U3D8 T Combining Transformations

Monday, March 18, 2019  7:35 PM

U3D8

Warm Up:

Describe the transformations that have occurred to \( h(x) \) to obtain the function \( y = -5h\left(\frac{2}{3}x\right) \)

1. **reflection in x-axis**

2. **vertical stretch factor 5**

3. **horizontal stretch factor 1.5**

If \((-1, 4)\) is a point on the function \( h(x) \), what would be the new point on the transformed function?

\[
(x, y) \rightarrow (1.5x, -5y)
\]

\[
(-1, 4) \rightarrow (1.5(-1), -5(4))
\]

\[
(-1, 4) \rightarrow (-1.5, -20)
\]

\(-1, 4\) reflected in x-axis becomes \((-1, -4)\)

\(-1, -4\) with vertical stretch factor 5 becomes \((-1, 20)\)

\((-1, -20)\) with horizontal stretch factor \(\frac{3}{2}\) becomes \((-1.5, -20)\)
Combining Transformations
When a function has a combination of transformations, apply them in order left to right when in the form:

\[ y = \pm af[\pm b(x - h)] + k \]

1. Reflection in x-axis with Vertical Stretches and Compressions.
2. Reflection in y-axis with Horizontal Stretches and Compressions.
3. Translations (Horizontal and Vertical Shifts)
Example 1:

a) Describe, in order, how the graph of \( y = -2f(4(x - 2)) \) can be obtained from the graph of \( y = f(x) \).

1. Reflection in \( x \)-axis
2. Vertical stretch factor 2
3. Horizontal compression factor \( \frac{1}{4} \)
   (or horizontal compression by 4)
4. Shift right 2.

\[ \begin{align*}
\text{multiply by} \quad \frac{1}{4} \\
\text{divide by} \quad 4
\end{align*} \]

b) If \((x, y)\) was a point on \( f(x) \), what would the value of the coordinates be after the 4 transformations?

\[ (x, y) \rightarrow \left( \frac{1}{4} x + 2, -2y \right) \]

\[ \text{or} \quad \left( \frac{x}{4} + 2, -2y \right) \]

Our textbook interchanges these ... some textbooks consistently do it one way or the other. As long as I see the word compression, I am not concerned whether you write \( \frac{1}{4} \) or 4.

\[ \text{the word is most important: compression factor 4} \]

\[ \text{means you are dividing by 4} \]

\[ \text{but if you say compression factor 4, it means you are multiplying by 4} \]

\[ \text{both give same result.} \]
Example 2:

Given \( f(x) = x^2 + 3 \)

a) Describe how the graph of \( y = \frac{1}{2} f(2x + 6) - 2 \) can be obtained from \( f(x) \).

*First factor out the coefficient on \( x \) if \( b \neq 1 \) and \( h \neq 0 \).

\[
y = -\frac{1}{2} f \left[ 2(x+3) \right] - 2
\]

1. reflect in \( x \)-axis.
2. vert. comp. factor \( \frac{1}{2} \) \( \Rightarrow \) \(-\frac{1}{2} \) \( \times \) \( y \)
3. hor. comp. factor \( \frac{1}{2} \)
4. shift left 3
5. shift down 2

b) What is the new “image equation”? \( y = -\frac{1}{2} f \left[ 2(x+3) \right] - 2 \)

\[
y = -\frac{1}{2} \left[ (2)^2 (x+3)^2 + 3 \right] - 2
\]

\[
y = -\frac{1}{2} \left[ (4)(x+3)^2 + 3 \right] - 2
\]

\[
y = -\frac{1}{2} (4)(x+3)^2 - \frac{3}{2} - \frac{4}{2}
\]

\[
y = -2(x+3)^2 - \frac{7}{2}
\]
c) Graph.

\[ f(x) = x^2 + 3 \]

\[ y = -\frac{1}{2} f \left[ 2(x+3) \right] - 2 \]

\[ y = -2 (x+3)^2 - \frac{7}{2} \]

\[ V(-3, -3.5) \]

U3D8 Practice: p. 240 #7(odd), 8-9(odd, sketch one from each), 14