



Problem of the Week

Problem A and Solution

Finding the Intersection

Problem

Isla starts with \$12 in her bank account. She adds \$12 to her account at the end of every two weeks from collecting recycled items. Javier starts with \$32 in his bank account. He earns \$4 at the end of every week for doing odd jobs for his neighbour, and adds that to his savings.

After how many weeks will they both have the same amount of money in their bank accounts?

Solution

We can use a table to show the pattern of savings for Isla and Javier. Each week we will add \$4 to Javier's total savings. Every two weeks we will add \$12 to Isla's savings. This means every odd week, Isla's savings will not change.

Week	Isla's Savings (in \$)	Javier's Savings (in \$)
Start	12	32
1	12	36
2	24	40
3	24	44
4	36	48
5	36	52
6	48	56
7	48	60
8	60	64
9	60	68
10	72	72

After 10 weeks of savings, Isla and Javier have the same amount of money in their bank accounts. However, there may be other weeks where they have the same savings.

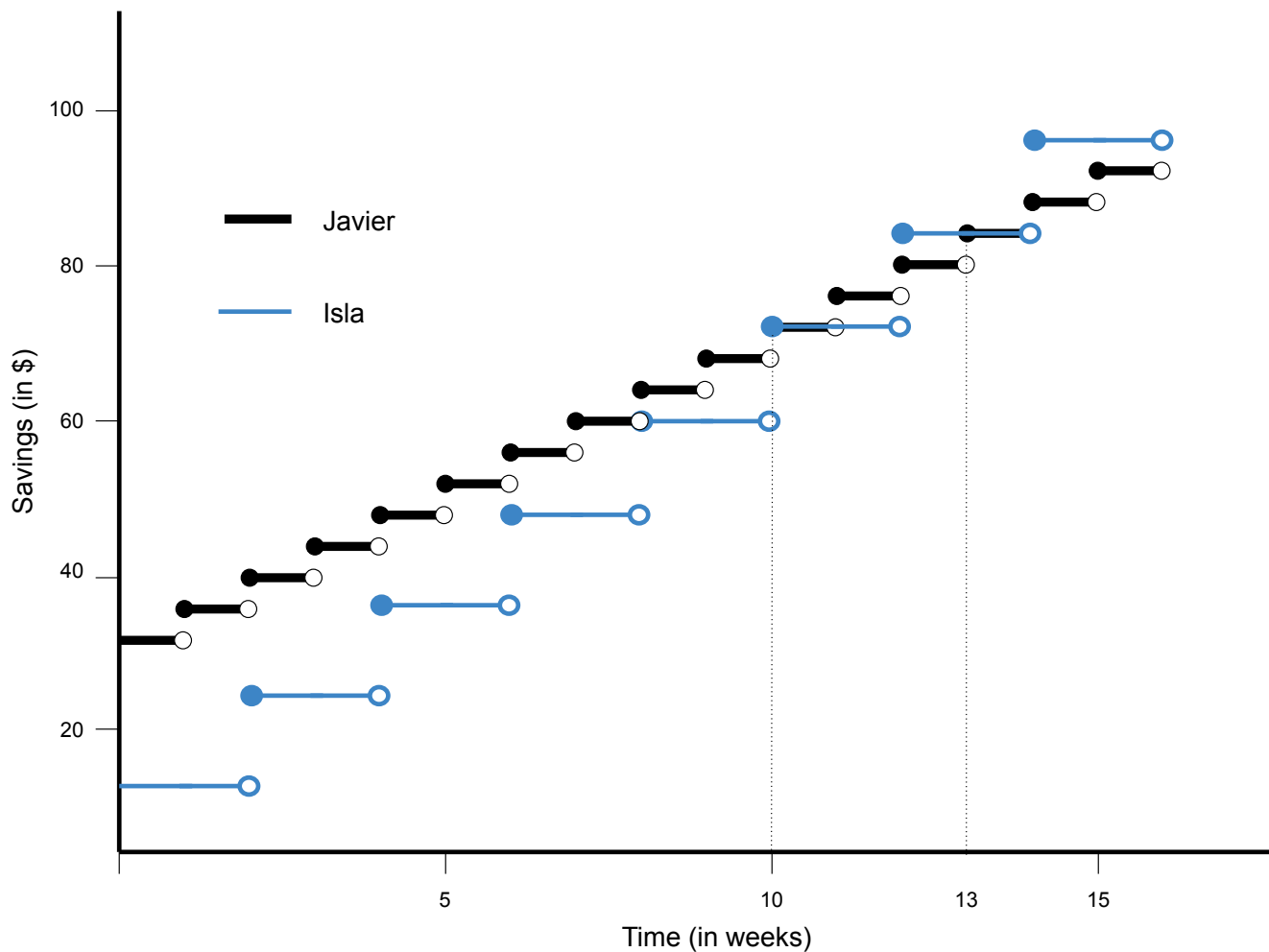
Week	Isla's Savings (in \$)	Javier's Savings (in \$)
11	72	76
12	84	80
13	84	84
14	96	88
15	96	92
16	108	96

At the end of the 13th week, Isla and Javier again have the same amount of money in their bank accounts. At the end of week 16, the difference between Isla's savings and Javier's savings is more than \$8. Since Javier only saves \$8 every two weeks and Isla saves more than Javier, his savings can no longer match Isla's. Therefore, at the end of week 13 they make the last contributions that give the same amount of money in their accounts.



Teacher's Notes

We can use a *step function* to describe Isla and Javier's savings. If we plot their savings over time, the amount in their bank accounts does not change between deposits. We can draw a graph with time on the x-axis and amount saved in dollars on the y-axis. The result would look like this:



As time passes, we can see that the graph for each person's savings looks like equally spaced steps. Where there is a jump between steps, the filled circle indicates that the actual value is at that point, and the empty circle indicates that the step does not include the value at that point of the graph. From the graph, we can see that the savings amounts overlap at week 10 and week 13. It may also look like there is an overlap at week 8. However at that point we can see that Javier's savings are actually at the level of the next step.

