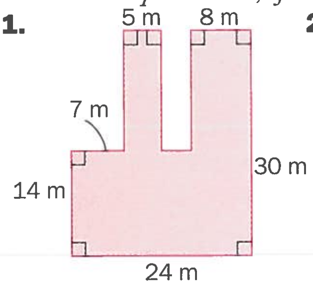
3. Canada's role in mapping the Earth using satellite imaging

4. the angles whose measures are a location's latitude and longitude

Review

9.1 *Calculate the area of each composite figure. Round to the nearest square unit, if necessary.*

1.



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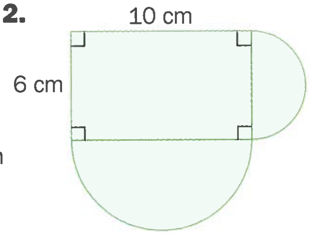
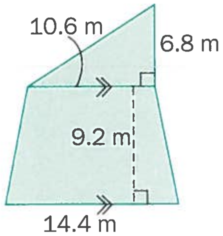
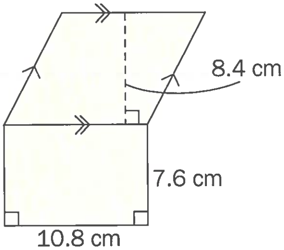


3.

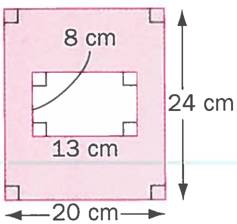


4.

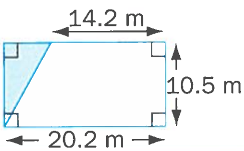


Calculate the area of each shaded region.

5.

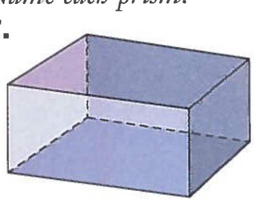


6.



9.2 *Name each prism.*

7.



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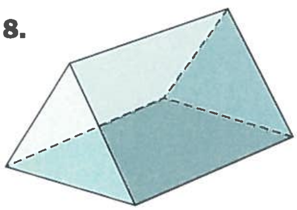
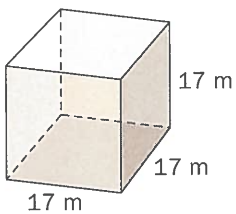
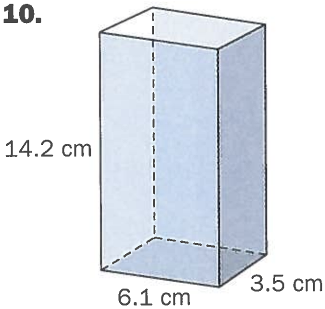


Calculate the surface area.

9.

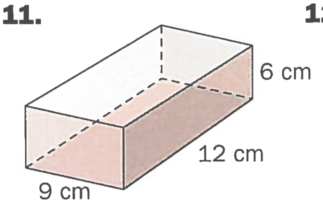


10.

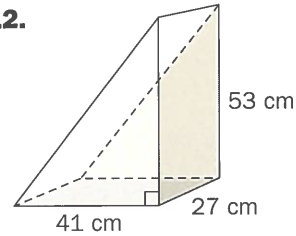


Calculate the volume.

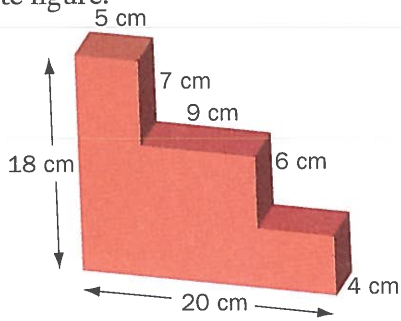
11.



12.

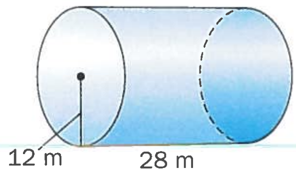


13. Calculate the surface area and volume of the composite figure.

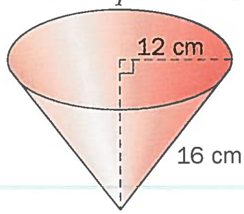


9.3 *Calculate the surface area, to the nearest square unit.*

14.

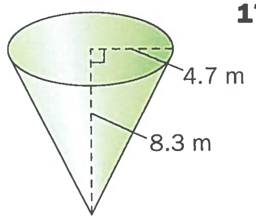


15.

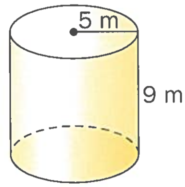


Calculate the volume, to the nearest cubic metre.

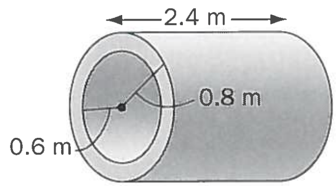
16.



17.

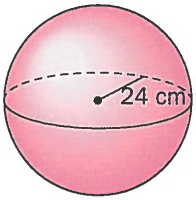


18. Concrete pipe Calculate the volume of concrete that is needed to make this section of pipe, to the nearest tenth of a cubic metre.

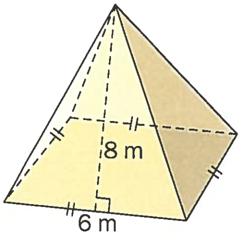


9.4 *Calculate the surface area. Round to the nearest square unit, if necessary.*

19.

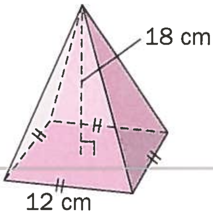


20.

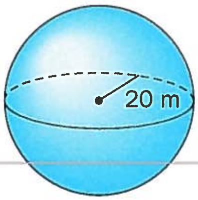


Calculate the volume. Round to the nearest cubic unit, if necessary.

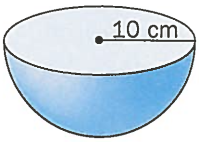
21.



22.



23. Calculate the surface area and volume of the hemisphere, to the nearest square or cubic centimetre.



24. How would the volume and surface area of a prism change if all the dimensions were tripled? Explain and justify your reasoning.

9.5 **25. Container** A container has the shape of a rectangular prism. Its inside dimensions are 12 cm by 10 cm by 15 cm.

a) Find the volume of water the container holds, in cubic centimetres; in cubic decimetres.

b) Find the mass of water, in kilograms, the container holds.

c) Find the capacity, in litres, of the container.

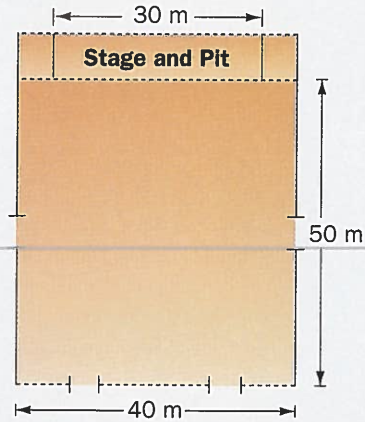
26. Tomato juice If you empty three 540-mL cans of tomato juice into a pitcher, how many litres of juice are in the pitcher?

Exploring Math

Designing a Theatre

You have been asked to submit a design for the seating in a theatre that will be located in an old movie house. The floor already slopes to the stage and orchestra pit, so all you need to do is to place the seats and the aisles.

The seating area measures 40 m wide by 50 m deep. The stage and orchestra pit are centred at the front of the theatre and measure 30 m across. There are two entrance/exit doors evenly spaced at the back of the theatre, and an extra exit door in the middle of each side. There is no balcony.



Each seat measures 50 cm by 50 cm, including arm rests. There is to be a 50-cm space in front of each seat for leg room and access. There should be no more than 20 seats in a row. Aisles must be at least 2 m wide.

The owners of the theatre want you to design a pleasant, comfortable seating arrangement. However, when you decide how many seats to include, keep in mind that the owners are in the business of selling tickets. If you include too few seats, your design may not be accepted.

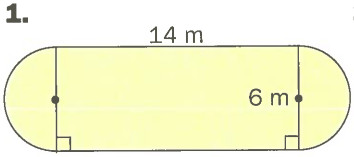
1. Design the placement of the seats and aisles so that people have easy access to their seats.

2. Label each aisle and each seat for the purpose of ticket sales.

Chapter Check

Calculate the area. Round to the nearest square metre, if necessary.

1.



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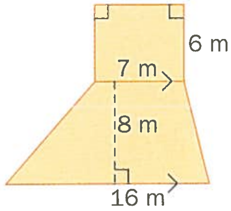
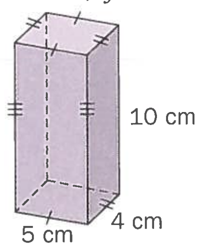
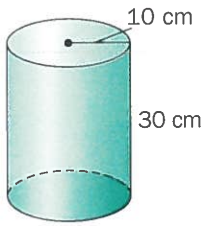


Calculate the surface area. Round to the nearest square unit, if necessary.

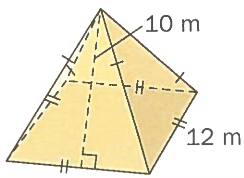
3.



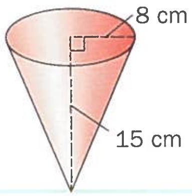
4.



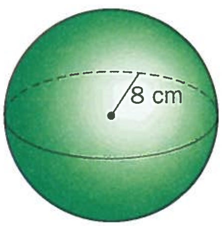
5.



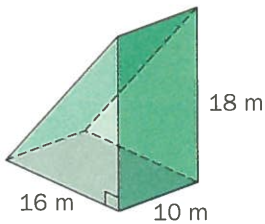
6.



7.

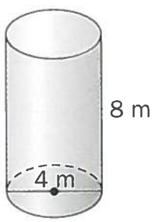


8.

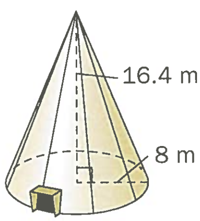


Calculate the volume. Round to the nearest cubic unit, if necessary.

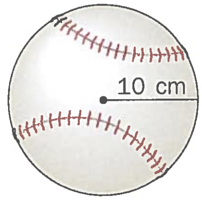
9.



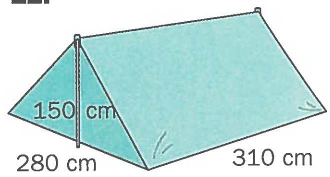
10.



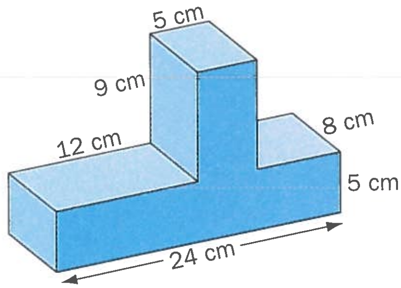
11.



12.



13. Calculate the surface area and volume of the composite solid.



14. How would the volume and surface area of a prism change if all the dimensions were doubled?

15. Tuna can A tuna can is a cylinder with a diameter of 8 cm and a height of 3.5 cm. Calculate the volume and surface area, to the nearest cubic or square centimetre.

16. Juice box A juice box measures 10 cm × 6 cm × 3 cm.

a) What is its surface area?

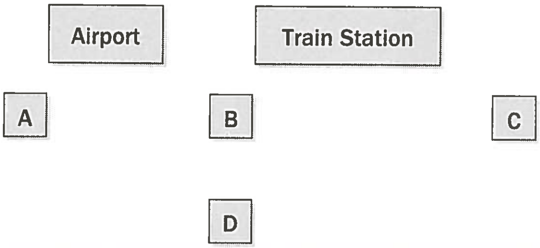
b) How many millilitres of juice can the juice box hold?

17. Wooden block A wooden block is a rectangular prism of volume 600 cm^3. What is the height of the block if its length is 20 cm and its width is 10 cm?

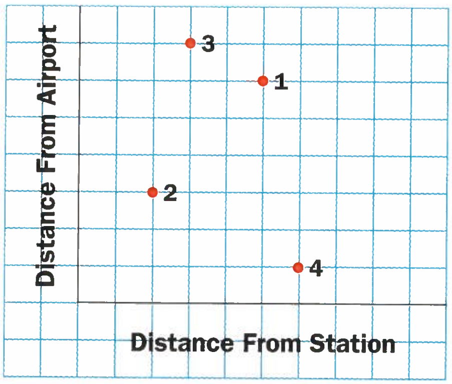
Using the Strategies

1. Whole numbers Jason chose a whole number less than 10. He multiplied the number by 6 and added 1. The result was a perfect square. What numbers could he have chosen?

2. Comparing models The map shows the locations of 4 houses, the train station, and the airport.

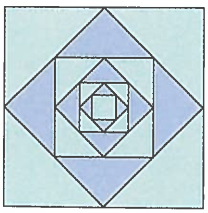


Match each house with the correct number on the distance graph. Give reasons for your answer.



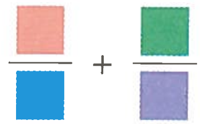
3. TV watching Sketch a graph to show the length of time you spend watching TV versus the day of the week.

4. Square design The area of the large square is 64 cm^2.



Each smaller square is formed by joining the midpoints of the sides of the next larger square. What is the area of the smallest square?

5. Copy the diagram. Fill in the boxes with the digits 2, 3, 4, and 5 to make the greatest possible sum



6. Consecutive integers Find 3 consecutive integers whose product is 1716.

7. Junk mail Estimate the number of pieces of junk mail delivered in Ontario in a year.

8. Perfect squares The number 2601 is a 4-digit number that is a perfect square, because $51^2 = 2601$. What is the smallest 4-digit number that is a perfect square and that has all even digits?

9. Compact discs Tania has a collection of compact discs. When she puts them in piles of 2, she has 1 left over. She also has 1 left over when she puts them in piles of 3 and piles of 4. She has none left over when she puts them in piles of 7. What is the smallest number of compact discs she can have?

10. Floor tiles A rectangular floor is tiled with 36 square tiles. The tiles around the outside edge of the rectangle are red, and the tiles on the inside are white. How many red tiles are there? Is there more than one possible answer?

11. Making a cube What is the edge length of a cube that can be made with 294 cm^2 of cardboard? What assumptions have you made?

12. Purchase price The amount of a purchase is \$12.43. How can the exact amount be paid without using a \$10.00 bill, but using the smallest number of bills and coins?

13. Populations What is the mean of the populations of all the countries in the world?