MCR 3UI-03
Mrs. Behnke

/55

UNIT 5 TEST: TRIGONOMETRY

Name:_____

PART A: TRIGONOMETRIC RATIOS, CAST RULE AND SPECIAL TRIANGLES

1. A point P(3, -2) lies on the terminal arm of an angle θ in standard position.
   (a) Draw the triangle on the Cartesian Plane. [1]
   (b) Determine the exact length of the terminal arm (i.e. Find *r*) [1]

   (b) State the 3 primary trigonometric ratios in fraction form (exact answers – no decimals). [3]

   (c) State the 3 reciprocal trigonometric ratios in fraction form. [3]

   (d) Calculate the related acute angle, β, and the principal angle, θ to the nearest tenth of a degree. [2]

2. Find all values of $\angle\theta$, to the nearest degree if $0° \leq \theta \leq 360°$. **For full marks**, draw a diagram to support your answers. [6]

   a) $\cos A = \frac{1}{\sqrt{2}}$

   b) $csc\theta = -2$

3. State two values of θ to the nearest degree for the following trigonometric ratio. [3]
   $csc\theta = -1.155$

4. Evaluate. Give an exact answer. You must show your work for full marks. [3]
   $cos225°sin45°tan120°$

5. Use the CAST rule and your knowledge of special angles to determine values for all 3 primary trigonometric ratios of the following angles. **For full marks**, leave answers **exact** and in fraction form and provide a diagram to support your answer. [6]

   a) 150°

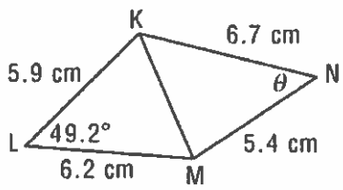
   b) 315°

PART B: APPLICATIONS OF TRIGONOMETRY

1. Austin, Ben and Celeste are spread out, standing in a field. The distance between Austin and Celeste is 7 m, the distance between Austin and Ben is 9 m, From Ben's point of view, the others are separated by an angle of 18°. From Celeste's point of view, by what angle are Austin and Ben separated? Include a diagram with your solution. Answer to the nearest degree. [5]

2. The buoys to mark the triangular course for a yacht race are located at points Y, T, and C. If $YT = 3.5\,km$, $< Y = 65°$, and $< T = 71°$, determine the total length of the course, correct to one decimal place. [5]

3. Determine the value of angle θ, to the nearest degree. NOTE: The diagram is NOT to scale. [5]



PART C: TRIGONOMETRIC IDENTITIES [12]

For the following questions, **prove ONLY 3** of the 5 trigonometric identities. **Be sure to state which identity was used.** Each proof is out of 4 for a total of 12 marks. Two bonus marks will be awarded if you answer all five. ☺ Place a checkmark in the corner of the proofs you want marked out of 4.

$cos\theta = sin\theta cot\theta$

Mark this out of 4? ☐

Mark this out of 4? ☐

$sec^2\theta + csc^2\theta = sec^2\theta csc^2\theta$

Mark this out of 4? ☐

$\frac{sin\theta}{1 - cos\theta} = csc\theta(1 + cos\theta)$

Mark this out of 4? ☐

$cos^2\theta - cos^4\theta = cos^2\theta sin^2\theta$

Mark this out of 4? ☐

$\frac{cot\theta - tan\theta}{csc\theta - sec\theta} = sin\theta + cos\theta$

Essential Skills:

- ☐ Solve problems using the Sine Law (oblique triangles)
- ☐ Solve problems using the Cosine Law (oblique triangles)
- ☐ Determine measures in triangles involving the ambiguous case of the Sine Law
- ☐ Given an angle, determine trigonometric ratios using special triangles and CAST rule
- ☐ Given a trigonometric ratio, determine two principal angles using special triangles and CAST rule
- ☐ Prove simple trigonometric identities (involving PI,QI,RI)

MCR 3UI-04
Mrs. Behnke

/55

UNIT 5 TEST: TRIGONOMETRY

Name:_____

PART A: TRIGONOMETRIC RATIOS, CAST RULE AND SPECIAL TRIANGLES

1. A point P(2, -5) lies on the terminal arm of an angle θ in standard position.
   (a) Draw the triangle on the Cartesian Plane. [1]
   (b) Determine the exact length of the terminal arm (i.e. Find *r*) [1]

   (c) State the 3 primary trigonometric ratios in fraction form (exact answers – no decimals). [3]

   (d) State the 3 reciprocal trigonometric ratios in fraction form. [3]

   (e) Calculate the related acute angle, β, and the principal angle, θ to the nearest tenth of a degree. [2]

2. Find all values of $\angle\theta$, to the nearest degree if $0° \leq \theta \leq 360°$. **For full marks**, draw a diagram to support your answers. [6]
   a) $sin\theta = \frac{-1}{\sqrt{2}}$
   b) $csc\theta = \frac{2}{\sqrt{3}}$

3. State two values of θ to the nearest degree for the following trigonometric ratio. [3]
   $sec\theta = -1.155$

4. Evaluate. Give an exact answer.
   You must show your work for full marks. [3]
   $cos30°sin315°tan150°$

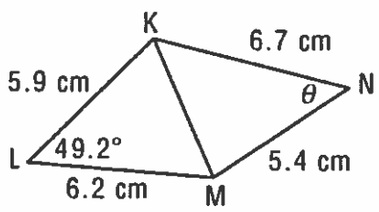
4. Use the CAST rule and your knowledge of special angles to determine values for all 3 primary trigonometric ratios of the following angles. **For full marks**, leave answers **exact** and in fraction form and provide a diagram to support your answer. [6]
   a) 225°
   b) 330°

PART B: APPLICATIONS OF TRIGONOMETRY

1. Austin, Ben and Celeste are spread out, standing in a field. The distance between Austin and Celeste is 7 m, the distance between Austin and Ben is 9 m, From Ben's point of view, the others are separated by an angle of 18°. From Celeste's point of view, by what angle are Austin and Ben separated? Include a diagram with your solution. Answer to the nearest degree. [5]

2. Two buildings are separated by an alleyway that is 21 metres wide. The distance from the top of the shorter building to the top of the taller building is 35 metres. The angle of depression from the top of the shorter building to the bottom of the taller building is 50°. What is the height of the taller building to the nearest metre? Include a diagram with your solution. [5]

3. Determine the value of angle θ, to the nearest degree. NOTE: The diagram is NOT to scale. [5]



PART C: TRIGONOMETRIC IDENTITIES [12]

For the following questions, **prove ONLY 3** of the 5 trigonometric identities. **Be sure to state which identity was used.** Each proof is out of 4 for a total of 12 marks. Two bonus marks will be awarded if you answer all five. ☺ Place a checkmark in the corner of the proofs you want marked out of 4.

$csc\theta = sec\theta cot\theta$

Mark this out of 4? ☐

$cos\theta + \frac{sin^2\theta}{cos\theta} = sec\theta$

Mark this out of 4? ☐

Mark this out of 4? ☐

$(sin\theta - cos\theta)(sin\theta + cos\theta) = 2sin^2\theta - 1$

$tan\theta + cot\theta = sec\theta csc\theta$

Mark this out of 4? ☐

$\frac{tan\theta - sin\theta}{sin^3\theta} = \frac{sec\theta}{1 + cos\theta}$

Mark this out of 4? ☐

Essential Skills:

- ☐ Solve problems using the Sine Law (oblique triangles)
- ☐ Solve problems using the Cosine Law (oblique triangles)
- ☐ Determine measures in triangles involving the ambiguous case of the Sine Law
- ☐ Given an angle, determine trigonometric ratios using special triangles and CAST rule
- ☐ Given a trigonometric ratio, determine two principal angles using special triangles and CAST rule
- ☐ Prove simple trigonometric identities (involving PI,QI,RI)

MCR 3UI-05
Mrs. Behnke

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UNIT 5 TEST: TRIGONOMETRY

Name:_____

PART A: TRIGONOMETRIC RATIOS, CAST RULE AND SPECIAL TRIANGLES

1. A point P(-5, -2) lies on the terminal arm of an angle θ in standard position.
   (a) Draw the triangle on the Cartesian Plane. [1]
   (b) Determine the exact length of the terminal arm (i.e. Find *r*) [1]

   (c) State the 3 primary trigonometric ratios in fraction form (exact answers – no decimals). [3]

   (d) State the 3 reciprocal trigonometric ratios in fraction form. [3]

   (e) Calculate the related acute angle, β, and the principal angle, θ to the nearest tenth of a degree. [2]

2. Find all values of $\angle\theta$, to the nearest degree if $0° \leq \theta \leq 360°$. **For full marks**, draw a diagram to support your answers. [6]
   a) $cos\theta = \frac{-1}{\sqrt{2}}$
   b) $cot\theta = \sqrt{3}$

3. State two values of θ to the nearest degree for the following trigonometric ratio. [3]
   $tan\theta = 0.777$

4. Evaluate. Give an exact answer. You must show your work for full marks. [3]
   $cos315°sin30°tan120°$

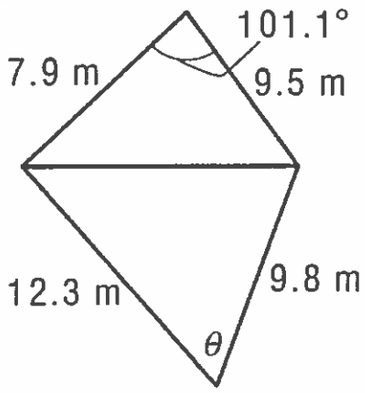
4. Use the CAST rule and your knowledge of special angles to determine values for all 3 primary trigonometric ratios of the following angles. **For full marks**, leave answers **exact** and in fraction form and provide a diagram to support your answer. [6]
   a) 225°
   b) 330°

PART B: APPLICATIONS OF TRIGONOMETRY

1. A forest ranger spots a fire on a bearing of 020° from her station. She estimates that the fire is about 10 km away. A second station is due east of the first. A ranger in the second station thinks that the fire is about 12 km away from him. How far apart are the two stations, to the nearest kilometre. [5]

2. The buoys to mark the triangular course for a yacht race are located at points Y, T, and C. If $y = 3.5\ km,\ t = 5.8\ km, and\ \angle Y = 68°$ (35°), determine the ~~total length of the course~~ size of angle C, correct to one decimal place. [5]

3. Determine the value of angle θ, to the nearest degree. NOTE: The diagram is NOT to scale. [5]



PART C: TRIGONOMETRIC IDENTITIES [12]

For the following questions, **prove ONLY 3** of the 5 trigonometric identities. **Be sure to state which identity was used.** Each proof is out of 4 for a total of 12 marks. Two bonus marks will be awarded if you answer all five. ☺ Place a checkmark in the corner of the proofs you want marked out of 4.

$sec\theta = csc\theta tan\theta$

Mark this out of 4? ☐

$cos\theta(sec\theta - 1) = 1 - cos\theta$

Mark this out of 4? ☐

$(sin\theta - cos\theta)^2 = 1 - 2sin\theta cos\theta$

Mark this out of 4? ☐

$\frac{cos^2\theta}{1 + sin\theta} = 1 - sin\theta$

Mark this out of 4? ☐

$\frac{tan\theta - cot\theta}{sec\theta - csc\theta} = sin\theta + cos\theta$

Mark this out of 4? ☐

Essential Skills:

- ☐ Solve problems using the Sine Law (oblique triangles)
- ☐ Solve problems using the Cosine Law (oblique triangles)
- ☐ Determine measures in triangles involving the ambiguous case of the Sine Law
- ☐ Given an angle, determine trigonometric ratios using special triangles and CAST rule
- ☐ Given a trigonometric ratio, determine two principal angles using special triangles and CAST rule
- ☐ Prove simple trigonometric identities (involving PI,QI,RI)

MCR 3UI-03
Mrs. Behnke

/55

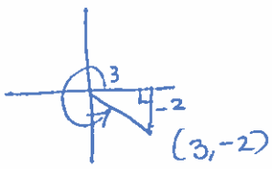
UNIT 5 TEST: TRIGONOMETRY

Name: Marker.

PART A: TRIGONOMETRIC RATIOS, CAST RULE AND SPECIAL TRIANGLES

1. A point P(3, -2) lies on the terminal arm of an angle θ in standard position.

(a) Draw the triangle on the Cartesian Plane. [1]



(b) Determine the exact length of the terminal arm (i.e. Find *r*) [1]

$r = \sqrt{9+4}$

$r = \sqrt{13}$

(b) State the 3 primary trigonometric ratios in fraction form (exact answers – no decimals). [3]

$\sin\theta = \frac{-2}{\sqrt{13}}$ $\cos\theta = \frac{3}{\sqrt{13}}$ $\tan\theta = \frac{-2}{3}$

(c) State the 3 reciprocal trigonometric ratios in fraction form. [3]

$\csc\theta = \frac{-\sqrt{13}}{2}$ $\sec\theta = \frac{\sqrt{13}}{3}$ $\cot\theta = \frac{-3}{2}$

(d) Calculate the related acute angle, β, and the principal angle, θ to the nearest tenth of a degree. [2]

$\beta = \tan^{-1}(2 \div 3)$

$\beta = 33.69\ldots$

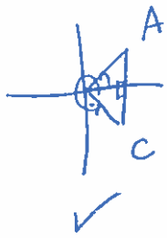
$\beta \doteq 33.7^\circ$

$\theta = 360^\circ - \beta$

$\theta \doteq 326.3^\circ$

2. Find all values of $\angle\theta$, to the nearest degree if $0° \le \theta \le 360°$. **For full marks**, draw a diagram to support your answers. [6]

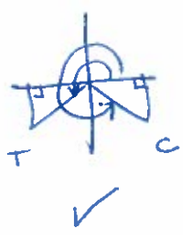
a) $\cos A = \frac{1}{\sqrt{2}}$ $\beta = 45^\circ$ ✓



$A = 45^\circ$ OR 315° ✓

✓

b) $\csc\theta = -2$ $\sin\theta = -\frac{1}{2}$ $\beta = 30^\circ$ ✓



$\theta = 210^\circ$ OR 330° ✓

✓

3. State two values of θ to the nearest degree for the following trigonometric ratio. [3]

$\csc\theta = -1.155$

S | A
T | C

$\sin\theta = \frac{-1}{1.155}$ $\beta = 59.974$

$\beta \doteq 60^\circ$

$\theta = 240^\circ$ OR 300°

4. Evaluate. Give an exact answer. You must show your work for full marks. [3]

$\cos 225° \sin 45° \tan 120°$

$= -\cos 45^\circ (\sin 45^\circ)(-\tan 60^\circ)$ S

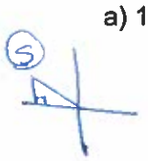
$= -\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \left(-\frac{\sqrt{3}}{1}\right)$

$= \frac{\sqrt{3}}{2}$

A
T C

5. Use the CAST rule and your knowledge of special angles to determine values for all 3 primary trigonometric ratios of the following angles. **For full marks**, leave answers **exact** and in fraction form and provide a diagram to support your answer. [6]

a) 150° $\beta = 30^\circ$

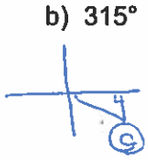


$\sin 150^\circ = \sin 30^\circ = \frac{1}{2}$

$\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

$\tan 150^\circ = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$

b) 315° $\beta = 45^\circ$



$\sin 315^\circ = -\sin 45^\circ = -\frac{1}{\sqrt{2}}$

$\cos 315^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 315^\circ = -\tan 45^\circ = -1$

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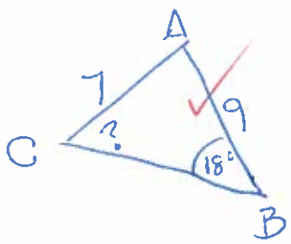
PART B: APPLICATIONS OF TRIGONOMETRY

1. Austin, Ben and Celeste are spread out, standing in a field. The distance between Austin and Celeste is 7 m, the distance between Austin and Ben is 9 m, From Ben's point of view, the others are separated by an angle of 18°. From Celeste's point of view, by what angle are Austin and Ben separated? Include a diagram with your solution. Answer to the nearest degree. [5]



$\frac{\sin C}{9} = \frac{\sin 18°}{7}$ ✓

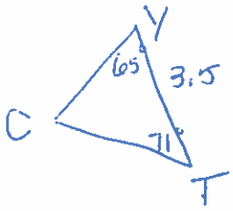
$C = \sin^{-1}\left(\frac{9\sin 18°}{7}\right)$

$C_1 = \sin^{-1}(0.3973...)$ OR $C_2 = 157°$ ✓

$C_1 = 23.40...$

$C_1 \doteq 23°$ ✓

2. The buoys to mark the triangular course for a yacht race are located at points Y, T, and C. If $YT = 3.5\,km$, $< Y = 65°$, and $< T = 71°$, determine the total length of the course, correct to one decimal place. [5]



$\angle C = 180° - 136°$

$\angle C = 44°$ ✓

$\frac{y}{\sin 65°} = \frac{3.5}{\sin 44°}$ ✓

$y = 3.5 \times \sin 65° \div \sin 44°$

$y = 4.56638...$ ✓

$t = \frac{3.5}{\sin 44°} \times \sin 71°$

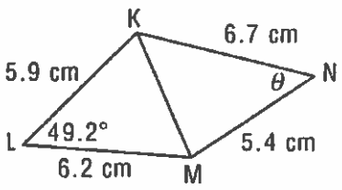
$t = 4.7639...$ ✓

$c + y + t = 12.82...$ ✓

∴ the total course was **12.8km**

(-½ if 12.9 km)

3. Determine the value of angle θ, to the nearest degree. NOTE: The diagram is NOT to scale. [5]



$l^2 = 5.9^2 + 6.2^2 - 2(5.9)(6.2)\cos 49.2°$

$\sqrt{l^2} = \sqrt{25.44574}$

$l \doteq 5.044...$ ✓

$N = \cos^{-1}\left[\frac{(6.7^2 + 5.4^2 - l^2)}{[2(6.7)(5.4)]}\right]$ ✓

$6.7^2 + 5.4^2 - 5.04^2 = 48.6484$

$N = \cos^{-1}\left(\frac{48.60425}{72.36}\right)$ ✓

$\cos^{-1}(0.67231) = 47.75$

$N = \cos^{-1}(0.6717...)$

$N \doteq 47.80...$ ✓

$\theta \doteq 48°$

-✓accuracy + rounding.

PART C: TRIGONOMETRIC IDENTITIES [12]

For the following questions, **prove ONLY 3** of the 5 trigonometric identities. **Be sure to state which identity was used.** Each proof is out of 4 for a total of 12 marks. Two bonus marks will be awarded if you answer all five. ☺ Place a checkmark in the corner of the proofs you want marked out of 4.

43
12
55

$\cos\theta = \sin\theta\cot\theta$

Mark this out of 4? ☐

RS

$\sin\theta \times \frac{1}{\tan\theta}$ (RI)

$= \sin\theta \times \frac{\cos\theta}{\sin\theta}$ (QI)

$= \cos\theta$

$= LS$

$\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta$

Mark this out of 4? ☐

LS

$\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$ (RI)

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$

$= \frac{1}{\cos^2\theta\sin^2\theta}$ (PI)

$= \frac{1}{\cos^2\theta} \cdot \frac{1}{\sin^2\theta}$ (RI)

$= \sec^2\theta\csc^2\theta$

$= RS.$

$\frac{\sin\theta}{1-\cos\theta} = \csc\theta(1+\cos\theta)$

Mark this out of 4? ☐

$\frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$ (PI)

$= \frac{1+\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}(1+\cos\theta)$

$= \csc\theta(1+\cos\theta)$ (RI)

$= RS.$

$\cos^2\theta - \cos^4\theta = \cos^2\theta\sin^2\theta$

Mark this out of 4? ☐

LS

$\cos^2\theta(1-\cos^2\theta)$

$= \cos^2\theta(\sin^2\theta)$ (PI)

$= RS$

OR.

RS

$\cos^2\theta(1-\cos^2\theta)$ PI.

$= \cos^2\theta - \cos^4\theta$

$= LS.$

$\frac{\cot\theta - \tan\theta}{\csc\theta - \sec\theta} = \sin\theta\cos\theta$

Mark this out of 4? ☐

LS

$\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) \div \left(\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\right)$ (QI / RI)

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta} \div \left(\frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}\right)$

$= \frac{(\cos\theta - \sin\theta)(\sin\theta + \cos\theta)(\sin\theta\cos\theta)}{(\cos\theta\sin\theta)(\cos\theta - \sin\theta)}$

$= \sin\theta + \cos\theta.$

Essential Skills:

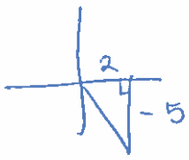
- ☐ Solve problems using the Sine Law (oblique triangles)
- ☐ Solve problems using the Cosine Law (oblique triangles)
- ☐ Determine measures in triangles involving the ambiguous case of the Sine Law
- ☐ Given an angle, determine trigonometric ratios using special triangles and CAST rule
- ☐ Given a trigonometric ratio, determine two principal angles using special triangles and CAST rule
- ☐ Prove simple trigonometric identities (involving PI,QI,RI)

MCR 3UI-04 /55 **UNIT 5 TEST: TRIGONOMETRY**
Mrs. Behnke **Name:**_____

PART A: TRIGONOMETRIC RATIOS, CAST RULE AND SPECIAL TRIANGLES

1. A point P(2, -5) lies on the terminal arm of an angle θ in standard position.

(a) Draw the triangle on the Cartesian Plane. [1]



(b) Determine the exact length of the terminal arm (i.e. Find *r*) [1]

$r = \sqrt{25+4}$

$r = \sqrt{29}$

(c) State the 3 primary trigonometric ratios in fraction form (exact answers – no decimals). [3]

$\sin\theta = \frac{-5}{\sqrt{29}}$ $\cos\theta = \frac{2}{\sqrt{29}}$ $\tan\theta = \frac{-5}{2}$

-1 if θ missing
-1 in both part c and in part d if missing negatives

(d) State the 3 reciprocal trigonometric ratios in fraction form. [3]

$\csc\theta = \frac{\sqrt{29}}{-5}$ $\sec\theta = \frac{\sqrt{29}}{2}$ $\cot\theta = -\frac{2}{5}$

10

(e) Calculate the related acute angle, β, and the principal angle, θ to the nearest tenth of a degree. [2]

$\beta = \tan^{-1}(2.5)$

$\beta \doteq 68.19...$

$\beta \doteq 68.2°$

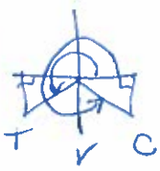
$\theta = 360° - \beta$

$\theta \doteq 291.8°$

2. Find all values of $\angle\theta$, to the nearest degree if $0° \le \theta \le 360°$. **For full marks**, draw a diagram to support your answers. [6]

a) $\sin\theta = \frac{-1}{\sqrt{2}}$ $\beta = 45°$ ✓

$\theta = 225°$ OR $315°$ ✓



b) $\csc\theta = \frac{\sqrt{3}}{2}$ $\sin\theta = \frac{2}{\sqrt{3}}$ $\frac{\sqrt{3}}{2}$.

$\beta = 60°$

$\theta = 60°$ OR $120°$

6

3. State two values of θ to the nearest degree for the following trigonometric ratio. [3]

$\sec\theta = -1.155$

$\cos\theta = \frac{-1}{1.155}$

$\beta = 30°$ ✓

$\theta = 150°$ OR $210°$

4. Evaluate. Give an exact answer. You must show your work for full marks. [3]

$\cos 30° \sin 315° \tan 150°$

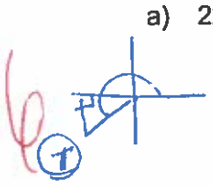
$= \frac{\sqrt{3}}{2}(-\sin 45°)(-\tan 30°)$

$= \frac{\sqrt{3}}{2}\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{3}}\right)$

$= \frac{1}{2\sqrt{2}}$

4. Use the CAST rule and your knowledge of special angles to determine values for all 3 primary trigonometric ratios of the following angles. **For full marks**, leave answers **exact** and in fraction form and provide a diagram to support your answer. [6]

a) 225° $\beta = 45°$ ✓



signs ✓

$\sin 225° = -\sin 45° = -\frac{1}{\sqrt{2}}$

$\cos 225° = -\cos 45° = -\frac{1}{\sqrt{2}}$

$\tan 225° = \tan 45° = 1$

ratios ✓

b) 330° $\beta = 30°$ ✓

$\sin 330° = -\sin 30° = -\frac{1}{2}$

$\cos 330° = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan 330° = -\tan 30° = -\frac{1}{\sqrt{3}}$

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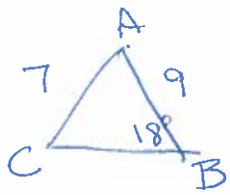
PART B: APPLICATIONS OF TRIGONOMETRY

1. Austin, Ben and Celeste are spread out, standing in a field. The distance between Austin and Celeste is 7 m, the distance between Austin and Ben is 9 m, From Ben's point of view, the others are separated by an angle of 18°. From Celeste's point of view, by what angle are Austin and Ben separated? Include a diagram with your solution. Answer to the nearest degree. [5]



$\frac{\sin C}{9} = \frac{\sin 18^\circ}{7}$

$\sin C = \frac{\sin 18^\circ}{7} \times 9$

$\sin C = 0.3973\ldots$

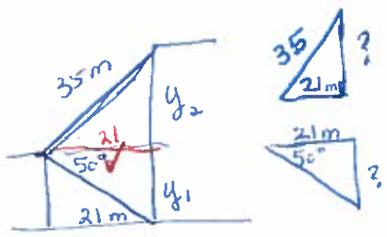
$C = 23.4099\ldots$

$C_1 \doteq 23^\circ$

$C_2 = 157^\circ$

∴ the boys are either 23° or 157° apart from Celeste's point of view.

2. Two buildings are separated by an alleyway that is 21 metres wide. The distance from the top of the shorter building to the top of the taller building is 35 metres. The angle of depression from the top of the shorter building to the bottom of the taller building is 50°. What is the height of the taller building to the nearest metre? Include a diagram with your solution. [5]



$y_2^2 = 35^2 - 21^2$ (OR use 3,4,5 Δ)

$y_2^2 = 784$

$y_2 = 28$

$y_1 = 21 \tan 50^\circ$

$y_1 = 25.026\ldots$

$y = 28 + 25.026$

$y = 53.026$

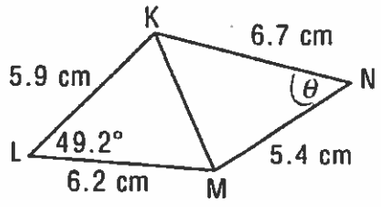
$y \doteq 53$ m

∴ the building (taller one) is 53 m tall.

(4) marks for: 17.62 + 28 ≐ 46 m (wrong angle of depression)

✓ 35/21

3. Determine the value of angle θ, to the nearest degree. NOTE: The diagram is NOT to scale. [5]



$n^2 = 5.9^2 + 6.2^2 - 2(5.9)(6.2)\cos 49.2^\circ$

$\sqrt{n^2} = \sqrt{25.445748\ldots}$

$n = 5.04437\ldots$

$\cos N = \frac{6.7^2 + 5.4^2 - n^2}{2(6.7)(5.4)}$

$\cos N = \frac{48.60425\ldots}{72.36}$

$\cos N = 0.6717\ldots$

$N = 47.80\ldots$

$N \doteq 48^\circ$

$\frac{6.7^2 + 5.4^2 - 5.04^2}{72.36}$

$= \frac{48.64154}{72.36}$

$= 0.67231$

$= 47.7\ldots$

PART C: TRIGONOMETRIC IDENTITIES [12]

For the following questions, **prove ONLY 3** of the 5 trigonometric identities. **Be sure to state which identity was used.** Each proof is out of 4 for a total of 12 marks. Two bonus marks will be awarded if you answer all five. ☺ Place a checkmark in the corner of the proofs you want marked out of 4.

$\csc\theta = \sec\theta\cot\theta$

Mark this out of 4? ☐

LS

$\frac{1}{\sin\theta}$

RS

$\frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$ (RI)

$= \frac{1}{\sin\theta}$

$\cos\theta + \frac{\sin^2\theta}{\cos\theta} = \sec\theta$

Mark this out of 4? ☐

LS

$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$ (PI)

$= \sec\theta$ (RI)

$= RS$

$(\sin\theta - \cos\theta)(\sin\theta + \cos\theta) = 2\sin^2\theta - 1$

Mark this out of 4? ☐

$= \sin^2\theta - \cos^2\theta$

$= \sin^2\theta - (1 - \sin^2\theta)$ (PI)

$= \sin^2\theta - 1 + \sin^2\theta$

$= \boxed{2\sin^2\theta - 1}$

$\tan\theta + \cot\theta = \sec\theta\csc\theta$

Mark this out of 4? ☐

LS

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ (QI, RI)

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta}$ (PI)

$= \sec\theta\csc\theta$.

$= RS$.

$\frac{\tan\theta - \sin\theta}{\sin^3\theta} = \frac{\sec\theta}{1 + \cos\theta}$

Mark this out of 4? ☐

$\left(\frac{\sin\theta}{\cos\theta} - \frac{\sin\theta}{1}\right)\frac{1}{\sin^3\theta}$ (QI)

$= \frac{\sin\theta - \sin\theta\cos\theta}{\cos\theta\sin^3\theta}$

$= \frac{\sin\theta(1 - \cos\theta)}{\sin^3\theta\cos\theta}$

$= \frac{(1 - \cos\theta)}{\sin^2\theta\cos\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$

$= \frac{1 - \cos^2\theta}{\sin^2\theta\cos\theta(1 + \cos\theta)}$

$= \frac{\sin^2\theta}{\sin^2\theta\cos\theta(1 + \cos\theta)}$ (PI)

$= \frac{1}{\cos\theta(1 + \cos\theta)}$ $= \frac{\sec\theta}{1 + \cos\theta}$ (RI) $= RS$

Essential Skills:

- ☐ Solve problems using the Sine Law (oblique triangles)
- ☐ Solve problems using the Cosine Law (oblique triangles)
- ☐ Determine measures in triangles involving the ambiguous case of the Sine Law
- ☐ Given an angle, determine trigonometric ratios using special triangles and CAST rule
- ☐ Given a trigonometric ratio, determine two principal angles using special triangles and CAST rule
- ☐ Prove simple trigonometric identities (involving PI,QI,RI)

MCR 3UI-05
Mrs. Behnke

/55

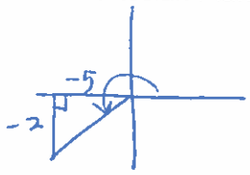
UNIT 5 TEST: TRIGONOMETRY

Name: _____

PART A: TRIGONOMETRIC RATIOS, CAST RULE AND SPECIAL TRIANGLES

1. A point P(-5, -2) lies on the terminal arm of an angle θ in standard position.

(a) Draw the triangle on the Cartesian Plane. [1]



(b) Determine the exact length of the terminal arm (i.e. Find *r*) [1]

$r = \sqrt{25+4}$

$r = \sqrt{29}$

(c) State the 3 primary trigonometric ratios in fraction form (exact answers – no decimals). [3]

$\sin\theta = \frac{-2}{\sqrt{29}}$ $\cos\theta = \frac{-5}{\sqrt{29}}$ $\tan\theta = \frac{2}{5}$

(d) State the 3 reciprocal trigonometric ratios in fraction form. [3]

$\csc\theta = -\frac{\sqrt{29}}{2}$ $\sec\theta = -\frac{\sqrt{29}}{5}$ $\cot\theta = \frac{5}{2}$

(e) Calculate the related acute angle, β, and the principal angle, θ to the nearest tenth of a degree. [2]

$\beta = \tan^{-1}(0.4)$

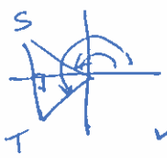
$\beta = 21.801°$

$\theta = 180° + \beta$

$\theta \doteq 201.8°$

2. Find all values of $\angle\theta$, to the nearest degree if $0° \le \theta \le 360°$. **For full marks**, draw a diagram to support your answers. [6]

a) $\cos\theta = \frac{-1}{\sqrt{2}}$ $\beta = 45°$ ✓



$\theta = 135°$ OR $225°$ ✓ ✓

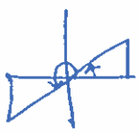
b) $\cot\theta = \sqrt{3}$ $\beta = 30°$ ✓

$\tan\theta = \frac{1}{\sqrt{3}}$

✓ $\theta = 30°$ OR $210°$ ✓

3. State two values of θ to the nearest degree for the following trigonometric ratio. [3]

$\tan\theta = 0.777$



$\beta = \tan^{-1}(0.777)$

$\theta = 38°$ OR $218°$

4. Evaluate. Give an exact answer. You must show your work for full marks. [3]

$\cos 315° \sin 30° \tan 120°$

$= \cos 45° \sin 30° \quad (-\tan 60°)$

$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left(-\sqrt{3}\right)$

$= -\frac{\sqrt{3}}{2\sqrt{2}}$

4. Use the CAST rule and your knowledge of special angles to determine values for all 3 primary trigonometric ratios of the following angles. **For full marks**, leave answers **exact** and in fraction form and provide a diagram to support your answer. [6]

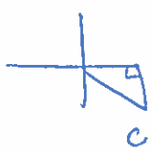
a) 225° $\beta = 45°$

$\sin 225° = -\sin 45° = -\frac{1}{\sqrt{2}}$

$\cos 225° = -\cos 45° = -\frac{1}{\sqrt{2}}$

$\tan 225° = \tan 45° = 1$

b) 330° $\beta = 30°$



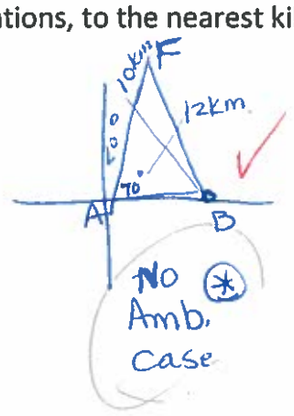
$\sin 330° = -\sin 30° = -\frac{1}{2}$

$\cos 330° = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan 330° = -\tan 30° = -\frac{1}{\sqrt{3}}$

PART B: APPLICATIONS OF TRIGONOMETRY

1. A forest ranger spots a fire on a bearing of 020° from her station. She estimates that the fire is about 10 km away. A second station is due east of the first. A ranger in the second station thinks that the fire is about 12 km away from him. How far apart are the two stations, to the nearest kilometre. [5]



$\frac{\sin B}{10} = \frac{\sin 70°}{12}$ ✓

$\sin B = \frac{10 \sin 70°}{12}$

$B = \sin^{-1}(0.783077...)$

$B \doteq 51.543189°$ ✓

$\angle F = 180° - 70° - B$

$\angle F = 58.45681°$ ✓

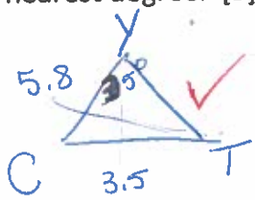
$\frac{f}{\sin F} = \frac{12}{\sin 70°}$

$f = \frac{12}{\sin 70°} \times \sin 58.45681°$

$f = 10.883...$ ✓

$f \doteq 11$ km

2. The buoys to mark the triangular course for a yacht race are located at points Y, T, and C. If $y = 3.5\ km$, $t = 5.8\ km, and\ \angle Y = 35°$, determine the size of angle C, to the nearest degree. [5]



$\frac{\sin T}{5.8} = \frac{\sin 35°}{3.5}$ ✓

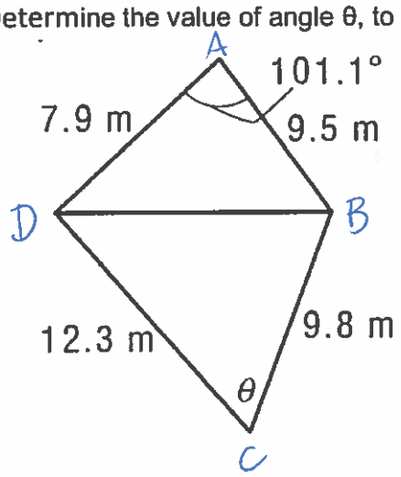
$\sin T = 0.950498...$

$T = 71.8967...°$

✓ $T_1 \doteq 72°$ $T_2 \doteq 108°$

✓ $C_1 = 73°$ $C_2 \doteq 37°$ ✓

3. Determine the value of angle θ, to the nearest degree. NOTE: The diagram is NOT to scale. [5]



$a^2 = 7.9^2 + 9.5^2 - 2(7.9)(9.5)\cos 101.1°$ ✓

$\sqrt{a^2} = \sqrt{181.5575}$

$a \doteq 13.474$ ✓

$\cos C = \frac{(9.8^2 + 12.3^2 - 181.5575)}{[2(9.8)(12.3)]}$ ✓

$C = \cos^{-1}\left(\frac{65.77245}{241.08}\right)$ ✓

$C \doteq \cos^{-1}(0.272824...)$

$C \doteq 74.1676$

$C \doteq 74°$ ✓

PART C: TRIGONOMETRIC IDENTITIES [12]

For the following questions, **prove ONLY 3** of the 5 trigonometric identities. **Be sure to state which identity was used.** Each proof is out of 4 for a total of 12 marks. Two bonus marks will be awarded if you answer all five. ☺ Place a checkmark in the corner of the proofs you want marked out of 4.

$sec\theta = csc\theta tan\theta$ Mark this out of 4? ☐

LS: $\frac{1}{\cos\theta}$

RS: $\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$ RI, QI

$= \frac{1}{\cos\theta}$

$=$ LS.

$\cos\theta(sec\theta - 1) = 1 - \cos\theta$ Mark this out of 4? ☐

LS: $\cos\theta\left(\frac{1}{\cos\theta} - \frac{1}{\cos\theta}\right)$ RI

$= 1 - \cos\theta$

$=$ RS.

$(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta$ Mark this out of 4? ☐

LS

$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$ (PI)

$\frac{\cos^2\theta}{1+\sin\theta} = 1 - \sin\theta$ Mark this out of 4? ☐

LS

$\frac{\cos^2\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

$= \frac{\cos^2\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos^2\theta(1-\sin\theta)}{\cos^2\theta}$ (PI)

$= 1 - \sin\theta$

$=$ RS

OR

LS

$\frac{1-\sin^2\theta}{1+\sin\theta}$ (PI)

$= \frac{(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta}$

$= 1 - \sin\theta$

$=$ RS.

$\frac{\tan\theta - \cot\theta}{\sec\theta - \csc\theta} = \sin\theta\cos\theta$ Mark this out of 4? ☐

LS QI, RI

$\left(\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}\right) \div \left(\frac{1}{\cos\theta} - \frac{1}{\sin\theta}\right)$

$= \frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta} \div \left(\frac{\sin\theta - \cos\theta}{\cos\theta\sin\theta}\right)$

$= \frac{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)(\cos\theta\sin\theta)}{\cos\theta\sin\theta(\sin\theta - \cos\theta)}$

$= \sin\theta + \cos\theta$

Essential Skills:

- ☐ Solve problems using the Sine Law (oblique triangles)
- ☐ Solve problems using the Cosine Law (oblique triangles)
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Abby Yost
* organize solutions X