Problem of the Week
Problem A and Solution
Time to Get Up

Problem

On the days she goes to work, Amy has a morning routine. The table below shows each activity she does and the time it takes to complete.

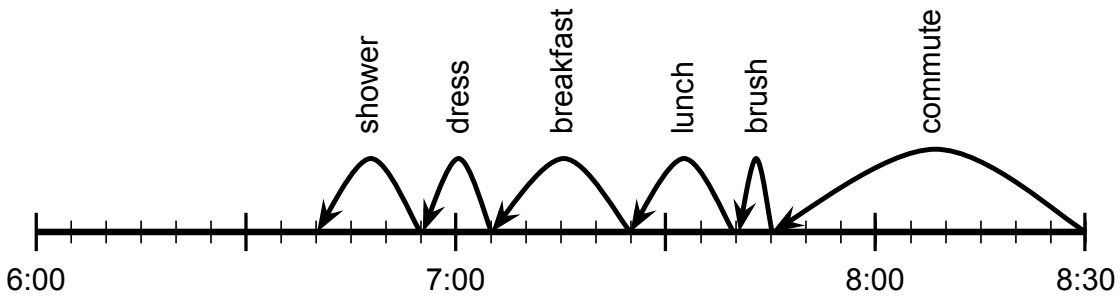
Activity	Time to Complete
Shower	15 minutes
Get dressed	10 minutes
Make and eat breakfast	20 minutes
Make lunch	15 minutes
Brush teeth and hair	5 minutes
Walk to work	45 minutes

Amy needs to be at work by 8:30 a.m. What is the latest time she could wake up in the morning, complete all of the activities in her routine, and get to work on time?

Justify your answer.

Solution

One way to solve this problem is to use a timeline. However in this case, we know the end time and we are trying to find the start time. This means we work backwards on the timeline. Each tick on the following timeline represents a 5 minute interval.



From the timeline, we can see that the latest Amy can wake up and make it to work on time is 6:40 a.m.

To solve the problem a second way, we can find the total time to complete all of her morning activities. Amy needs $15 + 10 + 20 + 15 + 5 + 45 = 110$ minutes.

Converting 110 minutes to hours and minutes, Amy needs 1 hour and 50 minutes. This is 10 minutes less than 2 hours. If she needed 2 hours, she would have to be up by 6:30 a.m. Since she requires 10 minutes less than 2 hours, she can get up 10 minutes after 6:30, or at 6:40 a.m.

Therefore, the latest Amy can wake up and make it to work on time is 6:40 a.m.

Teacher's Notes

The timeline in the solution is a variation of a number line. The spacing between tick marks on a number line normally represents integer or fraction intervals. In this case, the spacing between tick marks represents 5 minute (or $\frac{1}{12}$ of an hour) intervals.

Number lines are helpful tools for learning to add and subtract. Adding values means moving to the right on a number line and subtracting values means moving to the left. Many people find that addition involving time is more difficult than working with simple numbers, since the values that represent later times do not continuously increase. For example, if we were using a 12-hour clock, when we add 160 minutes to 11:50, the result is 2:30. In this case, the resulting time has a smaller number of hours and minutes compared to the original time. Using a number line can make these kinds of calculations easier.

We can calculate a sum like this without a number line by using quotient and remainder values.

- Calculate the total number of minutes represented by the starting time, 11:50, by multiplying the number of hours in the time by 60 and adding the number of minutes: $(60 \times 11) + 50 = 710$
- Add the number of minutes the time is increased by: $710 + 160 = 870$
- Calculate the quotient and remainder when dividing the sum by 60: $870 \div 60$ has quotient of 14 and a remainder of 30.
- The quotient represents the resulting hour and the remainder represents the minutes.
- If the new hour is more than 12, calculate the remainder when dividing the hour by 12: $14 \div 12$ has a quotient of 1 and a remainder of 2
- If the remainder is not 0, it represents the new hour. In this example, the resulting time is 2:30. Note that if the remainder is 0, then the hour would be 12.

This gets tricker though, when subtracting times rather than adding times. For example suppose we have a starting time of 2:30, and we want to know what time is 160 minutes earlier. If we try to follow the same steps, we start with the following calculations:

- Original time in minutes: $(2 \times 60) + 30 = 150$
- Subtract the number of minutes the time is decreased by: $150 - 160 = -10$

This gives us a negative number which is not easily translated into hours and minutes. It turns out that you can use quotient and remainder calculations to determine that -10 minutes represents the time of 11:50. However most students would not learn about quotient and remainder calculations involving negative numbers until after secondary school mathematics. So, using a number line is a good strategy when subtracting time.