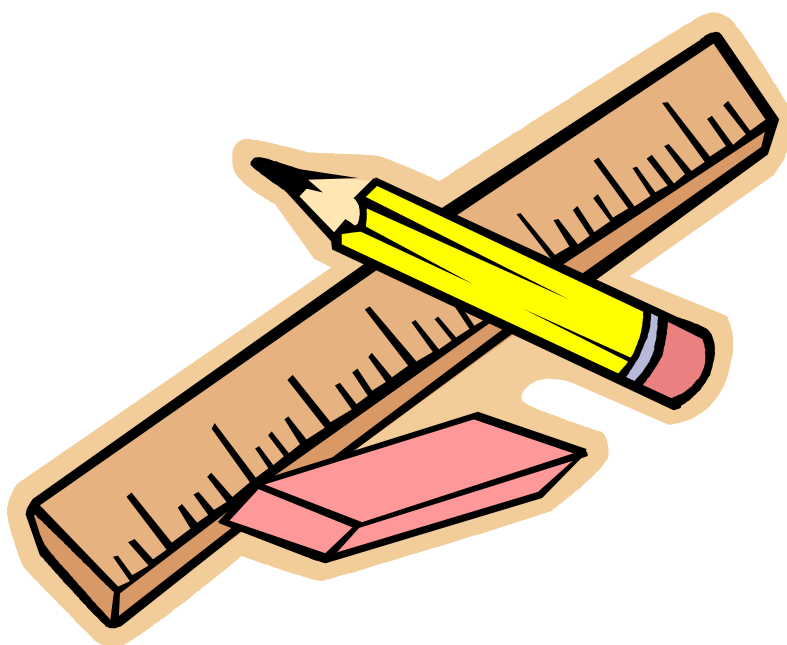


MCR 3UI

Course REVIEW



Unit 1: Algebraic Tools for Operating with Functions: Rational Expressions

1. Simplify. State any restrictions on the variables.
- a) $(4x^2 - 7x - 7) - (8x^2 - 5x - 9)$ b) $2(x-3)^2 - (2x+1)(3x+2)$
- c) $\frac{3x-3y}{5x-5y}$ d) $\frac{x^2-16}{x^2-x-12}$
- e) $\frac{x^2+2x-3}{x^2+6x+8} \times \frac{x^2+2x-8}{x^2+x-6}$ f) $\frac{2x^2-x-1}{3x^2+x-2} \div \frac{2x^2-3x-2}{3x^2-11x+6}$
- g) $\frac{x+2}{3} + \frac{2x-1}{4} - \frac{3x+1}{2}$ h) $\frac{4}{2x-3} - \frac{1}{3-2x}$
- i) $\frac{2}{x^2+5x+4} - \frac{3}{x^2-3x-4}$ j) $\frac{x+1}{3x^2+4x+1} + \frac{2x-1}{3x^2-5x-2}$
- k) $\frac{2x+2}{x^2-1} + \frac{x^2-1}{2x^2-x-1} \div \frac{9x+6}{12x+6}$ l) $\frac{2x+4}{4x} - \frac{7x+7}{3x} \times \frac{5x^2}{14x^2+14x}$

Unit 2: Radical Mathematics and Quadratic Functions

2. Simplify.
- a) $\sqrt{50}$ b) $\sqrt{44}$ c) $2\sqrt{3} \times \sqrt{6}$ d) $\frac{\sqrt{72}}{\sqrt{6}}$
- e) $5\sqrt{10} \times 3\sqrt{2}$ f) $(2\sqrt{5})^2$ g) $\frac{8-\sqrt{40}}{2}$ h) $\frac{15\sqrt{48}}{5\sqrt{3}}$
- i) $\sqrt{48} - \sqrt{27} + \sqrt{12}$ j) $\sqrt{6}(3\sqrt{2} + 2\sqrt{8})$ k) $(2-\sqrt{3})(1+3\sqrt{3})$
3. Solve by factoring.
- a) $2x^2 - 7x = 4$ b) $3x^2 = 6 - 7x$
4. Solve using the quadratic formula.
- a) $x^2 - 5x = 13$ b) $3x^2 = -3x + 7$
5. Complete the square and Partially Factor each of the following. State the maximum or minimum value of each function and the value of x when it occurs.
- a) $y = x^2 - 7x + 2$ b) $y = -4x^2 - 8x + 5$
- c) $y = -2x^2 + 5x + 5$ d) $y = \frac{1}{2}x^2 - 4x + 6$
6. Quadratic Story Questions.
- A. The function $h(t) = -4t^2 + 20t + 2$ gives the approximate height, h metres of a thrown football as a function of the time, t seconds since it was thrown. The ball hit the ground before a receiver could get near it.
- a) How long was the ball in the air, to the nearest tenth of second?
- b) For how many seconds was the height of the ball at least 17 m?
- c) What is the maximum height of the ball?

- B. The height of an object, $h(t)$, in metres, can be modelled by the equation $h(t) = 15t - 4t^2$, where t is the time in seconds after the object is released.
Can the object ever reach a height of 10 m? Explain.
- C. The profit function for a company is modelled by, $P(x) = -3x^2 + 7x + 9$, where x is the number of items produced, in thousands. Determine the break-even point(s).
7. Calculate the value of k such that $kx^2 - 3x + k = 0$ has: (a) one root. (b) two roots.
8. Does the linear function $g(x) = x + 2$ intersect the quadratic function $f(x) = 2x^2 - 2x - 3$? How can you tell? If it does intersect, determine the point(s) of intersection.
9. Determine the equation of the parabola with roots $3 + \sqrt{5}$ and $3 - \sqrt{5}$, and passing through the point $(3, 10)$

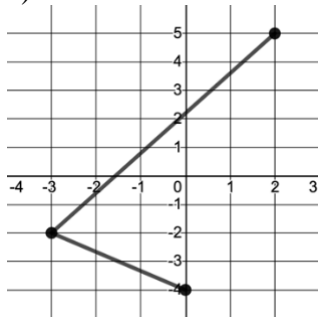
Unit 3: Transformations of Functions

10. For each of the following, state the domain, range and whether or not it is a function.

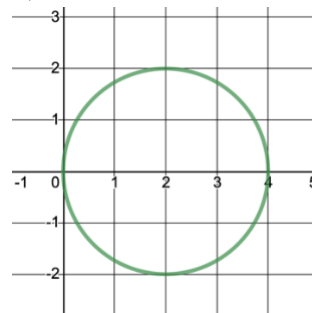
a) $\{ (2,4), (3,5), (7,9), (2,-5), (3,-7) \}$

b) $\{ (-1,6), (0,-6), (1,-6), (2,-6) \}$

c)



d)



11. If $f(x) = 3 - 2x^2$, find:

a) $f(5)$

b) $f\left(-\frac{1}{2}\right)$

12. Describe the transformations of the following functions from the graph of $f(x)$.

a) $y = f(x-2) - 3$

b) $y = -f(x+5) - 1$

c) $y = \frac{1}{3}f(-3x) + 5$

d) $y = -2f(2(x+3)) + 6$

13. Describe the transformations on $f(x) = x^2$ required to graph $y = -\frac{1}{4}\left[\frac{1}{2}(x-8)\right]^2$.

14. Find the inverse of each function. Is the inverse a function? Explain.

a) $y = 3x - 5$

b) $y = x^2 - 7$

c) $y = (x+2)^2$

d) $y = \sqrt{x-3}$

15. i) Use transformations to sketch the graphs of each of the following pairs of functions on the same set of axes. The first function is the Parent/Base Function.

a) $y = \sqrt{x}$ and $y = \sqrt{x} - 4$

b) $y = x^2$ and $y = -\frac{1}{2}(x+1)^2 - 3$

c) $y = 2^x$ and $y = -2^x$

d) $y = \frac{1}{x}$ and $y = \frac{3}{x+2}$

ii) State the domain and range of each function.

16. The graph of $y = x^2$ is stretched vertically by a factor of 2, translated 3 units to the left and translated 4 units upward. Write the equation of the transformed function and state its domain and range.

17. Given $f(x) = x^2 + 6x$

a) Write equations for $-f(x)$ and $f(-x)$.

b) Sketch the three graphs on the same set of axes.

c) Determine any points that are invariant for each reflection.

18. Copy and complete the chart below.

Relation	Rough Sketch	Domain	Range	Function? Yes or No
a) $y = 3x$				
b) $y = 2(x-1)^2 - 4$				
c) $y = -\sqrt{x} + 2$				
d) $y = \frac{1}{x}$				
e) $y = 3^x$				
f) $x^2 + y^2 = 25$				

Unit 4: Exponential Functions

19. Simplify. Express each answer with positive exponents.

a) $x^{-1} \cdot x^{-3} \cdot x^2$

b) $(x^{-1}y^2)^{-2}$

c) $5x^4 \cdot 3x^2$

d) $(6x^{-1}y^2)(-x^{-3}y^{-4})$

e) $\frac{3xy^3 \times 10x^4y^2}{15x^2y^6}$

f) $\left(\frac{4x^{-3}y^4}{8x^2y^{-2}}\right)^{-2}$

42. Solve each equation for $0 \leq x \leq 360^\circ$.

a) $\sin x = \frac{-\sqrt{3}}{2}$

b) $\sqrt{2} \cos x + 1 = 0$

c) $2 \sin x - 1 = 0$

d) $\tan x = \sqrt{3}$

e) $(\sqrt{2} \cos x + 1)(\sin x - 1) = 0$

f) $2 \cos^2 x + 3 \cos x = -1$

g) $\cos x + 1 = 2 \sin^2 x$

h) $\cos^2 x - 1 = \sin^2 x$

i) $15 \sin^2 x + \sin x = 2$

ANSWERS

UNIT 1

1a $-4x^2 - 2x + 2$

1b $-4x^2 - 19x + 16$

1c $\frac{3}{5}, x \neq y$

1d $\frac{x+4}{x+3},$
 $x \neq -3, 4$

1e $\frac{x-1}{x+2},$
 $x \neq -4, -3, -2, 2$

1f $\frac{(x-1)(x-3)}{(x+1)(x-2)},$
 $x \neq -1, -\frac{1}{2}, \frac{2}{3}, 2, 3$

1g $\frac{-8x-1}{12},$

1h $\frac{5}{2x-3},$
 $x \neq \frac{3}{2}$

1i $\frac{-x-20}{(x+1)(x+4)(x-4)},$
 $x \neq -4, -1, 4$

1j $\frac{3x-3}{(3x+1)(x-2)},$
 $x \neq -1, -\frac{1}{3}, 2$

1k $\frac{2x^2+6x+2}{(x-1)(3x+2)},$
 $x \neq \pm 1, -\frac{1}{2}, -\frac{2}{3}$

1l $\frac{-x+3}{3x},$
 $x \neq -1, 0$

UNIT 2

2a $5\sqrt{2}$

2b $2\sqrt{11}$

2c $6\sqrt{2}$

2d $2\sqrt{3}$

2e $30\sqrt{5}$

2f 20

2g $4 - \sqrt{10}$

2h 12

2i $3\sqrt{3}$

2j $14\sqrt{3}$

2k $5\sqrt{3} - 7$

3a $x = -\frac{1}{2}, 4$

3b $x = -3, \frac{2}{3}$

4a $x = \frac{5 \pm \sqrt{77}}{2}$

4b $x = \frac{-3 \pm \sqrt{93}}{6}$

5a $\min = \frac{-41}{4},$
 $x = \frac{7}{2}$

5b $\max = 9,$
 $x = -1$

5c $\max = \frac{65}{8},$
 $x = \frac{5}{4}$

5d $\min = -2,$
 $x = 4$

6.A a) 5.1 s
b) 3.2 s
c) 27 m

6B Yes,
Discriminant
=65

6C 3255 items

7a $k \in \left\{ \pm \frac{3}{2} \right\}$

7b $\left\{ k \in \mathbb{R}, -\frac{3}{2} < k < \frac{3}{2} \right\}$

8 D=49 so 2 points of intersection
 $(-1, 1), \left(\frac{5}{2}, \frac{9}{2} \right)$

9 $y = -2x^2 + 12x - 8$

UNIT 3

10a D: {2, 3, 7}
R: {-7, -5, 4, 5, 9}
Not a function

10b D: {-1, 0, 1, 2}
R: {-6, 6}
Function

10c D: $\{-3 \leq x \leq 2, x \in \mathbb{R}\}$
R: $\{-4 \leq y \leq 5, y \in \mathbb{R}\}$
Not a function

10d D: $\{0 \leq x \leq 4, x \in \mathbb{R}\}$
R: $\{-2 \leq y \leq 2, y \in \mathbb{R}\}$
Not a function

11a -47

11b $\frac{5}{2}$

12a Translated 2 units right
Translated 3 units down

12b Reflected in x-axis
Translated 5 units left
Translated 1 unit down

12c Reflected in y-axis
Vertical compression factor 1/3
Horizontal compression factor 1/3
Translated 5 units up

12d Reflected in x-axis
Vertical stretch by a factor of 2
Horizontal compression by 2
Translated 3 units left
Translated 6 units up

13 Reflected in x-axis
Vertical compression by factor $\frac{1}{4}$
Horizontal stretch by factor 2
Translated 8 units right

14a $f^{-1}(x) = \frac{x}{3} + \frac{5}{3}$
Function

14b $f^{-1}(x) = \pm \sqrt{x+7}$
Not a function

14c $f^{-1}(x) = \pm \sqrt{x-2}$
Not a function

14d $f^{-1}(x) = x^2 + 3$
Function

- 15a (i) Translated 4 units down
(ii) $D: \{x|x \in \mathbb{R}, x \geq 0\}$ $R: \{y|y \in \mathbb{R}, y \geq 0\}$
 $D: \{x|x \in \mathbb{R}, x \geq 0\}$ $R: \{y|y \in \mathbb{R}, y \geq -4\}$
- 15b (i) Reflected in x-axis
Vertical compression by factor of $1/2$
Translated 1 unit left, 3 units down
(ii) $D: \{x|x \in \mathbb{R}\}$ $R: \{y|y \in \mathbb{R}, y \geq 0\}$
 $D: \{x|x \in \mathbb{R}\}$ $R: \{y|y \in \mathbb{R}, y \leq -3\}$
- 15c (i) Reflected in x-axis
(ii) $D: \{x|x \in \mathbb{R}\}$ $R: \{y|y \in \mathbb{R}, y > 0\}$
 $D: \{x|x \in \mathbb{R}\}$ $R: \{y|y \in \mathbb{R}, y < 0\}$
- 15d (i) Vertical stretch by factor of 3
Translated 2 units left
(ii) $D: \{x|x \in \mathbb{R}, x \neq 0\}$ $R: \{y|y \in \mathbb{R}, y \neq 0\}$
 $D: \{x|x \in \mathbb{R}, x \neq -2\}$ $R: \{y|y \in \mathbb{R}, y \neq 0\}$
- 16 $f(x) = 2(x+3)^2 + 4$
 $D: \{x \in \mathbb{R}\}$
 $R: \{y \geq 4, y \in \mathbb{R}\}$
- 17a $-f(x) = -x^2 - 6x$
 $f(-x) = x^2 - 6x$
- 17c $-f(x): (0, 0), (-6, 0)$
 $f(-x): (0, 0)$
- 17a $D: \{x \in \mathbb{R}\}$
 $R: \{y \geq -4, y \in \mathbb{R}\}$
Function
- 18b $D: \{x \in \mathbb{R}\}$
 $R: \{y \geq -4, y \in \mathbb{R}\}$
Function
- 18c $D: \{x \geq 0, x \in \mathbb{R}\}$
 $R: \{y \leq 2, y \in \mathbb{R}\}$
Function
- 18d $D: \{x \neq 0, x \in \mathbb{R}\}$
 $R: \{y \neq 0, y \in \mathbb{R}\}$
Function
- 18e $D: \{x \in \mathbb{R}\}$
 $R: \{y > 0, y \in \mathbb{R}\}$
Function
- 18f $D: \{-5 \leq x \leq 5, x \in \mathbb{R}\}$
 $R: \{-5 \leq y \leq 5, y \in \mathbb{R}\}$
Not a function

UNIT 4

- 19a $\frac{1}{x^2}$ 19b $\frac{x^2}{y^4}$ 19c $15x^6$ 19d $\frac{-6}{x^4 y^2}$ 19e $\frac{2x^3}{y}$ 19f $\frac{4x^{10}}{y^{12}}$
- 20a $\frac{1}{25}$ 20b 1 20c $\frac{1}{81}$ 20d $\frac{2}{3}$ 20e 5 20f $\frac{1}{3}$
- 20g 16 20h $\frac{243}{32}$ 20i $\frac{25}{9}$ 21a $(-x)^{\frac{1}{3}}$ 21b $\frac{1}{x^3}$ 21c x^2
- 22a C 22b $p(n) = 50 \times (3)^n$
50: Initial population
3: rate of increase
n: number of days
- 23a $v(t) = 20000 \times (0.7)^t$
20000: Initial value of the car
0.7: percent of value carried to the next year
t: number of years
- 23b $\$14\,000$ 23bii $\$9800$ 23c 6.5 years
- 24a Yes 24b No 25a $y = 10^{\frac{1}{3}x}$: not listed 25b A 25c B 25d G
- 25e $y = 3(10^x)$: not listed 25f $y = 10^{x-3}$: not listed 25g C
- 26ai Vertical stretch of factor 5
Horizontal compression of factor $1/2$
Shift down 1 unit
- 26ci Domain: $\{x \in \mathbb{R}\}$
Range: $\{y > -1, y \in \mathbb{R}\}$
Asymptote: $y = -1$
x-intercept: $x \cong 0.73$
- 26aai Reflection over x-axis
Horizontal compression of factor $1/3$
Shift 4 units right
Shift 2 units up
- 26cii Domain: $\{x \in \mathbb{R}\}$
Range: $\{y < 2, y \in \mathbb{R}\}$
Asymptote: $y = 2$
x-intercept: $x \cong 4.21$

UNIT 5

27a	16.3 cm	27b	11.9 cm	27c	6.6 cm	28a	$A = 42.9^\circ$ $a = 9.0 \text{ cm}$ $b = 13.2 \text{ cm}$	28b	$L = 35.4^\circ$ $M = 54.6^\circ$ $k = 15.2 \text{ cm}$	29	146 m
30	$\sin \theta = \frac{21}{29}$ $\cos \theta = \frac{20}{29}$	31a	$A = 38.0^\circ$ or $A = 142.0^\circ$	31b	$A = 74.0^\circ$	31c	$A = 154.0^\circ$	32a	$C = 54.1^\circ$ $a = 30.3 \text{ cm}$ $c = 30.9 \text{ cm}$	32b	$S = 52.9^\circ$ $R = 60.9^\circ$ $T = 66.2^\circ$
32c	$G = 66.1^\circ$ $E = 46.1^\circ$ $g = 12.4 \text{ m}$	33a	20.3 cm	33b	13.5 cm ²	34	383 km	35	98 km		
36a	2 Triangles $H = 58.8^\circ$, $I = 101.2^\circ$, $i = 5.7 \text{ cm}$ or $H = 121.2^\circ$, $I = 38.8^\circ$, $i = 3.7 \text{ cm}$	36b	0 Triangles	36c	1 Triangle, $C = 20.3^\circ$, $A = 55.2^\circ$, $a = 3.3 \text{ m}$						

UNIT 6

37a	$\sin \theta = \frac{5}{\sqrt{41}}$, $\cos \theta = \frac{4}{\sqrt{41}}$, $\tan \theta = \frac{5}{4}$	37b	$\sin \theta = \frac{-4}{\sqrt{65}}$, $\cos \theta = \frac{7}{\sqrt{65}}$, $\tan \theta = -\frac{4}{7}$	38a	1
38b	$-\frac{\sqrt{3}}{2}$	39a	$A = 45^\circ, 315^\circ$	39b	$A = 120^\circ, 300^\circ$
40a	$D: \{0^\circ \leq x \leq 360^\circ, x \in R\}$ $R: \{-1 \leq y \leq 1, y \in R\}$ Amplitude=1 Period = 360° Phase Shift = none	40b	$D: \{0^\circ \leq x \leq 180^\circ, x \in R\}$ $R: \{0 \leq y \leq 4, y \in R\}$ Amplitude=2 Period = 180° Phase Shift = none Up 2 units	40c	$D: \{-45^\circ \leq x \leq 315^\circ, x \in R\}$ $R: \{-\frac{1}{2} \leq y \leq \frac{1}{2}, y \in R\}$ Amplitude = $\frac{1}{2}$ Period = 360° Phase Shift = left 45°
40d	$D: \{0^\circ \leq x \leq 360^\circ, x \in R\}$ $R: \{-1 \leq y \leq 1, y \in R\}$ Amplitude=1 Period = 360° Phase Shift = none	40e	$D: \{0^\circ \leq x \leq 1080^\circ, x \in R\}$ $R: \{-3 \leq y \leq 3, y \in R\}$ Amplitude=3 Period = 1080° Phase Shift = none	40f	$D: \{180^\circ \leq x \leq 900^\circ, x \in R\}$ $R: \{-1 \leq y \leq 3, y \in R\}$ Amplitude=2 Period = 720° Phase Shift = right 180° Up 1 unit
42a	$x = 240^\circ, 300^\circ$	42b	$x = 135^\circ, 225^\circ$	42c	$x = 30^\circ, 150^\circ$
42d	$x = 60^\circ, 240^\circ$	42e	$x = 90^\circ, 135^\circ, 225^\circ$	42f	$x = 120^\circ, 180^\circ, 240^\circ$
42g	$x = 60^\circ, 180^\circ, 300^\circ$	42h	$x = 0^\circ, 180^\circ, 360^\circ$	42i	$x = 19.5^\circ, 160.5^\circ, 203.6^\circ, 336.4^\circ$