CONTENTS

Using *McGraw-Hill Ryerson Calculus & Advanced Functions, Solutions*

McGraw-Hill Ryerson Calculus & Advanced Functions, *Solutions* provides complete model solutions to the following:
for each numbered section of *McGraw-Hill Ryerson Calculus & Advanced Functions*,
- every odd numbered question in the *Practise*
- all questions in the *Apply, Solve, Communicate*

Solutions are also included for all questions in these sections:
- Review
- Chapter Check
- Problem Solving: Using the Strategies

Note that solutions to the Achievement Check questions are provided in *McGraw-Hill Ryerson Calculus & Advanced Functions*, Teacher's Resource.

Teachers will find the completeness of the *McGraw-Hill Ryerson Calculus & Advanced Functions*, *Solutions* helpful in planning students' assignments. Depending on their level of ability, the time available, and local curriculum constraints, students will probably only be able to work on a selection of the questions presented in the *McGraw-Hill Ryerson Calculus & Advanced Functions* student text. A review of the solutions provides a valuable tool in deciding which problems might be better choices in any particular situation. The solutions will also be helpful in deciding which questions might be suitable for extra practice of a particular skill.

In mathematics, for all but the most routine of practice questions, multiple solutions exist. The methods used in *McGraw-Hill Ryerson Calculus & Advanced Functions*, *Solutions* are generally modelled after the examples presented in the student text. Although only one solution is presented per question, teachers and students are encouraged to develop as many different solutions as possible. An example of such a question is Page 30, Question 7, parts b) and c). The approximate values can be found by substitution as shown or by using the Value operation on the graphing calculator. Discussion and comparison of different methods can be highly rewarding. It leads to a deeper understanding of the concepts involved in solving the problem and to a greater appreciation of the many connections among topics.

Occasionally different approaches are used. This is done deliberately to enrich and extend the reader's insight or to emphasize a particular concept. In such cases, the foundations of the approach are supplied. Also, in a few situations, a symbol that might be new to the students is introduced. For example in Chapter 3 the dot symbol is used for multiplication. When a graphing calculator is used, there are often multiple ways of obtaining the required solution. The solutions provided here sometimes use different operations than the one shown in the book. This will help to broaden students' skills with the calculator.

There are numerous complex numerical expressions that are evaluated in a single step. The solutions are developed with the understanding that the reader has access to a scientific calculator, and one has been used to achieve the result. Despite access to calculators, numerous problems offer irresistible challenges to develop their solution in a manner that avoids the need for one, through the order in which algebraic simplifications are performed. Such challenges should be encouraged.

There are a number of situations, particularly in the solutions to Practise questions, where the reader may sense a repetition in the style of presentation. The solutions were developed with an understanding that a solution may, from time to time, be viewed in isolation and as such might require the full treatment.

The entire body of *McGraw-Hill Ryerson Calculus & Advanced Functions*, *Solutions* was created on a home computer in Te$_x$tures. Graphics for the solutions were created with the help of a variety of graphing software, spreadsheets, and graphing calculator output captured to the computer. Some of the traditional elements of the accompanying graphic support are missing in favour of the rapid capabilities provided by the electronic tools. Since many students will be working with such tools in their future careers, discussion of the features and interpretation of these various graphs and tables is encouraged and will provide a very worthwhile learning experience. Some solutions include a reference to a web site from which data was obtained. Due to the dynamic nature of the Internet, it cannot be guaranteed that these sites are still operational.

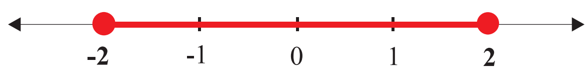
CHAPTER 1 Functions and Models

1.1 Functions and Their Use in Modelling

Practise

Section 1.1 Page 18 Question 1

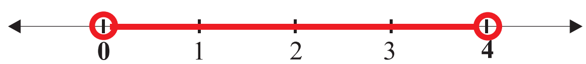
a) $x \in [-2, 2]$



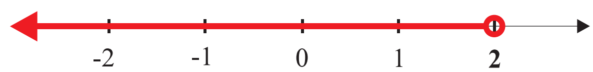
b) $x \in [4, 13]$

c) $x \in (-4, -1)$

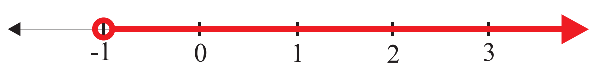
d) $x \in (0, 4)$



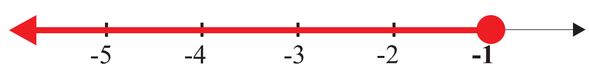
e) $x \in (-\infty, 2)$



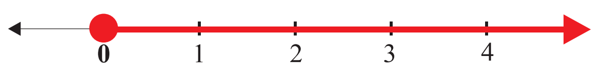
f) $x \in (-1, \infty)$



g) $x \in (-\infty, -1]$



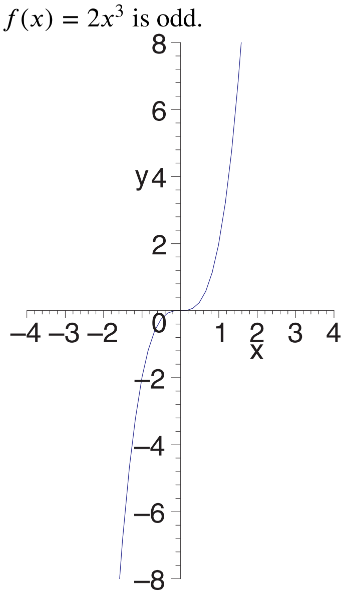
h) $x \in [0, \infty)$



Section 1.1 Page 18 Question 3

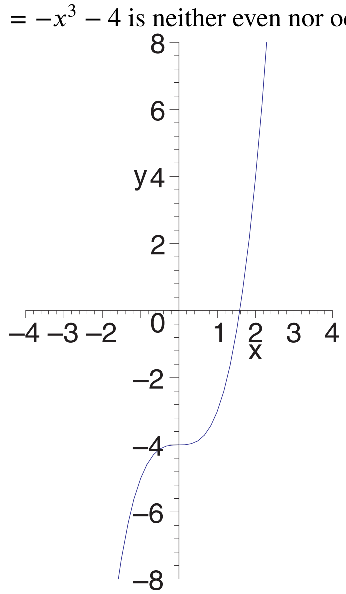
a)
$$\begin{aligned} f(-x) &= 2(-x)^3 \\ &= -2x^3 \\ &= -f(x) \end{aligned}$$

$f(x) = 2x^3$ is odd.



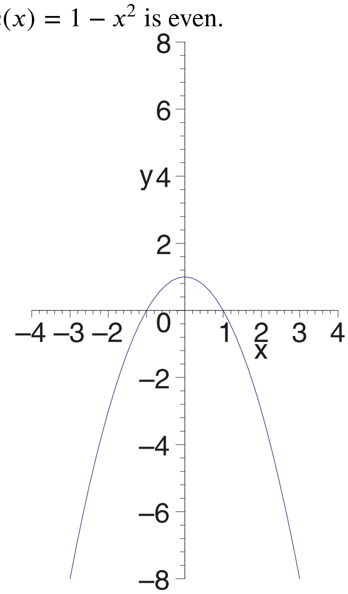
b)
$$\begin{aligned} g(-x) &= (-x)^3 - 4 \\ &= -x^3 - 4 \\ &\neq g(x) \text{ or } -g(x) \end{aligned}$$

$g(x) = -x^3 - 4$ is neither even nor odd.



c)
$$\begin{aligned} h(-x) &= 1 - (-x)^2 \\ &= 1 - x^2 \\ &= h(x) \end{aligned}$$

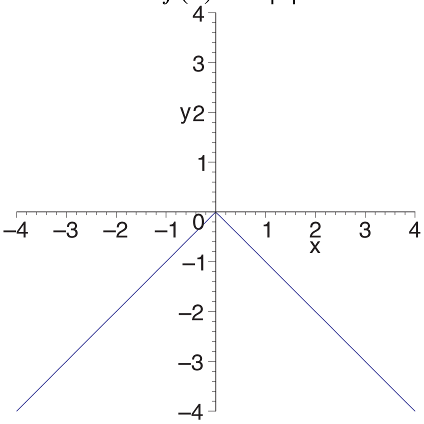
$h(x) = 1 - x^2$ is even.



Section 1.1 Page 19 Question 5

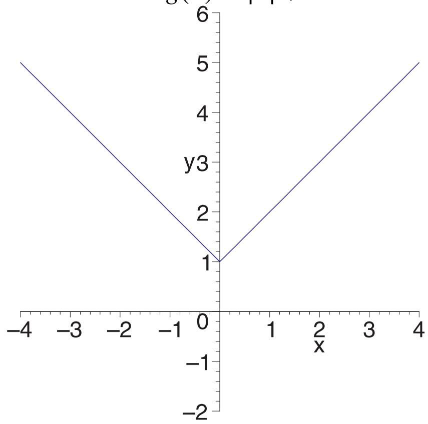
a)
$$\begin{aligned} f(-x) &= -|-x| \\ &= -|x| \\ &= f(x) \end{aligned}$$

The function $f(x) = -|x|$ is even.



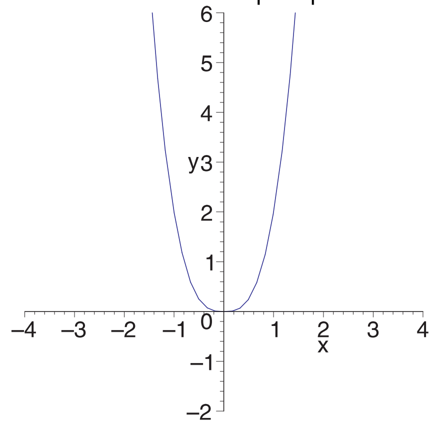
b)
$$\begin{aligned} g(-x) &= |-x| + 1 \\ &= |x| + 1 \\ &= g(x) \end{aligned}$$

The function $g(x) = |x| + 1$ is even.



c)
$$\begin{aligned} h(-x) &= \left|2(-x)^3\right| \\ &= \left|2x^3\right| \\ &= h(x) \end{aligned}$$

The function $h(x) = \left|2x^3\right|$ is even.



Section 1.1 Page 19 Question 7

a) Use $f(x) = \dfrac{1}{x^2}$.

i)
$$\begin{aligned} f(3) &= \frac{1}{3^2} \\ &= \frac{1}{9} \end{aligned}$$

ii)
$$\begin{aligned} f(-3) &= \frac{1}{(-3)^2} \\ &= \frac{1}{9} \end{aligned}$$

iii)
$$\begin{aligned} f\left(\frac{1}{3}\right) &= \frac{1}{\left(\frac{1}{3}\right)^2} \\ &= \frac{1}{\frac{1}{9}} \\ &= 9 \end{aligned}$$

iv)
$$\begin{aligned} f\left(\frac{1}{4}\right) &= \frac{1}{\left(\frac{1}{4}\right)^2} \\ &= \frac{1}{\frac{1}{16}} \\ &= 16 \end{aligned}$$

v)
$$\begin{aligned} f\left(\frac{1}{k}\right) &= \frac{1}{\left(\frac{1}{k}\right)^2} \\ &= \frac{1}{\frac{1}{k^2}} \\ &= k^2 \end{aligned}$$

vi)
$$\begin{aligned} f\left(\frac{k}{1+k}\right) &= \frac{1}{\left(\frac{k}{1+k}\right)^2} \\ &= \frac{1}{\frac{k^2}{(1+k)^2}} \\ &= \frac{(1+k)^2}{k^2} \end{aligned}$$

b) Use $f(x) = \dfrac{x}{1-x}$.

i)
$$\begin{aligned} f(3) &= \frac{3}{1-3} \\ &= -\frac{3}{2} \end{aligned}$$

ii)
$$\begin{aligned} f(-3) &= \frac{-3}{1-(-3)} \\ &= -\frac{3}{4} \end{aligned}$$

iii)
$$\begin{aligned} f\left(\frac{1}{3}\right) &= \frac{\frac{1}{3}}{1-\frac{1}{3}} \\ &= \frac{\frac{1}{3}}{\frac{2}{3}} \\ &= \frac{1}{2} \end{aligned}$$

iv) $f\left(\frac{1}{4}\right) = \dfrac{\frac{1}{4}}{1-\frac{1}{4}}$
$= \dfrac{\frac{1}{4}}{\frac{3}{4}}$
$= \dfrac{1}{3}$

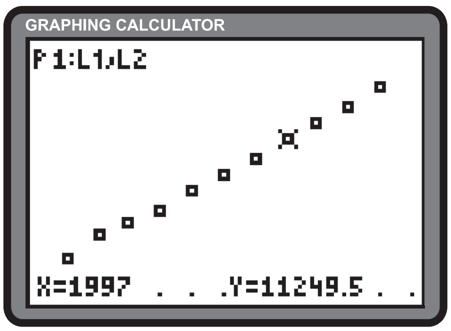
v) $f\left(\frac{1}{k}\right) = \dfrac{\frac{1}{k}}{1-\frac{1}{k}}$
$= \dfrac{\frac{1}{k}}{\frac{k-1}{k}}$
$= \dfrac{1}{k-1}$

vi) $f\left(\frac{k}{1+k}\right) = \dfrac{\frac{k}{1+k}}{1-\frac{k}{1+k}}$
$= \dfrac{\frac{k}{1+k}}{\frac{(1+k)-k}{1+k}}$
$= k$

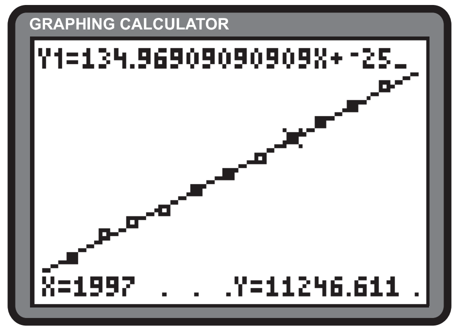
Section 1.1 Page 19 Question 9

Verbally: Answers will vary.

Visual representation with a scatter plot:



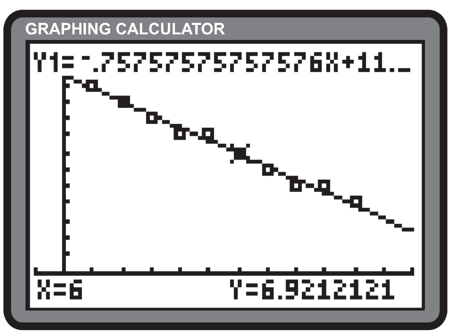
Algebraic representation using linear regression:



The graphing calculator suggests the function $f(x) \doteq 134.9690x - 258\,290$.

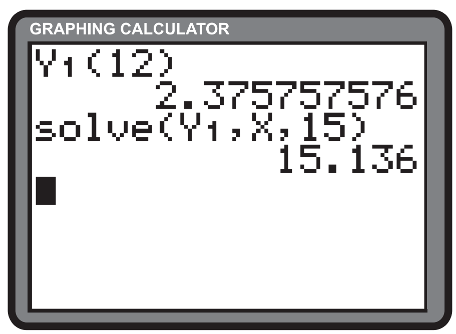
Section 1.1 Page 19 Question 11

a) The calculator suggests $y = -0.\overline{75}x + 11.4\overline{6}$ as the line of best fit.



b) $y(12) \doteq 2.3\overline{75}$

c) When $y = 0$, $x \doteq 15.136$.



Apply, Solve, Communicate

Section 1.1 Page 20 Question 13

Consider the function $h(x)$, where

$$h(x) = f(x)g(x)$$

If $f(x)$ and $g(x)$ are odd functions, then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= -f(x)[-g(x)] \\ &= f(x)g(x) \\ &= h(x) \end{aligned}$$

The product of two odd functions is an even function.

Section 1.1 Page 20 Question 14

Consider the function $h(x)$, where

$$h(x) = \frac{f(x)}{g(x)}$$

If $f(x)$ and $g(x)$ are even functions, then

$$\begin{aligned} h(-x) &= \frac{f(-x)}{g(-x)} \\ &= \frac{f(x)}{g(x)} \\ &= h(x) \end{aligned}$$

The quotient of two even functions is an even function.

Section 1.1 Page 20 Question 15

The product of an odd function and an even function is an odd function. Consider $h(x)$, where

$$h(x) = f(x)g(x)$$

If $f(x)$ is odd and $g(x)$ is even, then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= -f(x)g(x) \\ &= -h(x) \end{aligned}$$

Section 1.1 Page 20 Question 16

a) Since $y = f(x)$ is odd, $f(-x) = -f(x)$ for all x in the domain of f. Given that 0 is in the domain of f, we have $f(-0) = -f(0) \Rightarrow f(0) = -f(0) \Rightarrow 2f(0) = 0 \Rightarrow f(0) = 0$.

b) An odd function for which $f(0) \neq 0$ is $f(x) = \dfrac{|x|}{x}$

Section 1.1 Page 20 Question 17

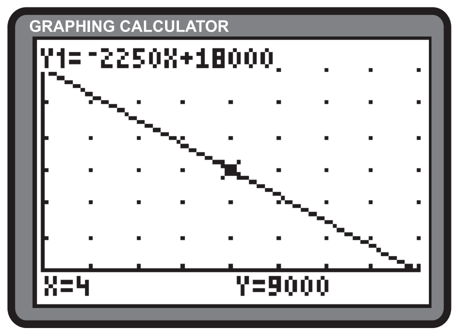
a) Let V be the value, in dollars, of the computer equipment after t years. From the given information, the points $(t, V) = (0, 18\,000)$ and $(t, V) = (4, 9000)$ are on the linear function. The slope of the line is $m = \dfrac{9000 - 18\,000}{4 - 0}$ or -2250. The linear model can be defined by the function $V(t) = 18\,000 - 2250t$.

b) $V(6) = 18\,000 - 2250(6)$ or \$4500

c) The domain of the function in the model is $t \in [0, 8]$. (After 8 years the equipment is worthless.)

d) The slope represents the annual depreciation of the computer equipment.

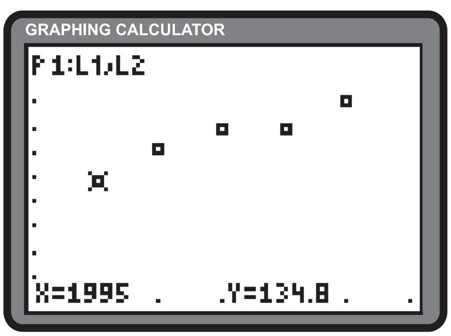
e)



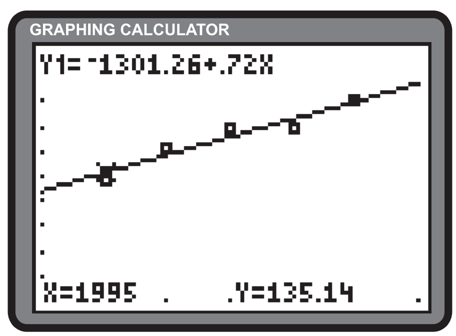
f) Since the function is given to be linear, its slope does not change.

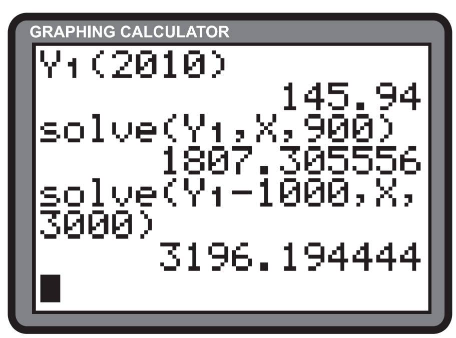
Section 1.1 Page 20 Question 18

a)



b) The line of best fit is $y = 0.72x - 1301.26$.





c) The model suggests the population reaches 145 940 in the year 2010.

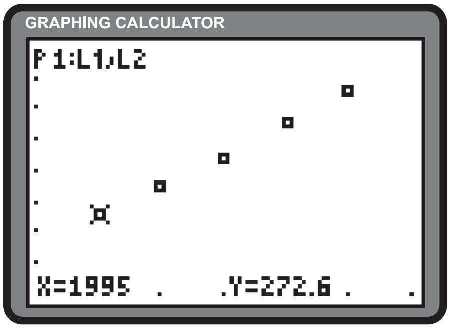
d) The model suggests the population was 0 in 1807. No.

e) The model suggests the population will reach 1 000 000 in the year 3196. Answers will vary.

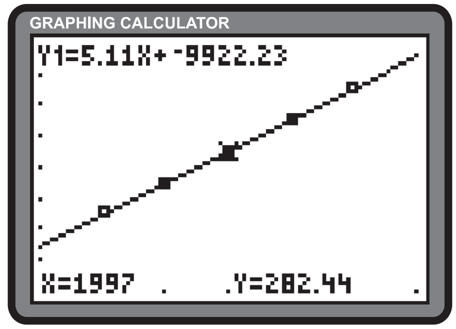
f) No. Explanations will vary.

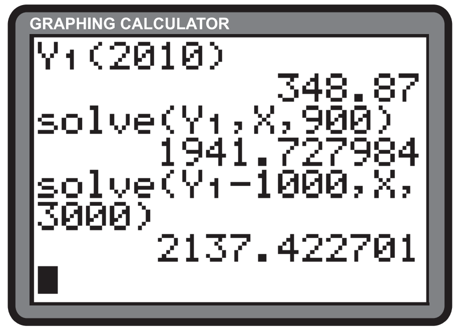
Section 1.1 Page 20 Question 19

a)



b) The line of best fit is $y = 5.11x - 9922.23$.





c) The model suggests the population will reach 348 870 in the year 2010.

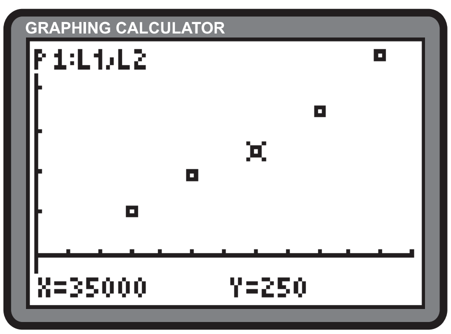
d) The model suggests the population was 0 in 1941.

e) The model suggests the population will reach 1 000 000 in the year 2137. No.

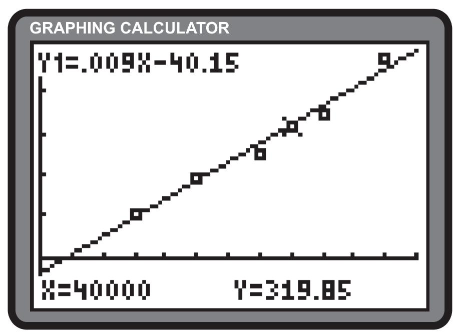
f) Explanations will vary.

Section 1.1 Page 20 Question 20

a)



b) The line of best fit is $y = 0.009x - 40.15$.



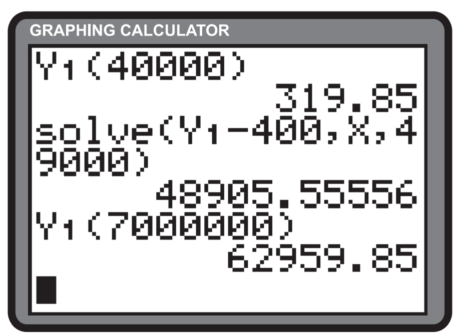
c) Estimates will vary. The model suggests annual pet expenses of approximately $319.85.

d) Estimates will vary. The model suggests an annual income of approximately $48 905.56 results in annual pet expenses of $400.

e) The model predicts pet expenses of approximately $62 959.85. Explanations will vary.

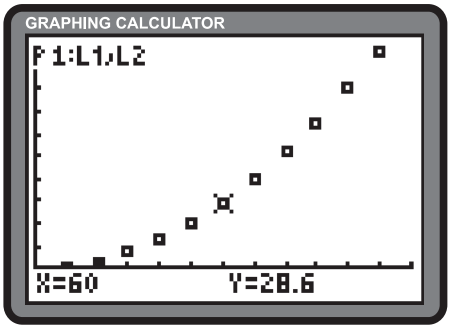
f) The model suggests an annual income of $ – 40.15. Explanations will vary.

g) No. Explanations will vary.

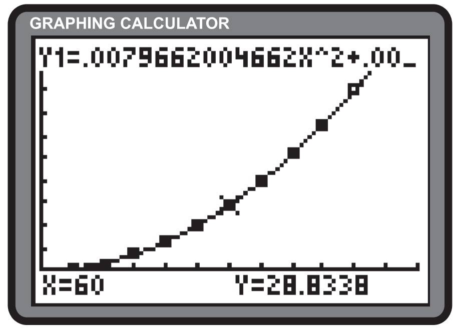


Section 1.1 Page 20 Question 21

a)



b) Answers will vary. Using the quadratic regression feature of the calculator yields an approximation of $d = 0.008s^2 + 0.002s + 0.059$.

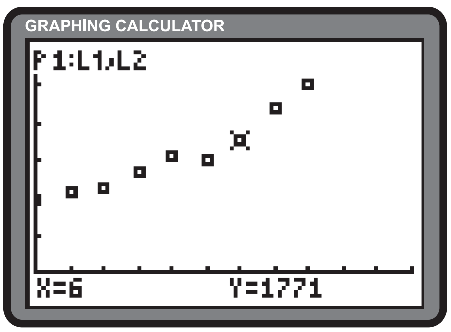


c) As speed increases the slope of the curve increases.

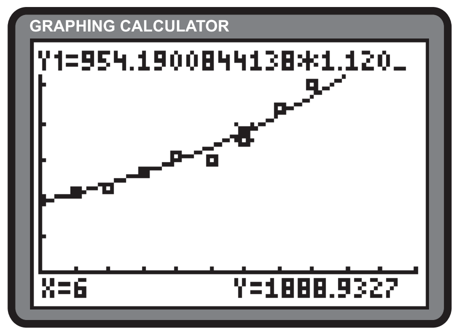
d) Answers will vary. This curve is steeper, and its slope increases more quickly.

Section 1.1 Page 20 Question 22

a) A scatter plot of the data appears below.

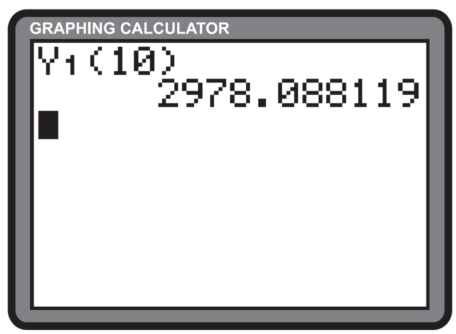


b) The **ExpReg feature** of the calculator approximates the data with $V = 954.19(1.12)^t$.



c) The slope increases with time.

d) The model predicts the value of the investment after 10 years to be approximately \$2978.09.



Section 1.1 Page 20 Question 23

All constant functions are even functions. The constant function $f(x) = 0$ is both even and odd.

Section 1.1 Page 20 Question 24

The sum of an odd function and an even function can be neither odd nor even, unless one of the functions is $y = 0$. Let $f(x)$ be even and $g(x)$ be odd, and let $h(x) = f(x) + g(x)$.

If $h(x)$ is even, then

$$\begin{aligned} h(-x) &= h(x) \\ f(-x) + g(-x) &= f(x) + g(x) \\ f(x) - g(x) &= f(x) + g(x) \\ 2g(x) &= 0 \\ g(x) &= 0 \end{aligned}$$

If $h(x)$ is odd, then

$$\begin{aligned} h(-x) &= -h(x) \\ f(-x) + g(-x) &= -f(x) - g(x) \\ f(x) - g(x) &= -f(x) - g(x) \\ 2f(x) &= 0 \\ f(x) &= 0 \end{aligned}$$

Section 1.1   Page 20   Question 25

Yes; only the function $f(x) = 0$. If a function $f(x)$ is both even and odd, then

$$\begin{aligned} f(-x) &= f(x) \\ f(-x) &= -f(x) \end{aligned}$$

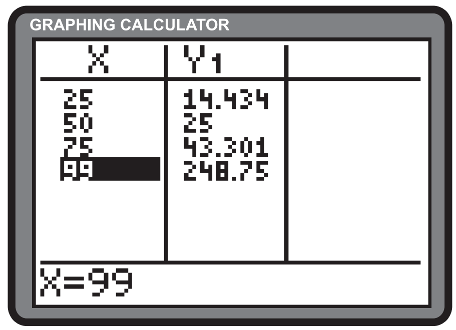
Thus,

$$\begin{aligned} f(x) &= -f(x) \\ 2f(x) &= 0 \\ f(x) &= 0 \end{aligned}$$

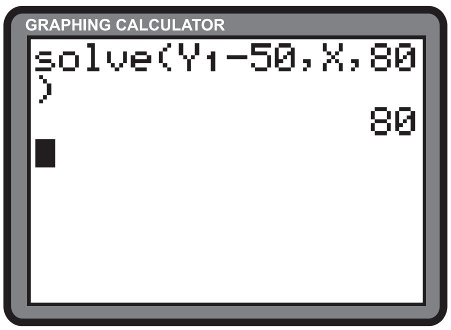
Section 1.1   Page 20   Question 26

a) A possible domain is $x \in [0, 100)$. Explanations may vary.

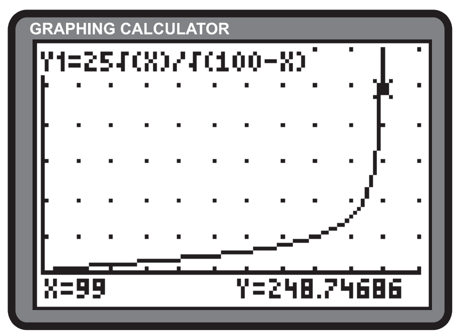
b) Define $y_1 = \dfrac{25\sqrt{x}}{\sqrt{100-x}}$. The costs of removal for the percent given appear in the table below as \$14 434, \$25 000, \$43 301 and \$248 750.



c) The calculator confirms that an estimate of 80% of pollutant can be removed for \$50 000.



d) Since an attempt to evaluate $C(100)$ results in division by zero, the model suggests that no amount of money will remove all the pollutant.

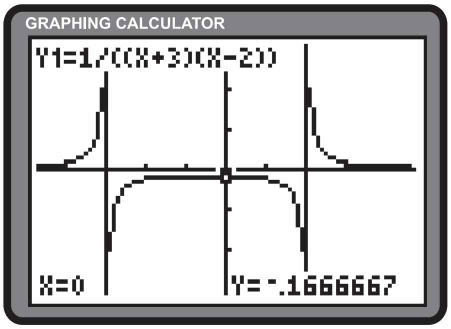


1.2 Lies My Graphing Calculator Tells Me

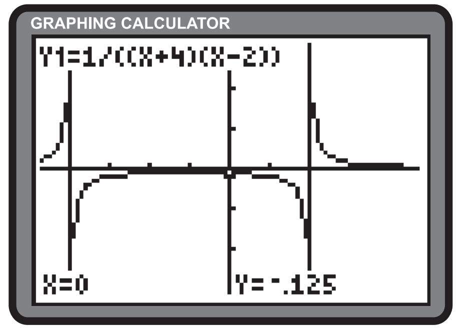
Apply, Solve, Communicate

Section 1.2 Page 28 Question 1

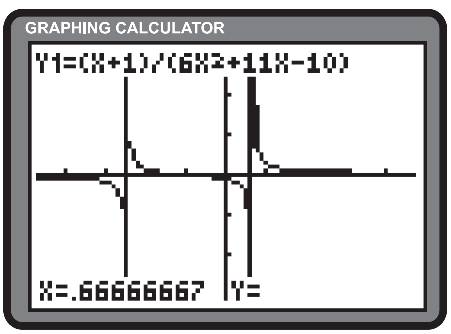
a) The equations of the two vertical asymptotes are $x = -3$ and $x = 2$. There are no x-intercepts. The y-intercept is $-\frac{1}{6}$.



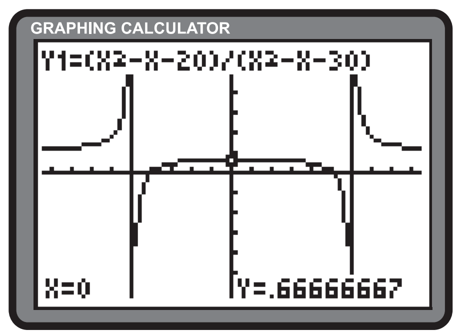
b) Factoring gives $y = \frac{1}{(x+4)(x-2)}$. The equations of the two vertical asymptotes are $x = -4$ and $x = 2$. There are no x-intercepts. The y-intercept is -0.125 or $-\frac{1}{8}$.



c) Factoring gives $y = \frac{x+1}{(2x+5)(3x-2)}$. The equations of the two vertical asymptotes are $x = -\frac{5}{2}$ and $x = \frac{2}{3}$. Setting $y = 0$ yields an x-intercept of -1. Setting $x = 0$ yields a y-intercept of $-\frac{1}{10}$.

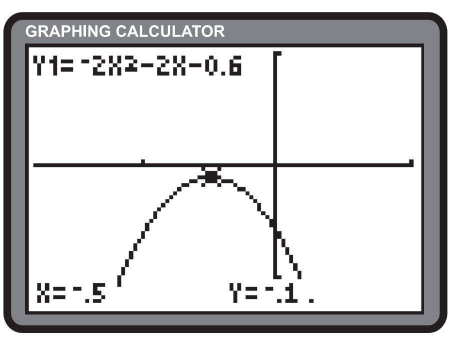


d) Factoring the numerator and denominator gives $y = \frac{(x-5)(x+4)}{(x-6)(x+5)}$. The equations of the two vertical asymptotes are $x = -5$ and $x = 6$. The x-intercepts are -4 and 5. The y-intercept is $\frac{2}{3}$.

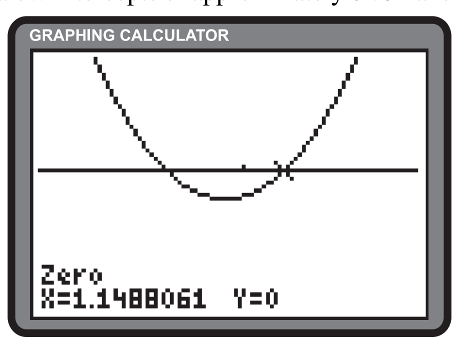


Section 1.2 Page 28 Question 2

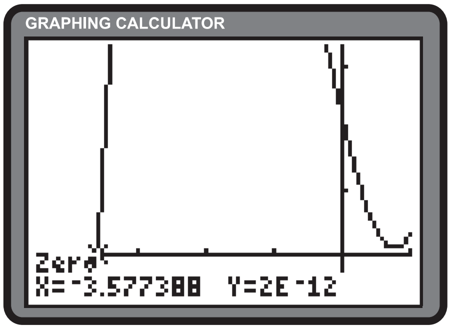
a) $y = -2x^2 - 2x - 0.6$ has no x-intercepts.



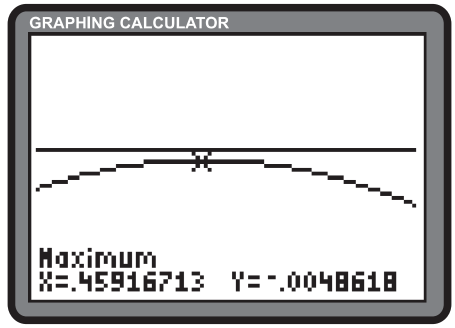
b) The **Zero operation** of the graphing calculator reveals x-intercepts of approximately 0.731 and 1.149.



c) $y = x^3 + 2x^2 - 5x + 2.3$ has an x-intercept of approximately -3.578.

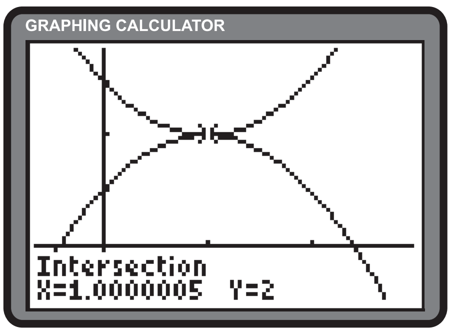


d) The **Zero operation** of the graphing calculator reveals an x-intercept of approximately 3.582. Zooming in on the interval around $x = 0.459$ several times reveals no point of intersection of the function with the x-axis.

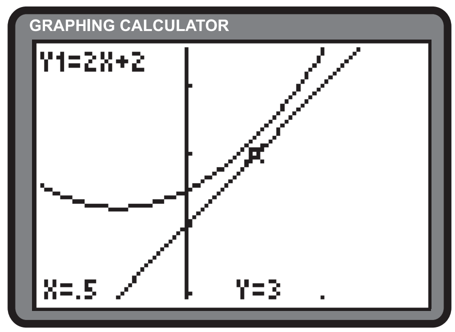


Section 1.2 Page 28 Question 3

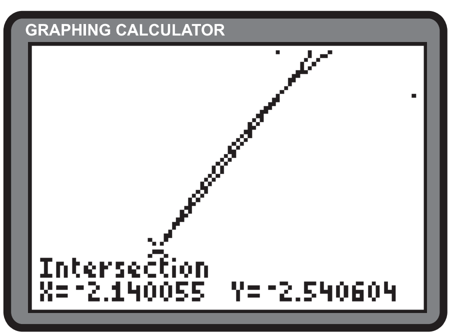
a) The **Intersect operation** suggests a point of intersection at $(1, 2)$. Substitution confirms this result.

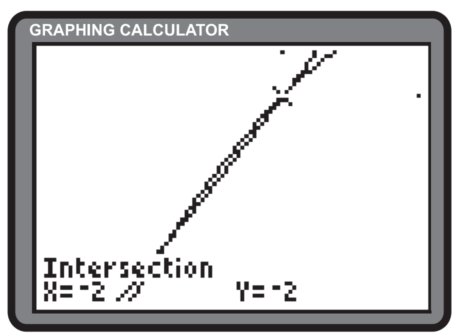


b) Zooming in on the interval around $x = 0.5$ reveals that the curves do not intersect.

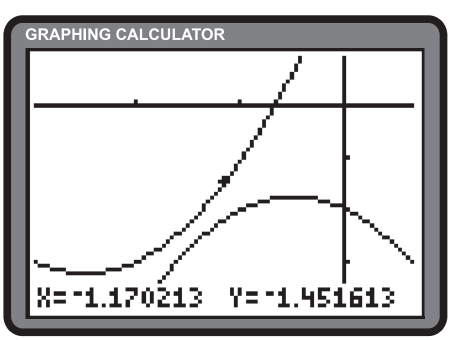


c) Zooming in on the interval around $x = -2$ reveals that the curves intersect twice in this neighbourhood. The coordinates are $(-2, -2)$ and approximately $(-2.1401, -2.5406)$. The calculator identifies the third point of intersection with the approximate coordinates $(5.1401, 77.541)$.



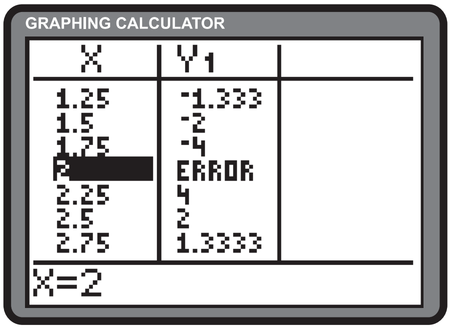


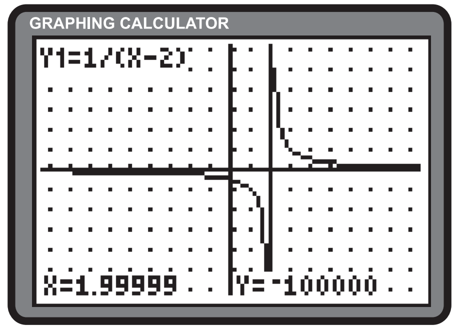
d) Zooming in on the interval around $x = -1$ reveals that the curves do not intersect.



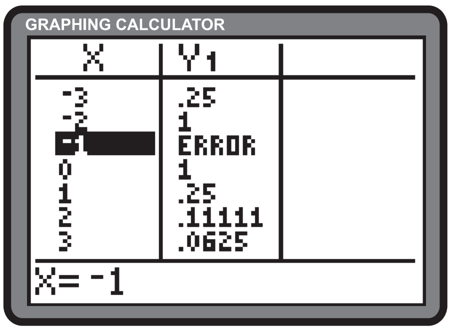
Section 1.2 Page 28 Question 4

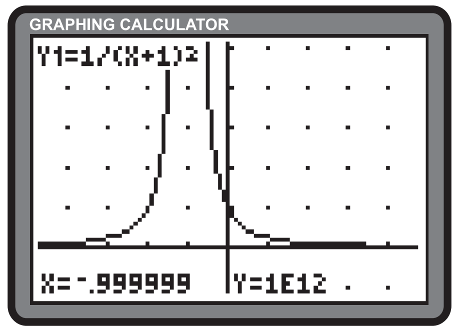
a) To avoid division by zero, $x - 2 \neq 0$, so the domain is $x \in \mathbb{R},\ x \neq 2$. The range is $y \in \mathbb{R},\ y \neq 0$.



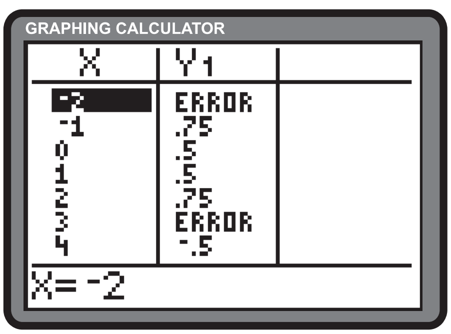


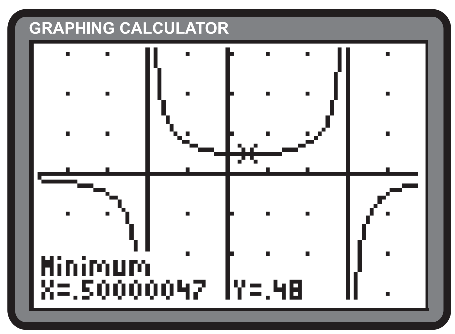
b) To avoid division by zero, $x + 1 \neq 0$, so the domain is $x \in \mathbb{R},\ x \neq -1$. The range is $y \in (0, \infty)$.



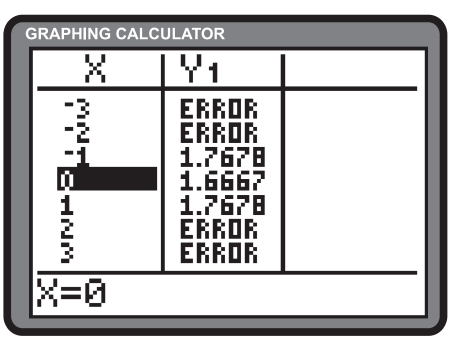


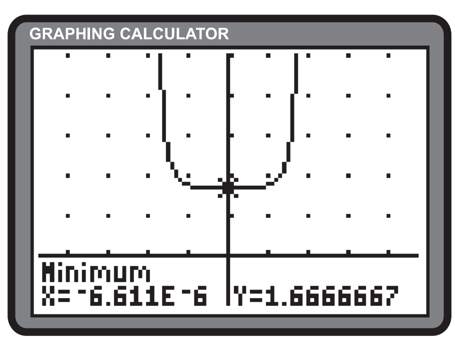
c) The function can be rewritten as $y = \dfrac{-3}{(x+2)(x-3)}$. To avoid division by zero, $(x+2)(x-3) \neq 0$, so the domain is $x \in \mathbb{R},\ x \neq -2, 3$. The range is $y \in (-\infty, 0)$ or $y \in [0.48, +\infty)$.





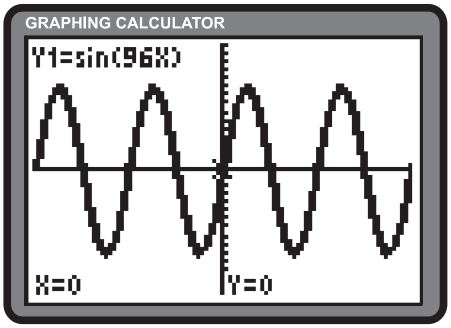
d) To avoid a negative radicand, $9 - x^4 \geq 0$, so $x \in \left[-\sqrt{3}, \sqrt{3}\right]$. To avoid division by zero, a further restriction limits the domain to $x \in \left(-\sqrt{3}, \sqrt{3}\right)$. The range is $y \in \left[\dfrac{5}{3}, +\infty\right)$.





Section 1.2 Page 28 Question 5

a)



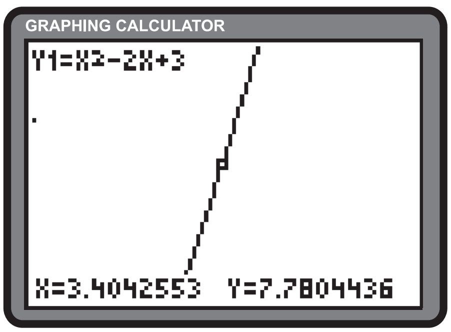
b) Answers may vary.

c) It will not work in any window. In the window given in part a), it will work for $y = \cos(2x)$ and $y = \cos(96x)$.

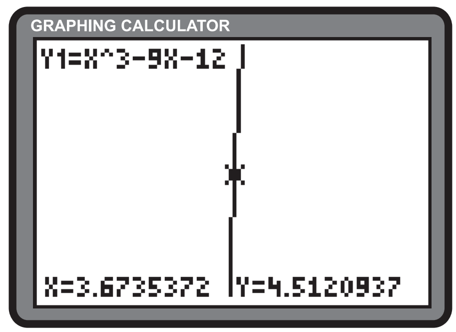
Section 1.2 Page 28 Question 6

For each of these solutions, use the **Zoom menu** until the segment of the graph appears linear.

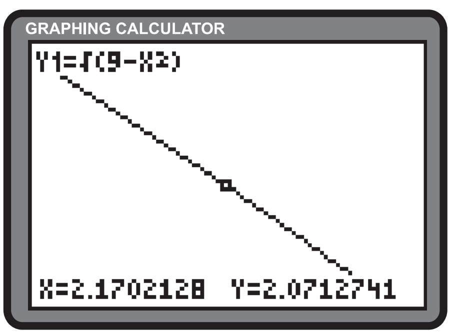
a) Answers may vary.



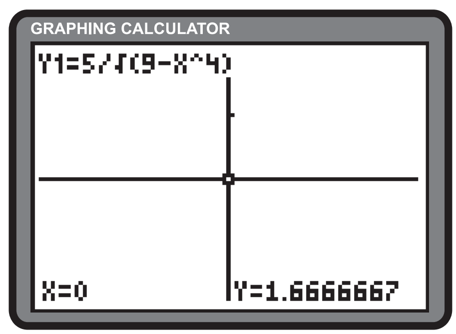
b) Answers may vary.



c) Answers may vary.

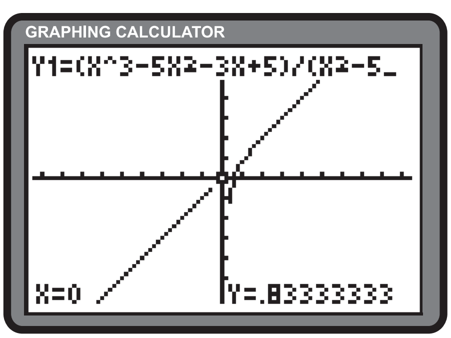


d) Answers may vary.

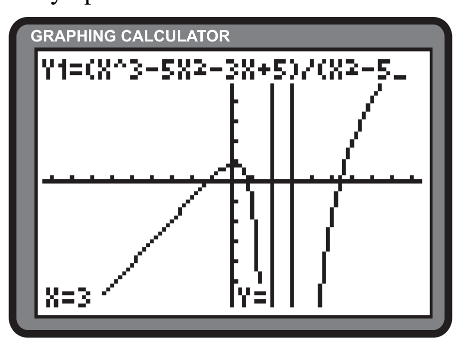


Section 1.2 Page 28 Question 7

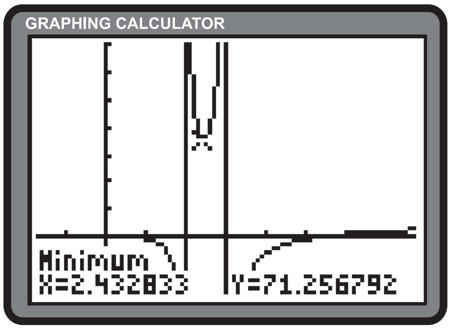
In the window $x \in [-94, 94]$, $y \in [-62, 62]$, the graph appears linear as $|x| \to \infty$.



In the window $x \in [-9.4, 9.4]$, $y \in [-6.2, 6.2]$, the vertical asymptotes of $x = 2$ and $x = 3$ are revealed.

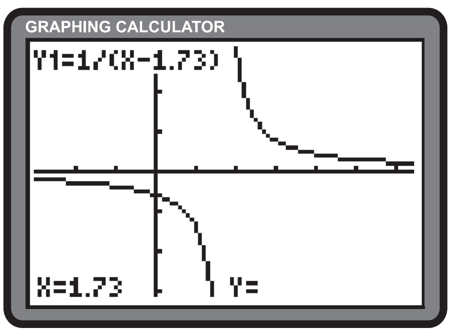


In the window $x \in [-1.7, 7.7]$, $y \in [-40, 140]$, the part of the function between the vertical asymptotes is highlighted.

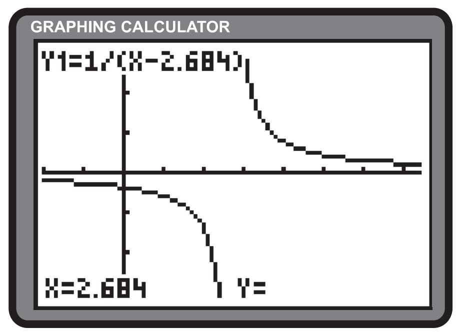


Section 1.2 Page 28 Question 8

a) The graphing of a function is due in part to the calculator sampling elements within the domain from Xmin to Xmax. The function $y = \dfrac{1}{x - 1.73}$ is undefined at $x = 1.73$. To graph the function properly, one sample must fall exactly at 1.73 so the discontinuity in the graph can be detected. Translating the **ZDecimal window** to the right 1.73 units results in a correct graph. Thus, use the window $x \in [-2.97, 6.43]$, $y \in [-3.1, 3.1]$



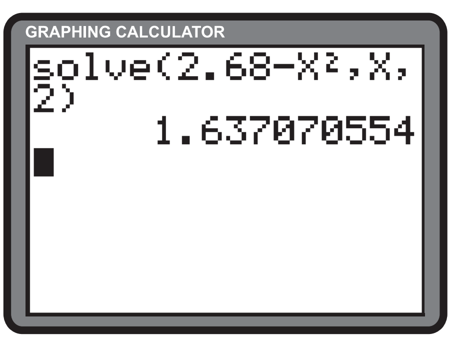
b) The graphing of a function is due in part to the calculator sampling elements within the domain from Xmin to Xmax. The function $y = \dfrac{1}{x - 2.684}$ is undefined at $x = 2.684$. To graph the function properly, one sample must fall exactly at 2.684 so the discontinuity in the graph can be detected. Translating the **ZDecimal window** to the right 2.684 units results in a correct graph. Thus, use the window $x \in [-2.016, 7.384]$, $y \in [-3.1, 3.1]$

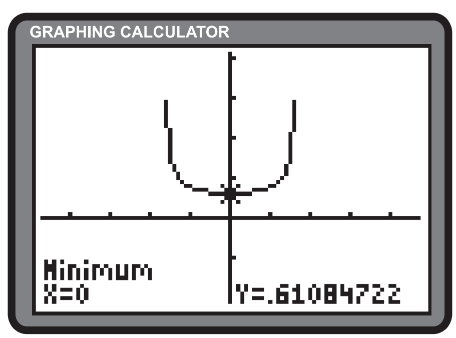


Section 1.2 Page 28 Question 9

The domain is restricted to $2.68 - x^2 > 0$ or $x^2 < 2.68$. Thus, $|x| < \sqrt{2.68}$, or $|x| < 1.6371$.

The **Minimum operation** of the calculator helps approximate the range: $y \in [0.6108, \infty)$.





Review of Key Concepts

1.1 Functions and Their Use in Modelling

Section Review Page 30 Question 1

a)

x	y
-2	0
-1	0.5
1	3
3	1.5

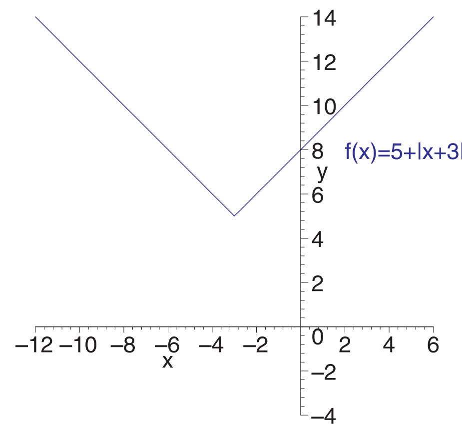
b) Domain: $x \in [-5, \infty)$

Section Review Page 30 Question 2

a)

x	y
-6	$5 + \lvert -6 + 3 \rvert = 8$
-5	$5 + \lvert -5 + 3 \rvert = 7$
-4	$5 + \lvert -4 + 3 \rvert = 6$
-3	$5 + \lvert -3 + 3 \rvert = 5$
-2	$5 + \lvert -2 + 3 \rvert = 6$
-1	$5 + \lvert -1 + 3 \rvert = 7$
0	$5 + \lvert 0 + 3 \rvert = 8$

b)



c) The function can be described as the sum of 5 and the distance from x to -3 on a number line.

Section Review Page 30 Question 3

a) $x \le 0$ is written as $x \in (-\infty, 0]$.
b) $-4 < x$ is written as $x \in (-4, +\infty)$.
c) $-5 \le x \le 5$ is written as $x \in [-5, 5]$.

Section Review Page 30 Question 4

a) For all but $x = \pm 2$, $f(x) = f(-x)$. $f(x)$ is neither even nor odd.
b) For each x-value, $g(x) = -g(-x)$. $g(x)$ is an odd function.
c) For each x-value, $h(x) = h(-x)$. $h(x)$ is an even function.

Section Review Page 30 Question 5

a)

$$\begin{aligned} f(-x) &= (-x)^2 + (-x) \\ &= x^2 - x \\ &\ne f(x) \text{ or } -f(x) \end{aligned}$$

$f(x)$ is neither even nor odd.

b)

$$\begin{aligned} g(-x) &= \left|(-x)^2 - 3\right| \\ &= \left|x^2 - 3\right| \\ &= g(x) \end{aligned}$$

$g(x)$ is an even function.

c)

$$\begin{aligned} h(-x) &= 5(-x) \\ &= -5x \\ &= -h(x) \end{aligned}$$

$h(x)$ is an odd function.

d)

$$\begin{aligned} r(-x) &= (-x)^2 - |-x| \\ &= x^2 - |x| \\ &= r(x) \end{aligned}$$

$r(x)$ is an even function.

e)

$$\begin{aligned} s(-x) &= \frac{1}{(-x)^3} \\ &= -\frac{1}{x^3} \\ &= -s(x) \end{aligned}$$

$s(x)$ is an odd function.

f)

$$\begin{aligned} t(-x) &= \left((-x)^3\right)^3 \\ &= -\left(x^3\right)^3 \\ &= -t(x) \end{aligned}$$

$t(x)$ is an odd function.

Section Review Page 30 Question 6

Consider the function $h(x)$, where

$$h(x) = f(x)g(x)$$

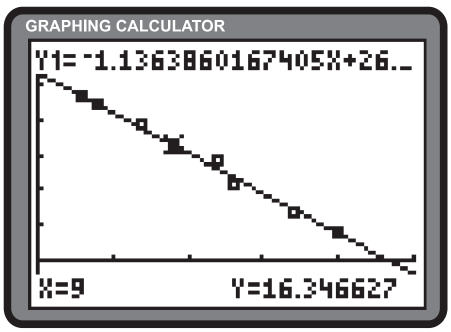
If $f(x)$ and $g(x)$ are even functions, then (1) becomes,

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \\ &= h(x) \end{aligned}$$

The product of two even functions is an even function.

Section Review Page 30 Question 7

a) The calculator suggests $d \doteq -1.14t + 26.57$ as the line of best fit.

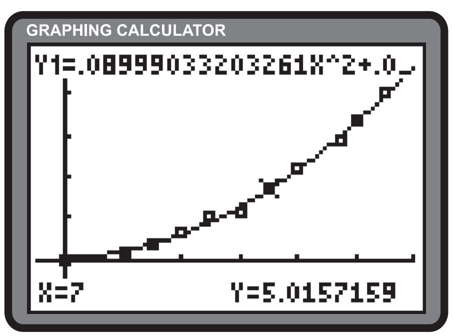


b) $d(14) = -1.14(14) + 26.57$
$= 10.61$

c) $d(23) = -1.14(23) + 26.57$
$= 0.35$

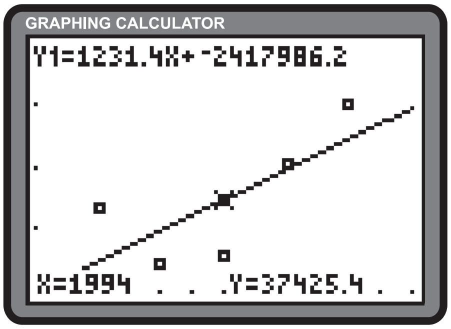
Section Review Page 30 Question 8

The **QuadReg feature** suggests the model $y \doteq 0.09x^2 + 0.08x + 0.03$.



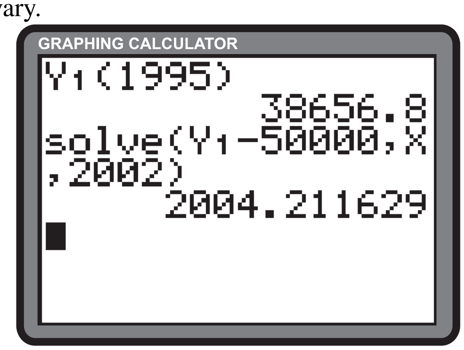
Section Review Page 31 Question 9

a) The calculator suggests $P \doteq 1231.4t - 2\,417\,986.2$ as the line of best fit, where P is the number of passengers, in thousands, and t is the year.



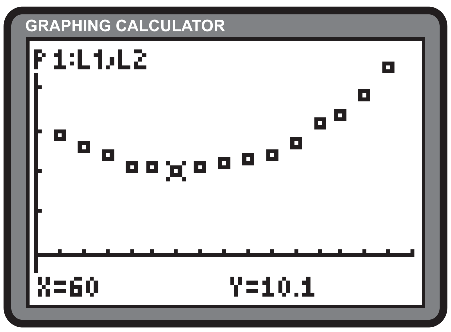
b) $P(1995) \doteq 38\,656\,800$. This result is higher than the actual value.

c) The model suggests that the number of passengers will reach 50 000 000 in 2004. No. Answers will vary.



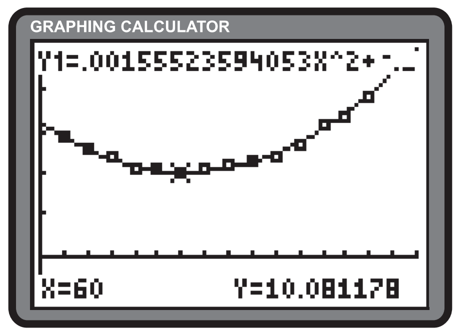
Section Review Page 31 Question 10

a)



b) The data suggest the best fuel economy of 10.1 L/100 km is achieved at 60 km/h.

c) The **QuadReg feature** suggests the model $F \doteq 0.001\,56s^2 - 0.192\,68s + 16.043\,16$.



d) The curve would be higher, and the slope would be steeper. The lowest point might change.

e) Translation of the curve downward would yield reduced fuel consumption for the same speeds.

Section Review Page 31 Question 11

a) To avoid division by zero, $x + 1 \neq 0$. The domain is $x \in \Re,\ x \neq -1$.

b) To avoid division by zero, $3 - x \neq 0$. The domain is $x \in \Re,\ x \neq 3$.

c) To avoid a negative radicand, $x + 1 \geq 0$. The domain is $x \in [-1, +\infty)$.

d) To avoid a negative radicand and division by zero, $3 - x > 0$. The domain is $x \in (-\infty, 3)$.

e) To avoid a negative radicand, $|x| - 1 \geq 0$. The domain is $x \in (-\infty, -1]$ or $x \in [1, +\infty)$.

f) $x^2 + 3 > 0$ for all real numbers. The domain is $x \in \Re$.

g) The denominator can be written as $(x-1)^2$. To avoid division by zero, $(x-1)^2 \neq 0$. The domain is $x \in \Re,\ x \neq 1$.

h) The denominator can be written as $(x+3)(x-2)$. To avoid division by zero, $(x+3)(x-2) \neq 0$. The domain is $x \in \Re,\ x \neq -3, 2$.

1.2 Lies My Graphing Calculator Tells Me

Section Review Page 31 Question 12

Answers will vary.

Section Review Page 31 Question 13

Answers will vary.

Chapter Test

Section Chapter Test Page 32 Question 1

a) For $f(x) = x^2$,

i) $f(1) = 1^2$ or 1

ii) $f(-1) = (-1)^2$ or 1

iii) $f(2) = (2)^2$ or 4

iv) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2$ or $\frac{1}{4}$

b) For $f(x) = 1 - x^3$,

i) $f(1) = 1 - 1^3$ or 0

ii) $f(-1) = 1 - (-1)^3$ or 2

iii) $f(2) = 1 - (2)^3$ or -7

iv) $f\left(\frac{1}{2}\right) = 1 - \left(\frac{1}{2}\right)^3$ or $\frac{7}{8}$

Section Chapter Test Page 32 Question 2

a) $-4 < x < 10$ is written $x \in (-4, 10)$. **b)** $x \le 5$ is written $x \in (-\infty, 5]$. **c)** $0 \le x$ is written $x \in [0, +\infty)$.

Section Chapter Test Page 32 Question 3

a) Since the graph of the function is rotationally symmetric with respect to the origin, the function is odd.
b) Since the graph of the function is symmetric with respect to neither the origin nor the y-axis, the function is neither odd nor even.
c) Since the graph of the function is symmetric with respect to the y-axis, the function is even.

Section Chapter Test Page 32 Question 4

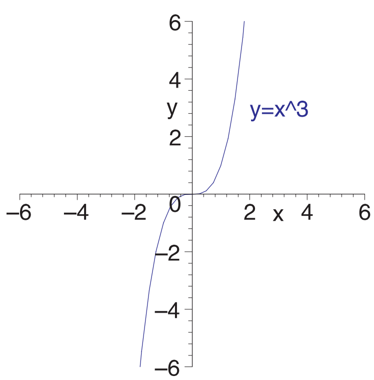
For each x-value, since $f(x) \ne f(-x)$ and $f(x) \ne -f(-x)$, $f(x)$ is neither even nor odd.

For each x-value, $g(x) = g(-x)$. $g(x)$ is an even function.

For each x-value, $h(x) = -h(-x)$. $h(x)$ is an odd function.

Section Chapter Test Page 32 Question 5

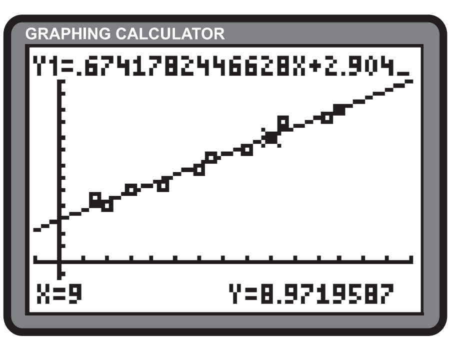
a)



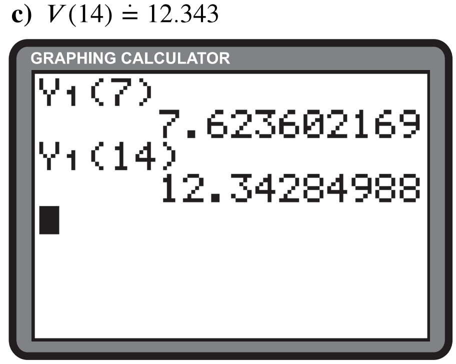
b) The slope has large positive values, decreases to 0 at $x = 0$, and then increases to large positive values.
c) The function is rotationally symmetric with respect to the origin (odd).
d) $y = x^5$ has a sharper turn on $(-1, 1)$, and is steeper outside this interval.

Section Chapter Test Page 32 Question 6

a) The calculator suggests the model $V \doteq 0.674r + 2.904$.

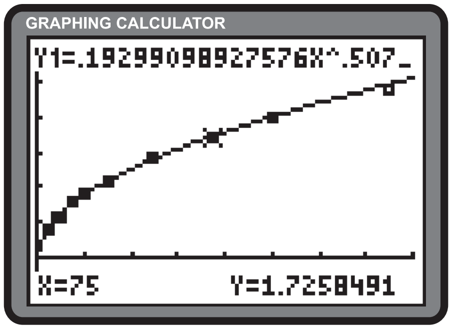


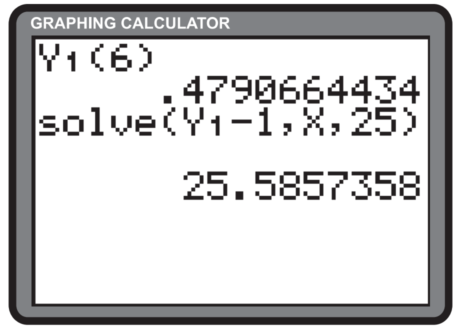
b) $V(7) \doteq 7.624$
c) $V(14) \doteq 12.343$



Section Chapter Test Page 32 Question 7

a)





b) Answers may vary. The **PwrReg feature** of the calculator suggests the model $P \doteq 0.193l^{0.507}$, where P is the period in seconds and l is the length of the pendulum in centimetres.

c) $P(6) \doteq 0.479$

d) The **Solve feature** of the calculator suggests a $l \doteq 25.586$ cm would yield a period of 1 s.

e) Shortening the pendulum reduces the period; the clock takes less time for each "tick".

Section Chapter Test Page 33 Question 8

a) To avoid division by 0, $x^2 - 1 \neq 0$. The domain of $f(x)$ is $x \in \Re,\ x \neq \pm 1$.

b) To avoid a negative radicand and division by 0, $x + 3 > 0$. The domain of $f(x)$ is $x \in (-3, +\infty)$.

c) The denominator can be written as $(x-3)(x+1)$. To avoid division by 0, $(x-3)(x+1) \neq 0$. The domain of $f(x)$ is $x \in \Re,\ x \neq -1, 3$.

d) Since $x^2 + x + 1 > 0$ for all real numbers, the domain of $f(x)$ is $\Re$.

Section Chapter Test Page 33 Question 9

a)
$$\begin{aligned} f(-x) &= (-x)^2 + (-x)^4 \\ &= x^2 + x^4 \\ &= f(x) \end{aligned}$$

$f(x)$ is an even function.

b)
$$\begin{aligned} g(-x) &= |(-x) - 1| \\ &= |-x - 1| \\ &= |x + 1| \\ &\neq g(x) \text{ or } -g(x) \end{aligned}$$

$g(x)$ is neither even nor odd.

c)
$$\begin{aligned} h(-x) &= -7(-x) \\ &= 7x \\ &= -h(x) \end{aligned}$$

$h(x)$ is an odd function.

d)
$$\begin{aligned} r(-x) &= (-x)^3 + |-x| \\ &= -x^3 + |x| \\ &\neq r(x) \text{ or } -r(x) \end{aligned}$$

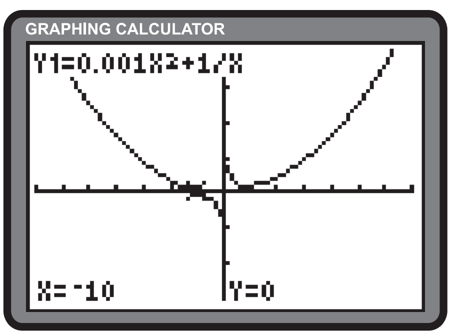
$r(x)$ is neither even nor odd.

e)
$$\begin{aligned} s(-x) &= \frac{1 + (-x)^2}{(-x)^2} \\ &= \frac{1 + x^2}{x^2} \\ &= s(x) \end{aligned}$$

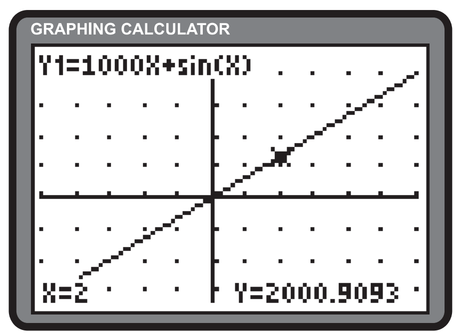
$s(x)$ is an even function.

Section Chapter Test Page 33 Question 10

a) Answers will vary. $x \in [-70, 70]$ and $y \in [-3, 4]$



b) $x \in [-5, 6]$ and $y \in [-5000, 7000]$



Challenge Problems

Section Challenge Problems Page 34 Question 1

Let x be the number of hits the player has made to this point in the season.

$$\begin{aligned} \frac{x}{322} &= 0.289 \\ x &= 0.289(322) \qquad (1) \end{aligned}$$

Let y be the number of hits remaining to achieve a batting average of .300.

$$\frac{x+y}{322+53} = 0.300 \qquad (2)$$

Substitute (1) into (2).

$$\begin{aligned} \frac{0.289(322)+y}{375} &= 0.3 \\ y &= 0.3(375) - 0.289(322) \\ &\doteq 19.4 \end{aligned}$$

The batter must have 20 hits to achieve a batting average of .300. This equates to a batting average of $\frac{20}{53}$ or 0.377 for the rest of the season.

Section Challenge Problems Page 34 Question 2

Let x be the date in the top left corner of the 3 by 3 square. Expressions for the remaining dates appear in the respective cells of the diagram. Let T be the total of the dates.

x	$x+1$	$x+2$
$x+7$	$x+8$	$x+9$
$x+14$	$x+15$	$x+16$

$$\begin{aligned} T &= x + (x+1) + (x+2) + (x+7) + (x+8) + (x+9) + (x+14) + (x+15) + (x+16) \\ &= 9x + 72 \\ &= 9(x+8) \end{aligned}$$

Section Challenge Problems Page 34 Question 3

Let T be the equivalent temperature on both scales.

$$\begin{aligned} \frac{T-32}{212-32} &= \frac{T-0}{100-0} \\ 100T - 3200 &= 180T \\ -80T &= 3200 \\ T &= -40 \end{aligned}$$

$-40°$ is an equivalent temperature on both scales.

Section Challenge Problems Page 34 Question 4

The amount of tape remaining on the reel is proportional to the area, A, of tape showing. Let r be the distance from the centre to the outer edge of the reel of tape.

a)
$$\begin{aligned} \frac{A_{\text{remaining}}}{A_{\text{original}}} &= \frac{3}{4} \\ \frac{\pi r^2 - \pi(2)^2}{\pi(6)^2 - \pi(2)^2} &= \frac{3}{4} \\ \frac{\pi r^2 - 4\pi}{36\pi - 4\pi} &= \frac{3}{4} \\ \frac{\pi r^2 - 4\pi}{32\pi} &= \frac{3}{4} \\ \pi r^2 - 4\pi &= 24\pi \\ \pi r^2 &= 28\pi \\ r &= \pm 2\sqrt{7} \end{aligned}$$

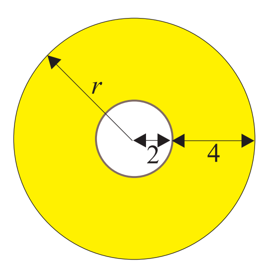
Since $r \geq 0$, $r = 2\sqrt{7}$.

b)
$$\begin{aligned} \frac{A_{\text{remaining}}}{A_{\text{original}}} &= \frac{1}{2} \\ \frac{\pi r^2 - \pi(2)^2}{\pi(6)^2 - \pi(2)^2} &= \frac{1}{2} \\ \frac{\pi r^2 - 4\pi}{36\pi - 4\pi} &= \frac{1}{2} \\ \frac{\pi r^2 - 4\pi}{32\pi} &= \frac{1}{2} \\ \pi r^2 - 4\pi &= 16\pi \\ \pi r^2 &= 20\pi \\ r &= \pm 2\sqrt{5} \end{aligned}$$

Since $r \geq 0$, $r = 2\sqrt{5}$.

c)
$$\begin{aligned} \frac{A_{\text{remaining}}}{A_{\text{original}}} &= \frac{1}{4} \\ \frac{\pi r^2 - \pi(2)^2}{\pi(6)^2 - \pi(2)^2} &= \frac{1}{4} \\ \frac{\pi r^2 - 4\pi}{36\pi - 4\pi} &= \frac{1}{4} \\ \frac{\pi r^2 - 4\pi}{32\pi} &= \frac{1}{4} \\ \pi r^2 - 4\pi &= 8\pi \\ \pi r^2 &= 12\pi \\ r &= \pm 2\sqrt{3} \end{aligned}$$

Since $r \geq 0$, $r = 2\sqrt{3}$.



Section Challenge Problems   Page 34   Question 5

Double the first number, triple the second number, and add the two resulting numbers together.

Section Challenge Problems   Page 34   Question 6

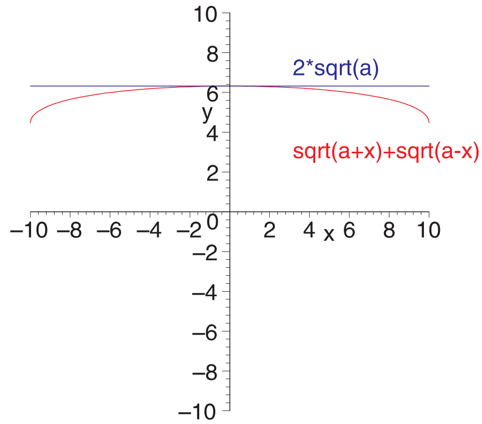
$$\begin{aligned}(\sqrt{a+x}+\sqrt{a-x})^2 &= (a+x)+(a-x)+2\sqrt{a-x}\sqrt{a+x}\\ &= 2a+2\sqrt{a-x}\sqrt{a+x}\\ &= 2(a+\sqrt{a^2-x^2})\end{aligned}$$

Since $x \in (0, a]$, $\sqrt{a^2-x^2} < \sqrt{a^2}$, thus,

$$\begin{aligned}(\sqrt{a+x}+\sqrt{a-x})^2 &< 2\left(a+\sqrt{a^2}\right)\\ &< 2(a+a)\\ &< 4a\\ &< (2\sqrt{a})^2 \qquad (1)\end{aligned}$$

Take the square root of both sides of (1).

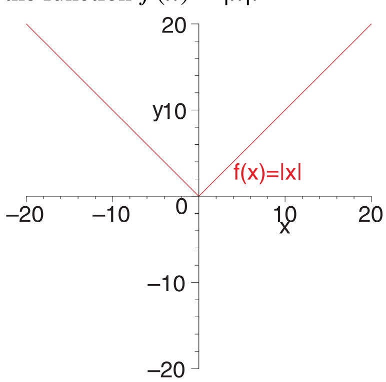
$$\sqrt{a+x}+\sqrt{a-x} < 2\sqrt{a}$$



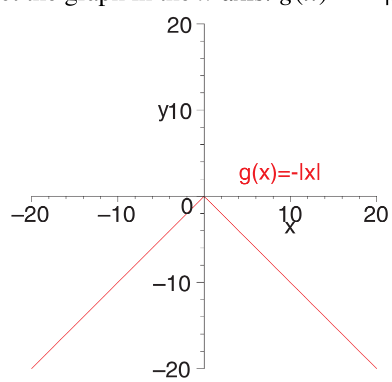
Section Challenge Problems   Page 34   Question 7

The graph can be developed by applying a sequence of transformations.

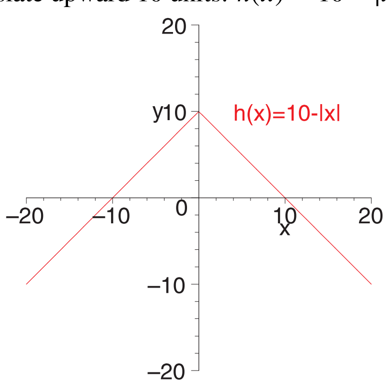
i) Graph the function $f(x) = |x|$.



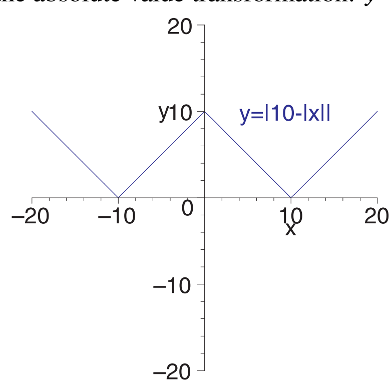
ii) Reflect the graph in the x-axis: $g(x) = -|x|$.



iii) Translate upward 10 units: $h(x) = 10 - |x|$.

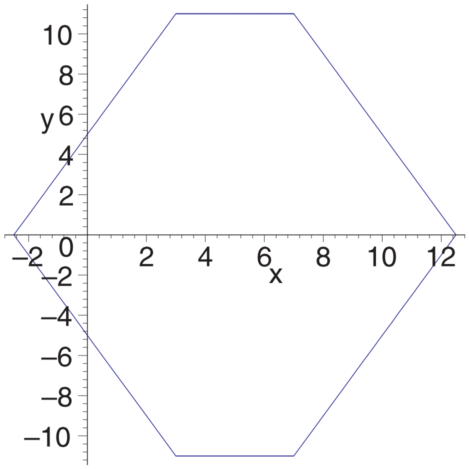


iv) Apply the absolute value transformation: $y = |10 - |x||$.



Section Challenge Problems Page 34 Question 8

Rewrite the relation as $|y| = 15 - |x-3| - |x-7|$. Use the definition of absolute value to reconstruct the relation as a piecewise set.



$$|y| = \begin{cases} 15 - |x-3| - |x-7| & ; y \geq 0 \\ -(15 - |x-3| - |x-7|) & ; y < 0 \end{cases} \quad (1)$$

$$|x-3| = \begin{cases} x-3 & ; x \geq 3 \\ 3-x & ; x < 3 \end{cases} \quad (2)$$

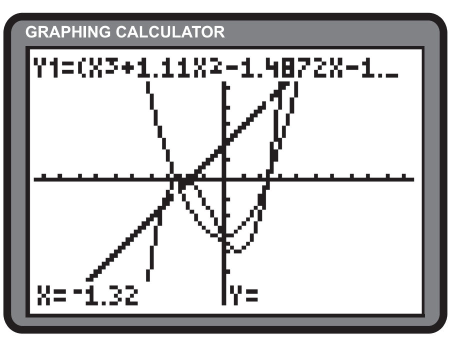
$$|x-7| = \begin{cases} x-7 & ; x \geq 7 \\ 7-x & ; x < 7 \end{cases} \quad (3)$$

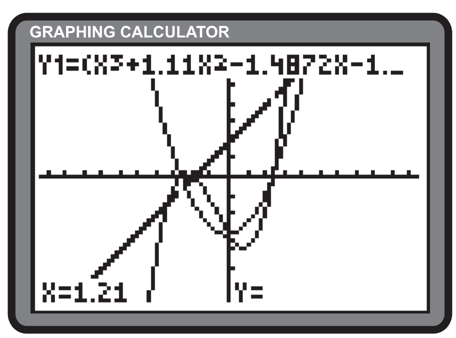
Assemble the pieces within the respective regions defined by the intersections of the intervals. The results are summarized in the table below.

Interval	***y* < 0**	***y* ≥ 0**
$(-\infty, 3)$	$y = (3-x) + (7-x) - 15$ $= -2x - 5$	$y = 15 - (3-x) - (7-x)$ $= 2x + 5$
$[3, 7)$	$y = (x-3) + (7-x) - 15$ $= -11$	$y = 15 - (x-3) - (7-x)$ $= 11$
$[7, \infty)$	$y = (x-3) + (x-7) - 15$ $= 2x - 25$	$y = 15 - (x-3) - (x-7)$ $= -2x + 25$

Section Challenge Problems Page 34 Question 9

The calculator assists in determining the roots of the numerator and denominator. The roots of $x^3 + 1.11x^2 - 1.4872$ are -1.32, -1 and 1.21. The roots of $x^2 + 0.11x - 1.5972$ are -1.32 and 1.21. These results define holes in $y = \dfrac{x^3 + 1.11x^2 - 1.4872}{x^2 + 0.11x - 1.5972}$ at $x = -1.32$ and $x - 1.21$. For the holes to appear visually, window settings must ensure that these domain values are sampled. Since both values are multiples of 0.11, suitable window settings for the domain would be $[-47(0.11), 47(0.11)]$ or $[-5.17, 5.17]$. To reflect proportionality, the range should be set to $[-31(0.11), 31(0.11)]$ or $[-3.41, 3.41]$. The holes are depicted in the thick graphs below.





Using the Strategies

Section Problem Solving   Page 37   Question 1

Once the female captain is identified, there are 3 remaining females from which 2 must be chosen. There are 3 ways this can satisfied. From the four males, 3 must be chosen. There are 4 possible combinations for the male members.

As a result there are 4 × 3 or 12 possible combinations of members that could represent the school.

Section Problem Solving   Page 37   Question 2

Fill the 5-L container and empty it into the 9-L container; then fill the 5-L container again and pour water into the 9-L container to fill it. There is now 1 L of water in the 5-L container. Empty the 9-L container, pour the 1 L of water from the 5-L container into the 9-L container, refill the 5-L container and pour it into the 9-L container. There are now 6 L of water in the 9-L container.

Section Problem Solving   Page 37   Question 3

Juan and Sue cross in 2 min; Sue returns in 2 min; Alicia and Larry cross in 8 min; Juan returns in 1 min; Juan and Sue cross in 2 min. The total time to cross is 15 min. Hint: Alicia and Larry must cross together, and someone must be on the opposite side to return the flashlight.

Section Problem Solving   Page 37   Question 4

From each vertex of convex n-gon, $n - 3$ diagonals can be drawn to non-adjacent vertices. Since each diagonal must be counted only once, a convex n-gon has $\frac{n(n-3)}{2}$ diagonals. Let the number of sides in the polygons be x and y respectively. Solve the following system of equations.

$$
\begin{aligned}
x + y &= 11 \\
y &= 11 - x \qquad (1) \\
\frac{x(x-3)}{2} + \frac{y(y-3)}{2} &= 14 \\
x^2 - 3x + y^2 - 3y &= 28 \qquad (2)
\end{aligned}
$$

Substitute (1) into (2).

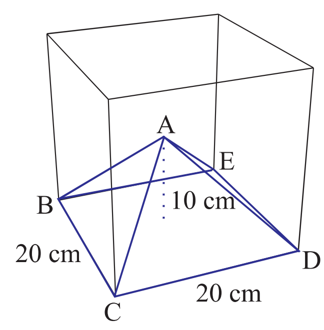
$$
\begin{aligned}
x^2 - 3x + (11-x)^2 - 3(11-x) &= 28 \\
x^2 - 3x + 121 - 22x + x^2 - 33 + 3x &= 28 \\
2x^2 - 22x + 60 &= 0 \\
x^2 - 11x + 30 &= 0 \\
(x-5)(x-6) &= 0 \\
x &= 5 \text{ or } 6
\end{aligned}
$$

the convex polygons that satisfy the requirements are a pentagon and a hexagon.

Section Problem Solving   Page 37   Question 5

a) The top vertex of each pyramid meets at the centre of the cube with each face of the cube being a base of the pyramid.

b) The dimensions are 20 cm by 20 cm by 10 cm.



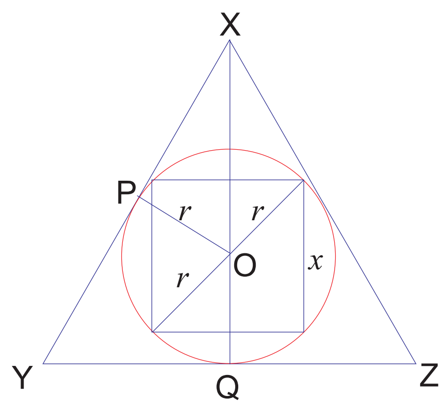
Section Problem Solving Page 37 Question 6

Yes. The following table gives the first month in the year offering a Friday the 13th.

January 1st	First month (not a leap year)	First month (leap year)
Sunday	Friday January 13	Friday January 13
Monday	Friday April 13	Friday September 13
Tuesday	Friday September 13	Friday June 13
Wednesday	Friday June 13	Friday March 13
Thursday	Friday February 13	Friday February 13
Friday	Friday August 13	Friday May 13
Saturday	Friday May 13	Friday October 13

Section Problem Solving Page 37 Question 7

Let r be the length of the diagonal of the inner square. The radius of the circle is then r. Determine the side length of the square.



$$\begin{aligned} x^2 + x^2 &= (2r)^2 \\ 2x^2 &= 4r^2 \\ x^2 &= 2r^2 \end{aligned}$$

Construct OP such that OP $\perp$ XY. In $\triangle$XOP,

$$\begin{aligned} \sin\angle\text{OXP} &= \frac{\text{OP}}{\text{OX}} \\ \text{OX} &= \frac{\text{OP}}{\sin\angle\text{OXP}} \\ &= \frac{r}{\sin 30°} \\ &= 2r \\ \tan\angle\text{OXP} &= \frac{\text{OP}}{\text{PX}} \\ \text{PX} &= \frac{\text{OP}}{\tan\angle\text{OXP}} \\ &= \frac{r}{\tan 30°} \\ &= \sqrt{3}r \end{aligned}$$

Since XQ = XO + OQ, the height of $\triangle$XYZ is $2r + r$ or $3r$. Since QY = PX = $\sqrt{3}r$, the ratio of the areas can be determined.

$$\begin{aligned} \frac{A\text{triangle}}{A\text{square}} &= \frac{(\sqrt{3}r)(3r)}{2r^2} \\ &= \frac{3\sqrt{3}}{2} \end{aligned}$$

The ratio of the area of the triangle to the area of the square is $\frac{3\sqrt{3}}{2}$.

CHAPTER 2 Polynomials

2.2 Dividing a Polynomial by a Polynomial

Practise

Section 2.2 Page 51 Question 1

a)

$$\begin{array}{r} x + 5 \\ x+3\overline{)x^2+8x+15} \\ \underline{x^2+3x} \\ 5x+15 \\ \underline{5x+15} \\ 0 \end{array}$$

Restriction: $x \neq -3$

b)

$$\begin{array}{r} a - 2 \\ a-5\overline{)a^2-7a+10} \\ \underline{a^2-5a} \\ -2a+10 \\ \underline{-2a+10} \\ 0 \end{array}$$

Restriction: $a \neq 5$

c)

$$\begin{array}{r} y + 3 \\ y-4\overline{)y^2-y-12} \\ \underline{y^2-4y} \\ 3y-12 \\ \underline{3y-12} \\ 0 \end{array}$$

Restriction: $y \neq 4$

d)

$$\begin{array}{r} t - 2 \\ t+2\overline{)t^2+0t-4} \\ \underline{t^2+2t} \\ -2t-4 \\ \underline{-2t-4} \\ 0 \end{array}$$

Restriction: $t \neq -2$

Section 2.2 Page 51 Question 3

a)

$$\begin{array}{r} x + 3 \\ 2x+5\overline{)2x^2+11x+15} \\ \underline{2x^2+5x} \\ 6x+15 \\ \underline{6x+15} \\ 0 \end{array}$$

Restriction: $x \neq -\frac{5}{2}$

b)

$$\begin{array}{r} y + 3 \\ 3y-1\overline{)3y^2+8y-3} \\ \underline{3y^2-y} \\ 9y-3 \\ \underline{9y-3} \\ 0 \end{array}$$

Restriction: $y \neq \frac{1}{3}$

c)

$$\begin{array}{r} r - 6 \\ 5r-1\overline{)5r^2-31r+6} \\ \underline{5r^2-r} \\ -30r+6 \\ \underline{-30r+6} \\ 0 \end{array}$$

Restriction: $r \neq \frac{1}{5}$

d)

$$\begin{array}{r} 2t - 3 \\ 2t+1\overline{)4t^2-4t-3} \\ \underline{4t^2+2t} \\ -6t-3 \\ \underline{-6t-3} \\ 0 \end{array}$$

Restriction: $t \neq -\frac{1}{2}$

e)

$$\begin{array}{r} 2r - 7 \\ 3r-2\overline{)6r^2-25r+14} \\ \underline{6r^2-4r} \\ -21r+14 \\ \underline{-21r+14} \\ 0 \end{array}$$

Restriction: $r \neq \frac{2}{3}$

f)

$$\begin{array}{r} 2x + 3 \\ 5x+3\overline{)10x^2+21x+9} \\ \underline{10x^2+6x} \\ 15x+9 \\ \underline{15x+9} \\ 0 \end{array}$$

Restriction: $x \neq -\frac{3}{5}$