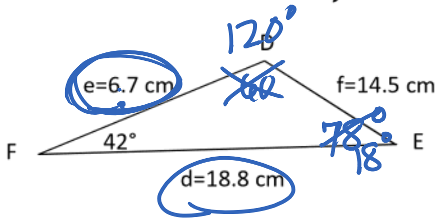
U5D3_T The Ambiguous Case

November 17, 2017 2:07 PM

U5D3 LESSON SHELL Ambiguous Case

Ambiguous Case of Sine Law

From last day, (Example 2 In ΔEFD, e = 6.7 cm, d = 18.8 cm, and F = 42 degrees.)



Using cosine law, we found $\boxed{D = 120°.}$
Now, try using the sine law:

$$\frac{\sin D}{d} = \frac{\sin F}{f}$$

$$\frac{\sin D}{18.8} = \frac{\sin 42^\circ}{14.5}$$

$$D = 60^\circ$$

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<u>Try these… Use calculator:</u>

Sin10°= 0.1736 sin20°= 0.3420 sin1°= 0.0175

Sin170°= 0.1736 sin160°= 0.3420 sin179°= 0.0175

What is the pattern? $\sin x = \sin(180° - x)$

Now, try:

$\sin^{-1}(0.1736) =$ 10°

$\sin^{-1}(0.3420) =$ 20°

$\sin^{-1}(0.0175) =$ 1°

*The calculator does not know whether you are looking for the acute angle or the obtuse angle so you

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MUST consider BOTH possibilities.

* The calculator always gives the acute angle when using the $\sin^{-1}$ button.

Back to example above,

> Recall: The largest angle is across from the largest side, the smallest angle is across from the smallest side, etc.

If D = 60° then E = 180° - 42° - 60° = 78°

e is smallest side so E must be smallest angle.

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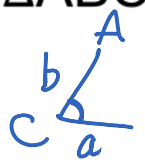
Example 1: b = 4, a = 3, A = 30° in ΔABC. Solve the triangle.

*We cannot use cosine law. Why?



*Side b is larger than side a so angle B will be larger than A. It is possible that B is Obtuse.

Solution in notes

* this is the "Ambiguous Case"

$$\frac{\sin B}{4} = \frac{\sin 30^\circ}{3}$$

$$\angle B = \sin^{-1}(0.6667)$$

$$\boxed{\angle B_1 \doteq 42^\circ} \longrightarrow B_2 \doteq 180^\circ - 42^\circ$$

(Ambiguous Case)

$$\boxed{B_2 \doteq 138^\circ}$$

$$\angle C_1 \doteq 180^\circ - 30^\circ - 42^\circ \text{ (ASTT)}$$

$$\boxed{\angle C_1 \doteq 108^\circ}$$

$$C_2 \doteq 180^\circ - 30^\circ - 138^\circ \text{ (ASTT)}$$

$$\boxed{C_2 \doteq 12^\circ}$$

$$c_1^2 = 3^2 + 4^2 - 2(3)(4)\cos 108^\circ$$

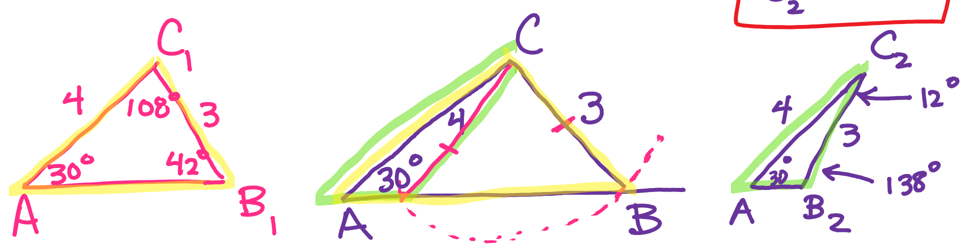
$$\sqrt{c_1^2} = \sqrt{32.4\ldots}$$

$$\boxed{c_1 \doteq 5.7}$$

$$c_2^2 = 3^2 + 4^2 - 2(3)(4)\cos 12^\circ$$

$$\sqrt{c_2^2} = \sqrt{1.5\ldots}$$

$$\boxed{c_2 \doteq 1.2}$$



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∴ there are two Δ's.

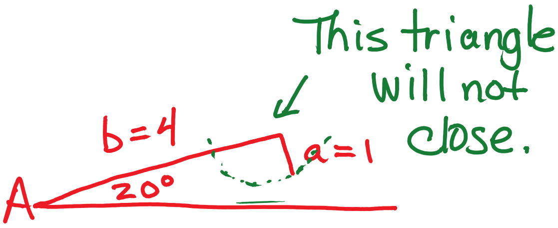
$A = 30^\circ, B_1 \doteq 42^\circ, C_1 \doteq 108^\circ, a = 3, b = 4, c \doteq 5.7$ OR $A = 30^\circ, B_2 \doteq 138^\circ, C \doteq 12^\circ, a = 3, b = 4, c \doteq 1.2$

Example 2: b = 4, a = 1, A = 20° in ΔABC. Solve the triangle. $b > a$ so $B > A$

Solution in notes

$$\frac{\sin B}{4} = \frac{\sin 20^\circ}{1}$$

$$B = \sin^{-1}(1.3681)$$



There is no such triangle possible.

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Example 3: In ΔPQR, Q = 38° , q = 28 cm, r = 45 cm. Determine the values of angles P and R. Try on your own after the lesson. $r>q$ so, $R>38^\circ$

Answer:

Δ_1: $R_1 \doteq 82^\circ$, $P_1 \doteq 60^\circ$

Δ_2: $R_2 \doteq 98^\circ$, $P_2 \doteq 44^\circ$

This is the Ambiguous Case. There are two possible triangles.

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Example 4: b = 4, a = 5, A = 53° in ΔABC. B = ? , C = ?

Solution in notes

$b < a$ so $B < 53°$

This cannot be the ambiguous case!

$$\frac{\sin B}{4} = \frac{\sin 53°}{5}$$

$$B = \sin^{-1}(0.6379)$$

$$\boxed{B \doteq 40°}$$

$$C \doteq 180° - 53° - 40° \text{ (ASTT)}$$

$$\boxed{C \doteq 87°}$$

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For homework number #19. Bearing is measured clockwise from North. So a bearing of 240° is the same as S60°W.

The Ambiguous Case:

If you are using the sine law and you are looking for an angle…

If there is ANY possibility that the angle you are looking for is obtuse then you MUST check for the ambiguous case (where two triangles are possible)…

To consider the ambiguous case given a, b, A…

- Solve for B_1 using the sine law
  Then use ASTT to solve for C_1
- The second case is:
  $B_2 = 180^{\circ} - B_1$
  $C_2 = 180^{\circ} - A - B_2$ (ASTT)
  If this gives you a positive C_2 value
  ($A + B_2 < 180^{\circ}$ and $C_2 > 0^{\circ}$)
  Then this IS the ambiguous case and there are two possible triangles: A, B_1, C_1 and A, B_2, C_2
  (if the C_2 value is negative then there is only one possible triangle: A, B_1, C_1.

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