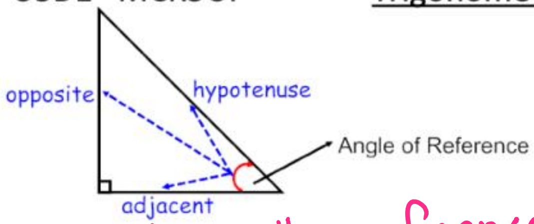
U5D1_T Trig of Right Triangles

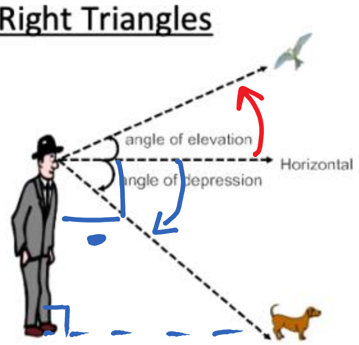
Wednesday, April 17, 2019 9:16 AM

U5D1_T Trig of Rig...

U5D1 MCR3UI

Trigonometry of Right Triangles





* You must use the reference angle to determine which side is the adjacent side and which is the opposite side.

The Primary Trig Ratios: SOH CAH TOA

Sine | Cosine | Tangent

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \frac{\text{Opp.}}{\text{Adj.}}$

The Reciprocal Trig Ratios:

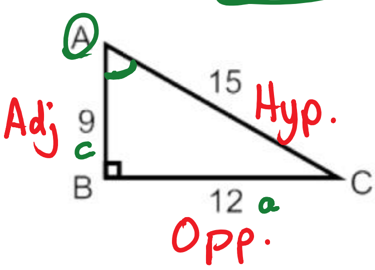
*see note under example 1 on next page.

Cosecant $= \frac{1}{\text{sine}}$ Secant $= \frac{1}{\text{cosine}}$ Cotangent $= \frac{1}{\text{tangent}}$

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\csc\theta = \frac{\text{Hyp.}}{\text{Opp.}}$ $\sec\theta = \frac{H}{A}$ $\cot\theta = \frac{A}{O}$

Example 1: Given the following right-triangle, state the 6 trig ratios of angle A. ← reference angle



$\sin A = \frac{O}{H}$ $\cos A = \frac{A}{H}$ $\tan A = \frac{O}{A}$

$\sin A = \frac{12}{15}$ $\cos A = \frac{3}{5}$ $\tan A = \frac{4}{3}$

$\sin A = \frac{4}{5}$

A : O : H

9 : 12 : 15 = 3 : 4 : 5

$\csc A = \frac{5}{4}$ $\sec A = \frac{5}{3}$ $\cot A = \frac{3}{4}$

reciprocal of sinA

Make sure your calculator is set to DEGREES!

To SOLVE a triangle means:

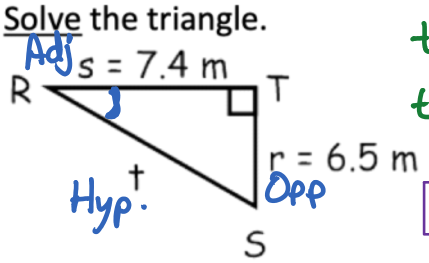
Determine the measure of all angles and all sides.

Hints: Use...

1. **Pythagorean Theorem** if you know 2 sides, & looking for 3rd side.
2. Angle Sum Triangle Theorem (ASTT) if you know **two angles.**
3. **Sine, Cosine, or Tangent** if you know one angle and one side, or if you know two sides and are looking for an angle.

Examples:

1) Solve the triangle.



$t^2 = r^2 + s^2$

$t^2 = 6.5^2 + 7.4^2$

$t^2 = 97.1$

$\boxed{t \doteq 9.8\text{m}}$

$(t>0)$

$\tan R = \frac{6.5}{7.4}$

$R = \tan^{-1}(6.5 \div 7.4)$

$R = 41.29537\ldots$

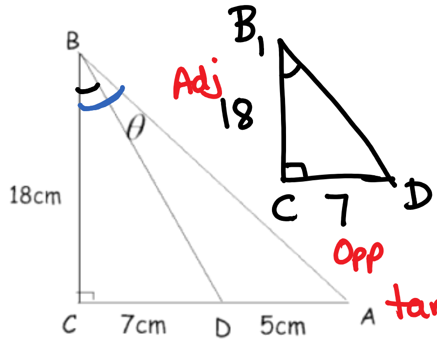
$\boxed{R \doteq 41^\circ}$

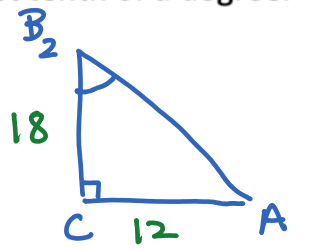
$S \doteq 90^\circ - 41^\circ$

$\boxed{S \doteq 49^\circ}$

RECIPROCAL RATIOS will have meaning as we work through the trig units. For now, you just need to know they exist and how to calculate them since csc, sec, cot are not buttons on your calculator.

2) Find the measure of $\angle\theta$, to the nearest tenth of a degree.





$\tan B_1 = \frac{7}{18}$

$B_1 = \tan^{-1}(7 \div 18)$

$B_1 \doteq 21.25^\circ$

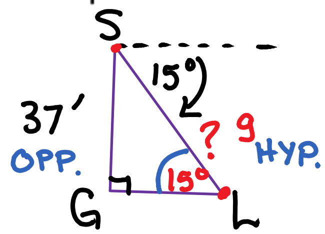
$B_2 = \tan^{-1}(12 \div 18)$

$B_2 \doteq 33.69$

$\theta = B_2 - B_1$

$\boxed{\theta \doteq 12.4^\circ}$

3) On the way to the landing spot, a skydiver got off track and got his parachute stuck amongst the branches of a tree. The skydiver notes that the angle of depression to the actual landing spot is 15°. If the skydiver is hanging 37 feet above ground, by how many feet did the skydiver miss the landing spot? Round to the nearest tenth of a foot.



$\frac{\sin 15^\circ}{1} = \frac{37}{g}$

using properties of proportions.

$\frac{g}{1} = \frac{37}{\sin 15^\circ}$

alternately, cross multiply

$\frac{\sin 15^\circ}{1} \times \frac{37}{g}$

$(\sin 15^\circ)g = 37$

$g = \frac{37}{\sin 15^\circ}$

$g = 142.957...$

$g \doteq 143.0$

∴ he is off by 143.0 feet.

4) Solve for x.

a) $\cot x = 1.2$

$\frac{1}{\tan x} = \frac{1.2}{1}$

So,

$\tan x = \frac{1}{1.2}$

$x = \tan^{-1}(1 \div 1.2)$

$x = 39.80557...$

$x \doteq 40^\circ$

b) $\sec x = 2$

$\cos x = \frac{1}{2}$

$x = \cos^{-1}(0.5)$

$x = 60^\circ$

5) $\sin 30° \sec 60° + \csc 245°$

$= \sin 30°\left(\frac{1}{\cos 60°}\right) + \frac{1}{\sin 245°}$

$= \frac{\sin 30°}{\cos 60°} + \frac{1}{\sin 245°}$

$= -0.1033779...$

$\doteq -0.1$

typo replace $\boxed{+\sec 20°}$ with $\boxed{(\sec 115°)}$

6) Prove: $\cot 245°(\sec 115°) = \csc 245°$

LS

$\cot 245°(\sec 115°)$

$= \frac{1}{\tan 245} \times \frac{1}{\cos 115°}$

$= -1.1033...$

RS

$\csc 245°$

$= \frac{1}{\sin 245°}$

$= -1.1033...$

L.S. = R.S.

Q.E.D.

NOTE:

Q.E.D. is Latin for "Quad Erad Demonstratum"

translation → "what was to be demonstrated"

It is used at the end of a mathematical proof as kind of a "Hip, Hip, Hooray! I proved what I set out to prove."

You will see Q.E.D. in some of my examples. It is not necessary to include - but, fun to give a little cheer!