Example 2:

Given $f(x) = x^2 + 3$ \quad $y = -\frac{1}{2}f(2x + 6) - 2$

b) What is the new "image equation"?

The image equation is the equation of the graph $f(x) = x^2 + 3$ with all the transformations applied to it. (reflection in $x$-axis, V.C.$\frac{1}{2}$, H.C.2, shift left 3, down 2 units).

1. Step 1. If the coefficient of $x$ inside the bracket is other than 1 then you must factor that number out of the bracket.

The image equation is $y = -\frac{1}{2}[f(2(x+3))] - 2$.

Inside this bracket put $y = -\frac{1}{2}f(2(x+3)) - 2$.

$y = -\frac{1}{2} \left[ (2(x+3))^2 + 3 \right] - 2$

This is the image equation.

We can simplify this equation into one of our normal quadratic forms. $\rightarrow$ expand it out for $y = ax^2 + bx + c$ form.
Use algebraic skills for \( y = a(x-h)^2 + k \) form.

\[
y = -\frac{1}{2} \left[ \frac{2}{4} (x+3)^2 + 3 \right] - 2
\]

\[
y = -\frac{1}{2} \left[ \frac{4}{4} (x+3)^2 + 3 \right] - 2
\]

\[
y = -\frac{1}{2} \left( \frac{4}{4} (x+3)^2 \right) - \frac{1}{2} \left( 3 \right) - \frac{2}{1}
\]

\[
y = -2(x+3)^2 - \frac{3}{2} - \frac{4}{2}
\]

\[
y = -2(x+3)^2 - \frac{7}{2}
\]

This is the simplified form of the image equation in vertex form.