U4D8_T Applications - Exponential Growth and Decay Problems

Thursday, April 11, 2019 10:45 AM

U4D8_T Applicatio...

U4D8 Warm Up:
A bacteria colony doubles every minute. If there are 10 bacteria in the colony initially, how many are there in 9 minutes?

time (mins)	number of bacteria
0	10
1	$20 = 10 \times 2$
2	$40 = (10 \times 2) \times 2$
3	$80 = (10 \times 2 \times 2) \times 2$
⋮	
9	$10 \times 2^9 = 5120$

You can fill in the table, doubling the previous value each time until 9 minutes have gone by OR you may notice the pattern so you can skip ahead to the 9th minute.

In general, for exponential growth / decay problems: $f(x) = a(b)^x$
where,

$f(x)$ is the FINAL value (y-value, for a given x-value)

a is the INITIAL value (y-intercept on graph)

b is the GROWTH FACTOR (if b > 1) OR

the DECAY FACTOR (if 0 < b < 1)

x is the number of GROWTH or DECAY periods

NOTE: If population is doubling every 5 years and t is in years then $x = \frac{t}{5}$

* every 1 year is $\frac{1}{5}$ of the time required for the population to double.

Important Notes:
If a growth rate is given (as a percent), then the base of the power in the equation (b) can be obtained by adding or subtracting the rate (as a decimal) from 1.

ex. A growth rate of 18% involves multiplying repeatedly by 1.18 Growth Factor

$118\% = 1.18$

Also, the units for the growth and decay rate and for the number of growth and decay periods MUST BE THE SAME!

ex. *monthly* interest rate of 0.05%, the growth period needs to be in months too!

Growth Problem

(in 2010) there is the initial 7500 people + 5% of 7500

At the end of the first year

In other words we have the original 100% + another 5%

105% total

1. Maryville had a population of about 7500 people in 2009. It is expected that the town's population will increase 5% each year.
   a. What is the initial population? 7500

   b. What is the growth rate, r? 5%/yr → Growth FACTOR $105\% = 1 + 0.05 = 1.05$

   c. Write the algebraic model for this situation using the above information. Include let statements.

   $P(t) = 7500(1.05)^t$, where P is the population t years after 2009.

   d. Use the model to predict the population in 2018.

   $t = 2018 - 2009$

   $t = 9$

   $P(9) = 7500(1.05)^9$

   $P(9) = 11\,634.96$

   ∴ we would expect 11634 people in 2018.

   e. In approximately what year will Maryville double its current population, assuming it continues to grow at this rate? Predict to the nearest tenth of a year.

   $7500(1.05)^t = 15000$ OR $1.05^t = 2$

t	P(t)
15	15591.96...
13	14142.36...
14	14849.
14.2	14995...
14.3	15068...

*to solve for t it is just trial and error on your calculator.

$7500 \times 1.05 \wedge \boxed{15} =$

$\boxed{\text{∴ the population would be double after about 14.2 years (in } 2009 + 14 = 2023)}$

initial sample

base of exponential is $\frac{1}{2}$

$x = \frac{t}{138}$, t in days

Decay Problems

2. A 200g sample of radioactive polonium-210 has a half-life of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount.

Decay FACTOR

$1 - 0.5 = 0.5$

a. What is the rate of decay? -50% every 138 days

b. Determine an equation to model this situation. Include let statements.

$M(t) = 200(0.5)^{t/138}$ where M is the mass remaining (g), after t days.

c. Determine the mass that remains after 5 years.

$t = 5 \times 365 + 1$

$t = 1825 + 1$

$t = 1826$

1 leap year in a five year span

$M(1826) = 200(0.5)^{\frac{1826}{138}}$

$M(1826) \doteq 0.02$

$\boxed{\therefore \text{ about } 0.02\text{g remains after 5 years.}}$

d. How much polonium-210 was there 414 days ago?

$t = -414$ days

$M(-414) = 200(0.5)^{-414/138}$

$M(-414) = 1600$

$\boxed{\therefore \text{ there was 1600g 414 days ago.}}$

e. Use your model to predict how long it would take for this 200g sample to decay to 110g.

$\rightarrow \quad 200(0.5)^{t/138} = 110$

* to solve for t it is trial & error on your calculator.

t	M(t)
120	109.46
119	110.01 ←

$\boxed{\text{After 119 days, there is about 110g remaining.}}$

3. A new car costs $24,000. It *loses* 18% of its value each **year** after it is purchased. This is called depreciation.

−18%

decay rate = 18%/yr   decay factor = $1 - 0.18$
$= 0.82$

a. Write an equation that models the decay/decline of the investment. Include let statements.

$V(t) = 24000(0.82)^t$, where V is $ value of the car after t years.

b. Use the equation to determine the value of the automobile after 20 value of the car after t years.

b. Use the equation to determine the value of the automobile after 30 ***months***.

$t = \frac{30}{12}$ $\quad V(2.5) = 24000(0.82)^{2.5}$

$t = 2.5$ $\quad V(2.5) \doteq 14613.22$

$\boxed{\therefore \text{ the car would be worth about \$14613.22 after 30 months.}}$

c. If the car was purchased June 3, 2015, during what **month** would the cars value first fall below \$10 000?

$24000(0.82)^t < 10000$

t	V(t)
5	8897
3	13232
4.5	9825.9
4.3	10223
4.4	10022
4.42	9983
4.43	9963
4.41	10003

* trial and error on calculator.

$.42 \times 12 = 5.04$ months.

4 years, 5.04 months ⇒ November is 5 months after June

$\boxed{\therefore \text{ in November, 2019 the value would first fall below \$10 000.}}$