MCR3UI-03 **Unit 4 Test: Exponents & Exponential Functions** NAME:______________________

For each of the following questions, assume we are working in the set of Real Numbers, unless otherwise specified.

Part A: Exponents [19 marks] Thursday, April 18, 2019 Period 1 /60

1. Simplify using exponent laws then evaluate when possible. Ensure all final answers are rewritten with positive exponents. Do not convert fraction answers to decimals. You must show steps for full marks.

[13]

(a) $x \times (x^4) \div (x^2)^3$ (b) $\left(\frac{-12x^{-4}y^7}{18x^4y^{-7}}\right)^{-3}$ (c) $\sqrt[5]{\sqrt{x^{24}} \times \sqrt[4]{x^{12}}}$

(d) $\frac{(256x^5)^{\frac{1}{2}}}{(8x^6)^{\frac{1}{3}}}$ (e) $\frac{5^{100}-5^{101}}{5^{102}-5^{101}}$

2. Rewrite in radical form then evaluate.
You must show steps for full marks.
Fraction answers must be expressed in fraction form.

$\left(\frac{2401}{256}\right)^{-\frac{3}{4}}$

[3]

3. Express $\sqrt[5]{81^7}$ as a power of 3.

[2]

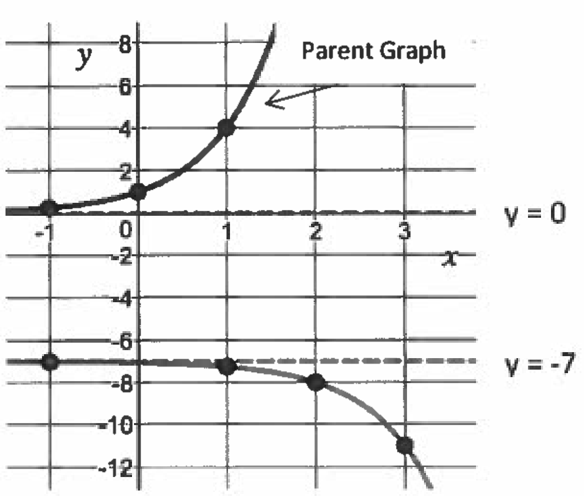
4. Solve.

(a) $4^x = \frac{1}{1024}$ (b) $11 \times 3^x = 297 \times 9^{x+1}$

[3]

5. Given the parent graph and a transformed image of the original, describe each of the transformations, write the equation of the parent graph and write the new image equation.

[4]



Equation of parent graph:

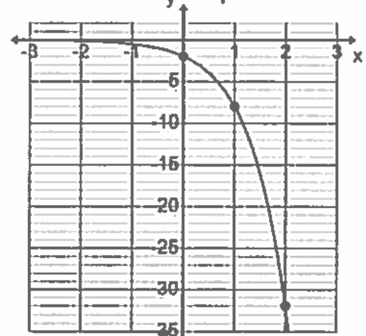
Description of transformations:

1.
2.
3.

Image Equation:

Part B: Exponential Functions [40 marks]

6. Match each equation with its graph below. [4]



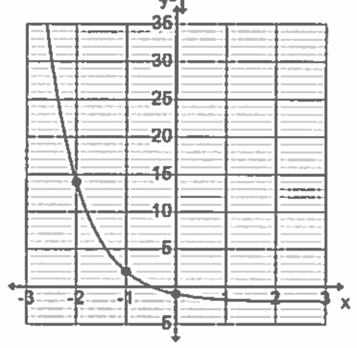
EQUATIONS

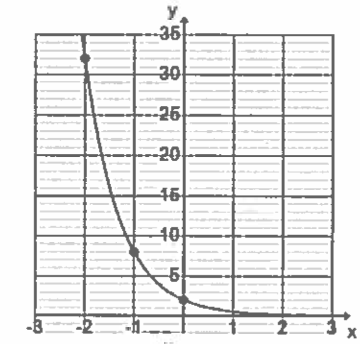
$y = (4)^{-x} - 2$

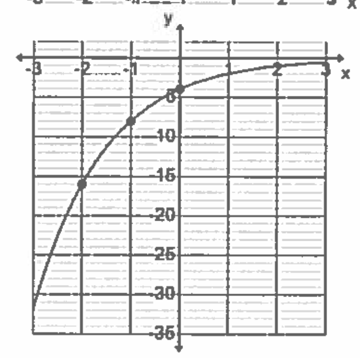
$y = 2\left(\frac{1}{4}\right)^{x}$

$y = -4\left(\frac{1}{2}\right)^{x}$

$y = -2(4)^{x}$







7. Given the base function of $y = 29^x$, write equation of the transformed function given it is stretched horizontally by factor 3, stretched vertically by factor 9,and shifted right 2 units. [2]

8. The temperature of Mrs. Behnke's tea, cooling over time, can be modelled by the exponential function $T(x) = 90\left(\frac{1}{2}\right)^{\frac{x}{32}}$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. [4]
   a. Determine the temperature of Mrs. Behnke's tea after 25 minutes, to the nearest tenth of a degree. [2]

   b. How many minutes (to the nearest tenth of a minute) will it take for the temperature of the tea to **first fall below** 28°C? [2]

9. The median house price in Waterloo Region increased by 3.6% from January 1, 2018 to January 1, 2019. A home was purchased in Waterloo Region on April 1, 2019 for $600 000. [6]
   a. Assuming this trend continues, write an exponential equation that models the Resale Value of this home over time. Include let statements. [2]

   b. At this rate, determine the date the resale price of the home would reach $1 million (Show your work accurate to the nearest month). [2]

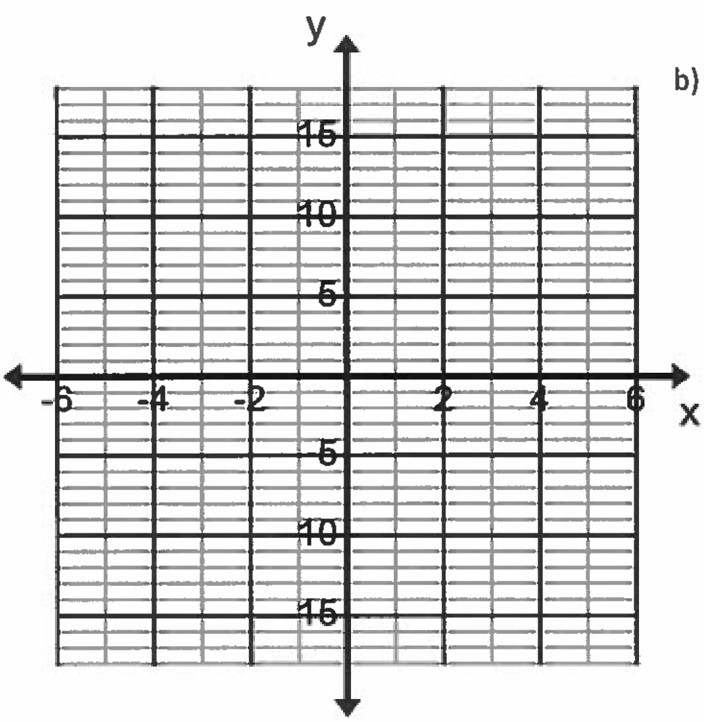
   c. Use your exponential equation to determine the expected resale value of the home on April 1, 2028. [2]

10. Complete the table below. Graph the parent AND transformed function for a) and b) on the grids provided. Use proper form. Intermediate graphs or tables may be used to show work. Be sure to show the y-intercept and any asymptotes.

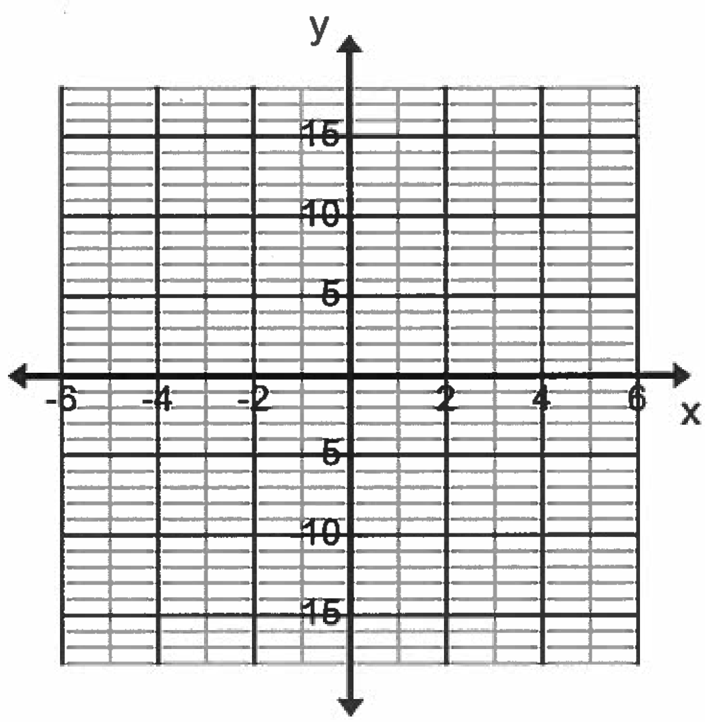
[19]

	a. $y=-2\left(\frac{1}{3}\right)^{-x}$	b. $y=(2)^{\frac{1}{2}x+1}-8$
Parent Function	$f(x)=$	$g(x)$
List the Transformations (in order)		
y-intercept		
x-intercept		
Equation of Asymptote		
Domain		
Range		
Increasing or Decreasing for $x \in R$?		

a)



b)



11. In each case, write an equation that models the situation described.

[2]

Situation	Equation
A bacteria colony has a population of 400 at 8:00a.m. The population triples every 16 hours. Let P(t) represent the population of the bacteria colony, t hours after 8:00a.m.	
The value of a $40 000 truck depreciates at 30% per year for tax purposes. Let V(t) represent the value of the truck after t years.	

[+1] BONUS: Simplify completely. Evaluate, if possible. $216^{\frac{4x^2y-16y}{24y-6x^2y}}$

MCR3UI-03 **Unit 4 Test: Exponents & Exponential Functions** NAME:

For each of the following questions, assume we are working in the set of Real Numbers, unless otherwise specified.

Part A: Exponents [19 marks] Thursday, April 18, 2019 Period 1

1. Simplify using exponent laws then evaluate when possible. Ensure all final answers are rewritten with positive exponents. Do not convert fraction answers to decimals. You must show steps for full marks.

[13]

(2) (a) $x \times (x^4) \div (x^2)^3$

$= x^{5-6}$

$= \frac{1}{x}$

(1) (b) $\left(\frac{-12x^{-4}y^7}{18x^4y^{-7}}\right)^{-3}$

$= \left(\frac{-3x^4y^{-7}}{2x^{-4}y^7}\right)^3$

$= \frac{(-3)^3(x^{4+4})^3(y^{-7-7})^3}{(2)^3}$

$= \frac{-27x^{24}}{8y^{42}}$

(2) (c) $\sqrt[5]{\sqrt{x^{24}} \times \sqrt[4]{x^{12}}}$

$= (x^{12} \times x^3)^{\frac{1}{5}}$

$= x^{15/5}$

$= x^3$

(2) (d) $\frac{(256x^5)^{\frac{1}{2}}}{(8x^6)^{\frac{1}{3}}}$

$= \frac{\sqrt{256}\, x^{\frac{5}{2}}}{\sqrt[3]{8}\, x^{6/3}}$

$= \frac{16}{2} x^{\frac{5}{2}-2}$

$= 8\sqrt{x}$

(2) (e) $\frac{5^{100}-5^{101}}{5^{102}-5^{101}}$

$= \frac{5^{100}(5^0-5)}{5^{101}(5^1-5^0)}$

$= \frac{-4}{5(4)}$

$= \frac{-1}{5}$

OR $\frac{5^{100}-5^{101}}{5^{102}-5^{101}} \times \frac{5^{-100}}{5^{-100}}$

$= \frac{5^0-5}{5^2-5}$

$= \frac{-4}{20} = \frac{-1}{5}$

2. Rewrite in radical form then evaluate. You must show steps for full marks. Fraction answers must be expressed in fraction form.

[3]
[2]

$\left(\frac{2401}{256}\right)^{-\frac{3}{4}}$

$= \frac{(\sqrt[4]{256})^3}{(\sqrt[4]{2401})^3}$

$= \frac{4^3}{7^3}$

$= \frac{64}{343}$ ✓

3. Express $\sqrt[5]{81^7}$ as a power of 3.

$= (3^{4})^{7/5}$

$= 3^{28/5}$ ✓

4. Solve.

[3]

(1) (a) $4^x = \frac{1}{1024}$

$4^x = 4^{-5}$

$x = -5$ ✓

(2) (b) $11 \times 3^x = 297 \times 9^{x+1}$

$3^x = 27 \times 9^{x+1}$

$3^x = 3^3 \times (3^2)^{x+1}$

$3^x = 3^{3+2x+2}$

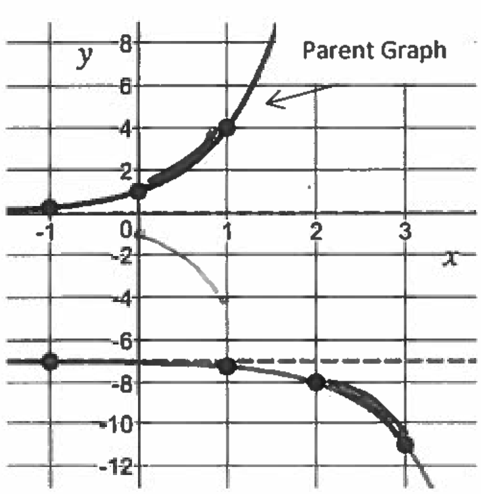
$3^x = 3^{2x+5}$

$2x+5 = x$

$x = -5$

5. Given the parent graph and a transformed image of the original, describe each of the transformations, write the equation of the parent graph and write the new image equation.

[4]



Equation of parent graph:

$f(x) = 4^x$

Description of transformations:

1. reflection in x-axis
2. shift right 2
3. shift down 7

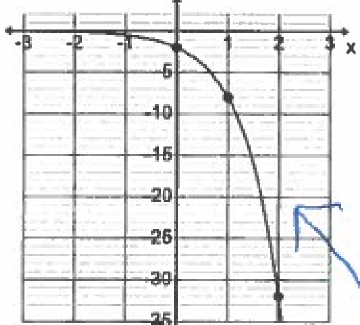
Image Equation:

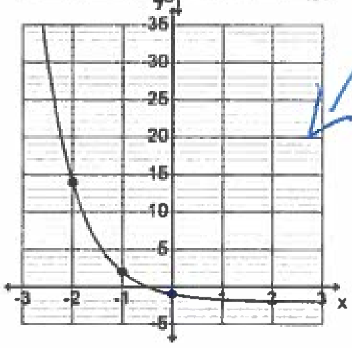
$y = -4^{x-2} - 7$

Part B: Exponential Functions [40 marks]

6. Match each equation with its graph below.

[4]





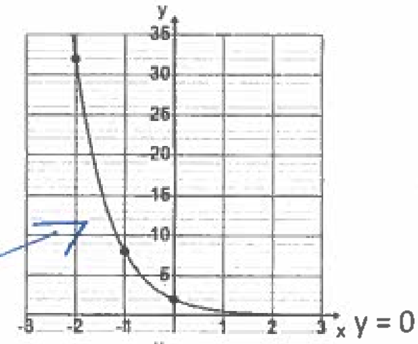
EQUATIONS

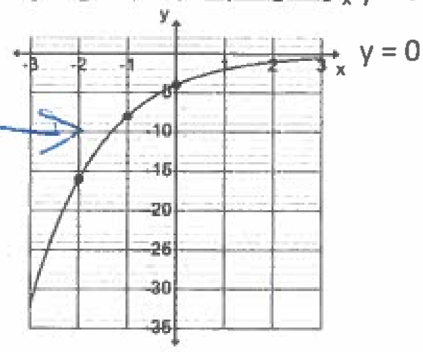
$y = (4)^{-x} - 2$

$y = 2\left(\frac{1}{4}\right)^x$

$y = -4\left(\frac{1}{2}\right)^x$

$y = -2(4)^x$





7. Given the base function of $y = 29^x$, write equation of the transformed function given it is stretched horizontally by factor 3, stretched vertically by factor 9,and shifted right 2 units.

[2]

$\frac{1}{3}x$ $9f(x)$ $x-2$

$y = 9(29)^{\frac{1}{3}(x-2)}$

8. The temperature of Mrs. Behnke's tea, cooling over time, can be modelled by the exponential function $T(x) = 90\left(\frac{1}{2}\right)^{\frac{x}{32}}$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes.

[4]

a. Determine the temperature of Mrs. Behnke's tea after 25 minutes, to the nearest tenth of a degree. [2]

$T(25) = 90(0.5)^{25/32}$

$= 52.367$

∴ the temp would be about 52.4°C.

b. How many minutes (to the nearest tenth of a minute) will it take for the temperature of the tea to **first fall below** 28°C? [2]

$90(0.5)^{t/32} < 28$

x	T(x)
50	30.47
55	27.3

x	T(t)
53	28.5
53.8	28.06
53.9	28.002

x	T(x)
54	27.94

After 54.[illegible] mins. the tea will first fall below 28°C.

+1 for .0

9. The median house price in Waterloo Region increased by 3.6% from January 1, 2018 to January 1, 2019. A home was purchased in Waterloo Region on April 1, 2019 for $600 000.

[6]

a. Assuming this trend continues, write an exponential equation that models the Resale Value of this home over time. Include let statements. [2]

$V(t) = 600000(1.036)^t$, where

b. At this rate, determine the date the resale price of the home would reach $1 million (Show your work accurate to the nearest month). [2]

$600\,000(1.036)^t = 1\,000\,000$

t	V(t)
14	984 436
14.4	998 962
14.5	1 001 999.

← 14.45 1 000 229.

0.45 × 12 = $5\frac{2}{5}$

2019 + 14 yrs = 2033

April + 5 months ⇒ Sept 1.

In September, 2033 home would be worth $1 M

accept Oct 1, 2033

c. Use your exponential equation to determine the expected resale value of the home on April 1, 2028. [2]

$V(9) = 600000(1.036)^9$

$V(9) = 824\,876.72$

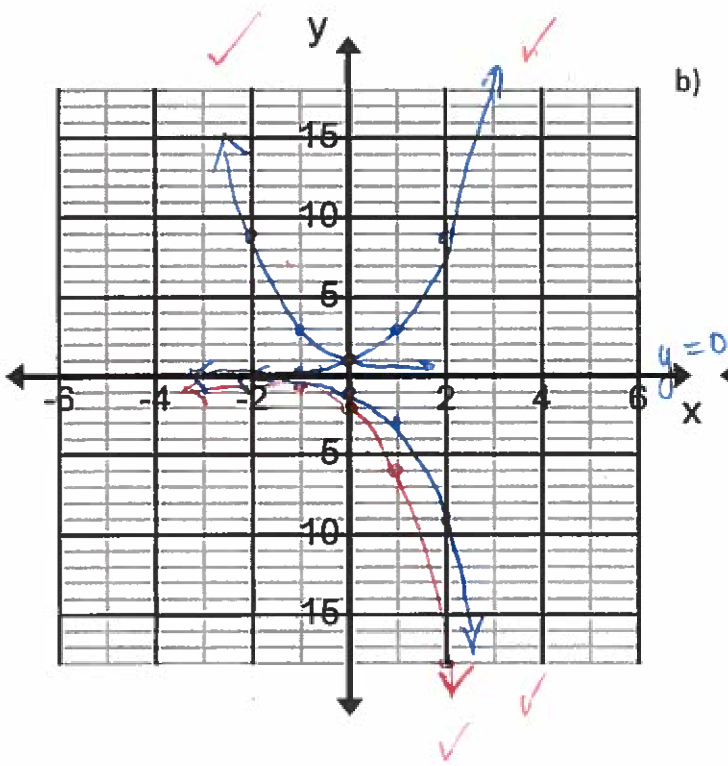
MCR3UI-03 **Unit 4 Test: Exponents & Exponential Functions** NAME:

10. Complete the table below. Graph the parent AND transformed function for a) and b) on the grids provided. Use proper form. Intermediate graphs or tables may be used to show work. Be sure to show the y-intercept and any asymptotes.

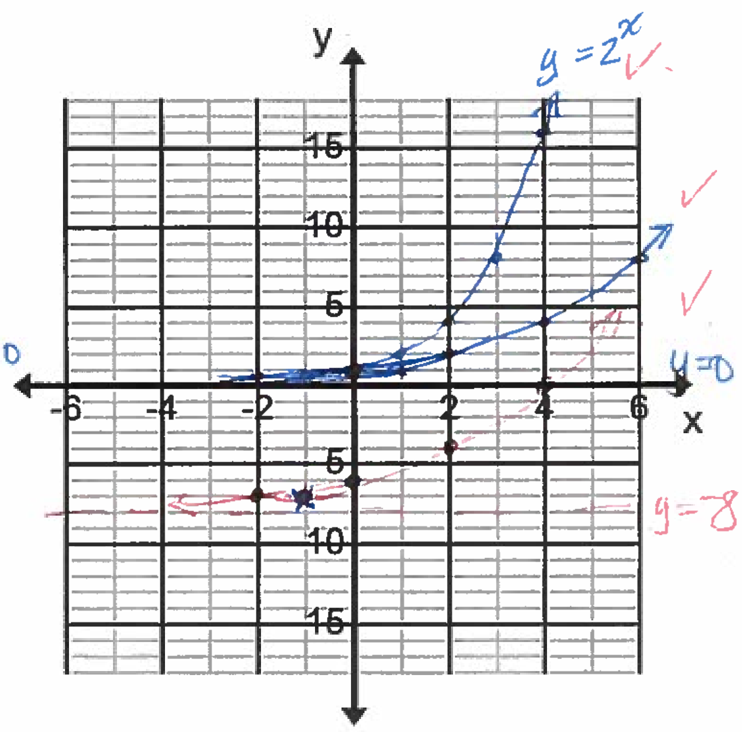
[19]

	(10) a. $y = -2\left(\frac{1}{3}\right)^{-x} = -2(3)^x$	(9) b. $y = (2)^{\frac{1}{2}x+1} - 8 = 2^{\frac{1}{2}(x+2)} - 8$
Parent Function	$f(x) = \left(\frac{1}{3}\right)^x$ OR 3^x	$g(x) = 2^x$
List the Transformations (in order)	reflection in → x-axis & → y-axis; V. Stretch factor 2 / reflection in x-axis	H. Stretch factor 2, Shift Left 2, Down 8
y-intercept	$y=-2$ OR $(0,-2)$	$y=-6$ OR $(0,-6)$
x-intercept		$x=4$
Equation of Asymptote	$y=0$	$y=-8$
Domain	$\{x \in R\}$	$\{x \in R\}$
Range	$\{y < 0\}$	$\{y > -8\}$
Increasing or Decreasing for $x \in R$?	Dec.	Inc.

a)



b)



11. In each case, write an equation that models the situation described.

[2]

Situation	Equation
A bacteria colony has a population of 400 at 8:00a.m. The population triples every 16 hours. Let P(t) represent the population of the bacteria colony, t hours after 8:00a.m.	$P(t) = 400(3)^{t/16}$
The value of a $40 000 truck depreciates at 30% per year for tax purposes. Let V(t) represent the value of the truck after t years.	$V(t) = 40000(0.7)^t$

[+1] BONUS: Simplify completely. Evaluate, if possible. $216^{\frac{4x^2y-16y}{24y-6x^2y}}$

$= (216)^{\frac{4y(x^2-4)}{-6y(x^2-4)}} = (216)^{-2/3} = \frac{1}{(\sqrt[3]{216})^2} = \frac{1}{6^2} = \frac{1}{36}$

MCR3UI-04 **Unit 4 Test: Exponents & Exponential Functions** NAME:________________

For each of the following questions, assume we are working in the set of Real Numbers, unless otherwise specified.

Part A: Exponents [19 marks] Thursday, April 18, 2019 Period 2 /60

1. Simplify using exponent laws then evaluate when possible. Ensure all final answers are rewritten with positive exponents. Do not convert fraction answers to decimals. You must show steps for full marks.

[13]

(a) $x \times (x^3) \div (x^2)^3$ (b) $\left(\frac{-12x^{-4}y^7}{28x^4y^{-7}}\right)^{-3}$ (c) $\sqrt[5]{\sqrt{x^{14}} \times \sqrt[4]{x^{12}}}$

(d) $\frac{(256x^{16})^{\frac{1}{2}}}{(8x^6)^{\frac{1}{3}}}$ (e) $\frac{5^{100}+5^{101}}{5^{102}+5^{101}}$

2. Rewrite in radical form then evaluate.
You must show steps for full marks.
Fraction answers must be expressed in fraction form.

$\left(\frac{2401}{256}\right)^{-\frac{3}{4}}$

[3]

3. Express $\sqrt[5]{27^7}$ as a power of 3.

[2]

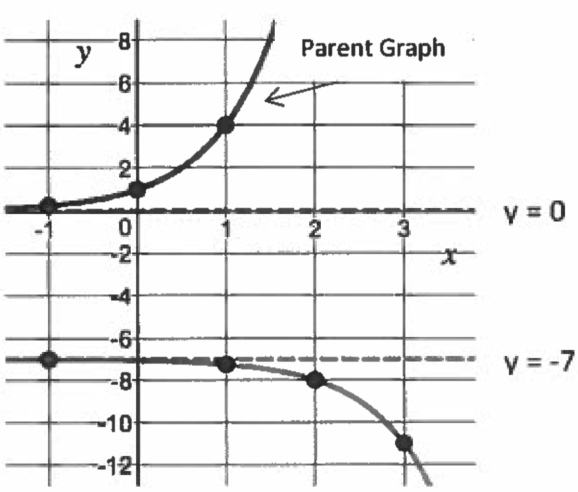
4. Solve.

(a) $2^x = \frac{1}{1024}$ (b) $11 \times 3^x = 33 \times 9^{x+1}$

[3]

5. Given the parent graph and a transformed image of the original, describe each of the transformations, write the equation of the parent graph and write the new image equation.

[4]



Equation of parent graph:

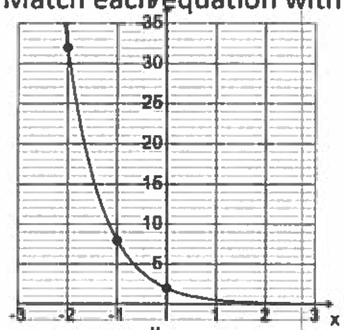
Description of transformations:

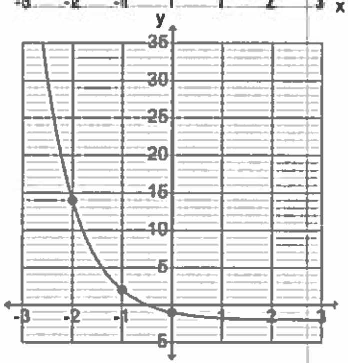
1.
2.
3.

Image Equation:

Part B: Exponential Functions [40 marks]

6. Match each equation with its graph below. [4]





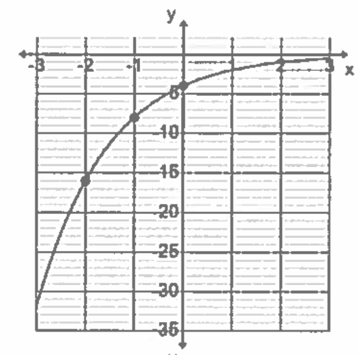
EQUATIONS

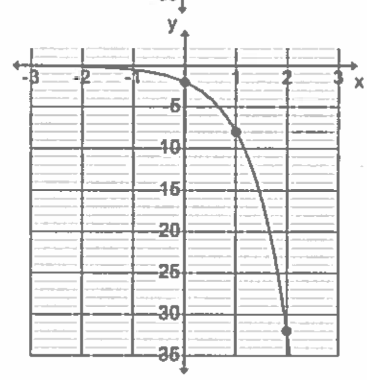
$y = 2\left(\frac{1}{4}\right)^x$

$y = -2(4)^x$

$y = -4\left(\frac{1}{2}\right)^x$

$y = (4)^{-x} - 2$





7. Given the base function of $y = 19^x$, write equation of the transformed function given it is stretched horizontally by factor 2, stretched vertically by factor 7,and shifted right 6 units. [2]

8. The temperature of Mrs. Behnke's tea, cooling over time, can be modelled by the exponential function $T(x) = 90\left(\frac{1}{2}\right)^{\frac{x}{35}}$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. [4]

   a. Determine the temperature of Mrs. Behnke's tea after 25 minutes, to the nearest tenth of a degree. [2]

   b. How many minutes (to the nearest tenth of a minute) will it take for the temperature of the tea to **first fall below** 28°C? [2]

9. The median house price in Waterloo Region increased by 3.6% from January 1, 2018 to January 1, 2019. A home was purchased in Waterloo Region on April 1, 2019 for $600 000. [6]

   a. Assuming this trend continues, write an exponential equation that models the Resale Value of this home over time. Include let statements. [2]

   b. At this rate, determine the date the resale price of the home would reach $1 million (Show your work accurate to the nearest month). [2]

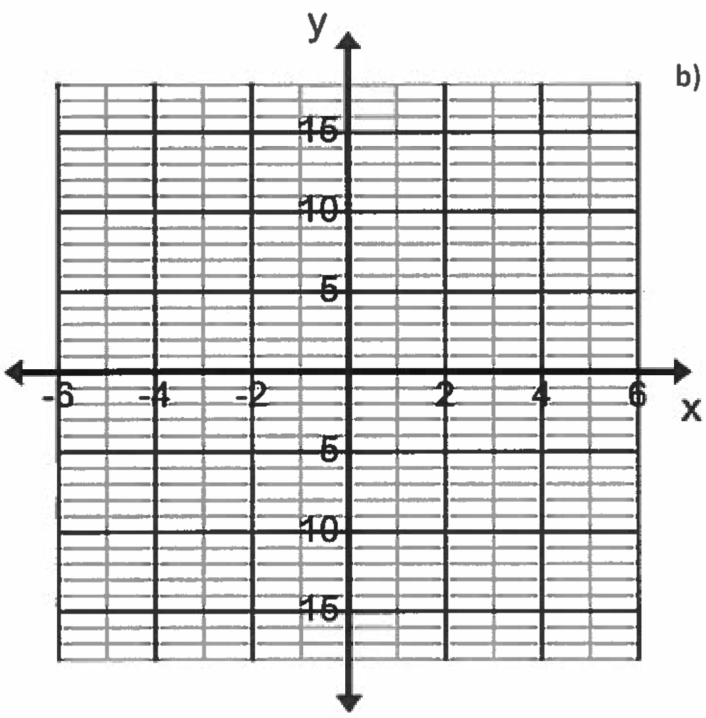
   c. Use your exponential equation to determine the expected resale value of the home on April 1, 2025. [2]

10. Complete the table below. Graph the parent AND transformed function for a) and b) on the grids provided. Use proper form. Intermediate graphs or tables may be used to show work. Be sure to show the y-intercept and any asymptotes.

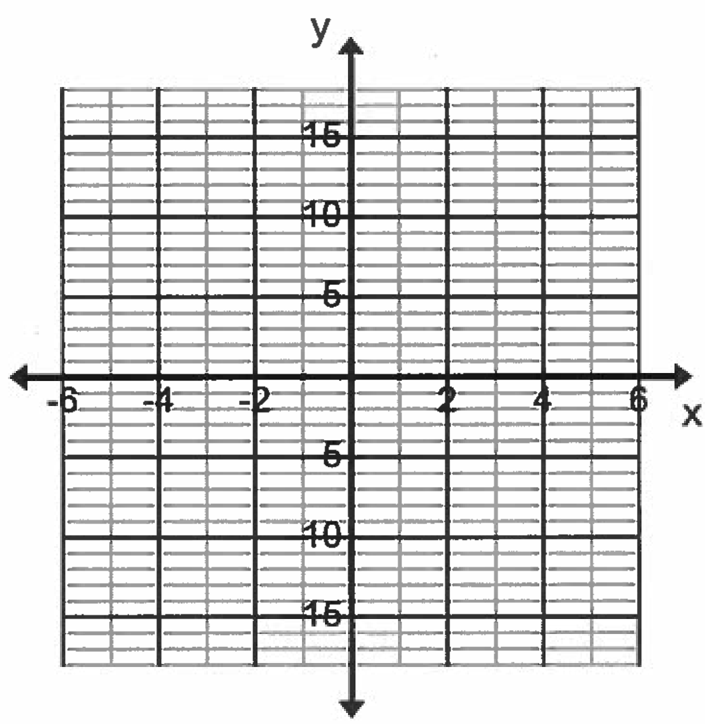
[19]

	a. $y = -2\left(\frac{1}{3}\right)^{-x}$	b. $y = (2)^{\frac{1}{2}x+1} - 4$
Parent Function	$f(x) =$	$g(x)$
List the Transformations (in order)		
y-intercept		
x-intercept		
Equation of Asymptote		
Domain		
Range		
Increasing or Decreasing for $x \in R$?		

a)



b)



11. In each case, write an equation that models the situation described.

[2]

Situation	Equation
A bacteria colony has a population of 800 at 10:00a.m. The population quadruples every 10 hours. Let P(t) represent the population of the bacteria colony, t hours after 10:00a.m.	
The value of a $60 000 truck depreciates at 28% per year for tax purposes. Let V(t) represent the value of the truck after t years.	

[+1] BONUS: Solve. $\sqrt{9x^6 + 30x^5 + 25x^4} = 0$

MCR3UI-04 **Unit 4 Test: Exponents & Exponential Functions** NAME:________________

For each of the following questions, assume we are working in the set of Real Numbers, unless otherwise specified.

Part A: Exponents [19 marks] Thursday, April 18, 2019 Period 2

1. Simplify using exponent laws then evaluate when possible. Ensure all final answers are rewritten with positive exponents. Do not convert fraction answers to decimals. You must show steps for full marks.

[13]

(a) $x \times (x^3) \div (x^2)^3$

$= x^4 \div x^6$

$= \frac{1}{x^2}$ (2)

(b) $\left(\frac{-12x^{-4}y^7}{28x^4y^{-7}}\right)^{-3}$

$= \left(\frac{-6x^{-8}y^{14}}{14}\right)^{-3}$

$= \left(\frac{-7x^8}{3y^{14}}\right)^3 = \frac{-343x^{24}}{27y^{42}}$

(4)

(c) $\sqrt[5]{\sqrt{x^{14}} \times \sqrt[4]{x^{12}}}$

$= (x^7 \times x^3)^{\frac{1}{5}}$

$= (x^{10})^{\frac{1}{5}}$

$= x^2$ (3)

(d) $\frac{(256x^{16})^{\frac{1}{2}}}{(8x^6)^{\frac{1}{3}}}$

$= \frac{\sqrt{256}\, x^{\frac{16}{2}}}{\sqrt[3]{8}\, x^{6/3}}$ (2)

$= \frac{16}{2} x^{8-2} = 8x^6$

(e) $\frac{5^{100}+5^{101}}{5^{102}+5^{101}} \times \frac{5^{-100}}{5^{-100}}$

$= \frac{5^0 + 5^1}{5^2+5^1}$ (2)

$= \frac{6}{30}$

$= \frac{1}{5}$

OR → factor rational expression

$\frac{5^{100}(1+5)}{5^{101}(5+1)}$

$= \frac{1}{5}$

2. Rewrite in radical form then evaluate.
You must show steps for full marks.
Fraction answers must be expressed in fraction form.

[3]
[2]

$\left(\frac{2401}{256}\right)^{-\frac{3}{4}}$

$= \left(\frac{256}{2401}\right)^{3/4}$ ✓

$= \frac{(\sqrt[4]{256})^3}{(\sqrt[4]{2401})^3}$ ✓

$= \frac{4^3}{7^3}$

$= \frac{64}{343}$ ✓

3. Express $\sqrt[5]{27^7}$ as a power of 3.

$= (3^3)^{7/5}$ ✓

$= 3^{21/5}$ ✓

4. Solve.

[3]

(a) $2^x = \frac{1}{1024}$

$2^x = 2^{-10}$

$x = -10$ ✓

(b) $11 \times 3^x = 33 \times 9^{x+1}$ ÷11

$3^x = 3^1 \times (3^2)^{x+1}$ ✓

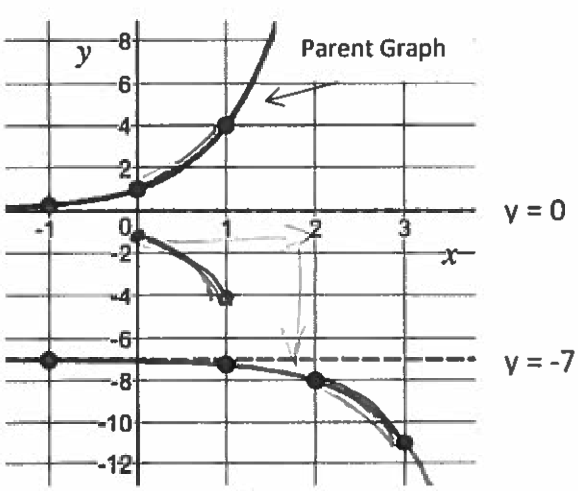
$3^x = 3^{2x+2+1}$

$x = 2x+3$ ✓

$-3 = x$

5. Given the parent graph and a transformed image of the original, describe each of the transformations, write the equation of the parent graph and write the new image equation.

[4]



Equation of parent graph:

$y = 4^x$

Description of transformations:

1. reflect in x-axis
2. shift right 2
3. down 7

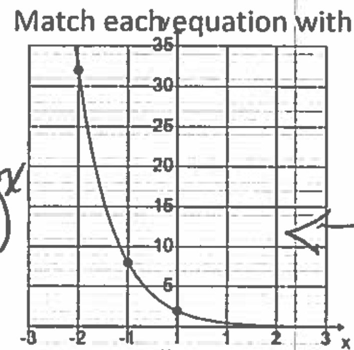
Image Equation:

$y = -4^{x-2} - 7$

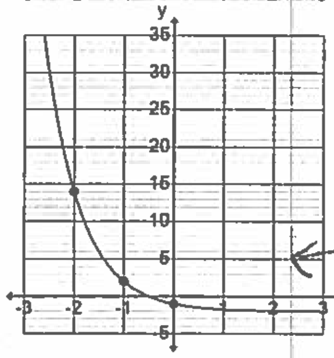
Part B: Exponential Functions [40 marks]

6. Match each equation with its graph below.

[4]



$y=2\left(\frac{1}{4}\right)^x$



$y=4^{-x}-2$

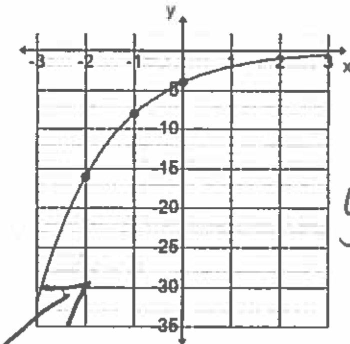
EQUATIONS

$y = 2\left(\frac{1}{4}\right)^x$

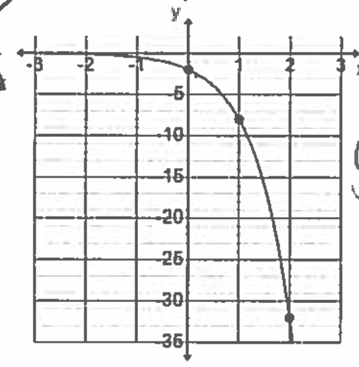
$y = -2(4)^x$

$y = -4\left(\frac{1}{2}\right)^x$

$y = (4)^{-x} - 2$



$y=-4\left(\frac{1}{2}\right)^x$



$y=-2(4)^x$

7. Given the base function of $y = 19^x$, write equation of the transformed function given it is stretched horizontally by factor 2, stretched vertically by factor 7,and shifted right 6 units.

[2]

$y=7(19)^{\frac{1}{2}(x-6)}$

8. The temperature of Mrs. Behnke's tea, cooling over time, can be modelled by the exponential function $T(x) = 90\left(\frac{1}{2}\right)^{\frac{x}{35}}$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes.

[4]

a. Determine the temperature of Mrs. Behnke's tea after 25 minutes, to the nearest tenth of a degree. [2]

$T(25)=90(0.5)^{25/35}$

$T(25)=54.8556$

∴ the tea would be 54.9°C after 25 mins.

b. How many minutes (to the nearest tenth of a minute) will it take for the temperature of the tea to **first fall below** 28°C? [2]

$90(0.5)^{t/35}<28$

t	V(t)
59	27.976
58.9	28.03

∴ after 59.0 minutes, the tea would first fall below 28°C.

+1 for 59.0

9. The median house price in Waterloo Region increased by 3.6% from January 1, 2018 to January 1, 2019. A home was purchased in Waterloo Region on April 1, 2019 for $600 000.

[6]

a. Assuming this trend continues, write an exponential equation that models the Resale Value of this home over time. Include let statements. [2]

$V(t)=600\,000(1.036)^t$, where $V(t)$ is the value of the home after t years.

b. At this rate, determine the date the resale price of the home would reach $1 million (Show your work accurate to the nearest month). [2]

$600\,000(1.036)^t=1\,000\,000$

$t=14.45$

$V(14.4)=998462$

$V(14.45)=1000229$ $V(14.5)=1001999$

$0.45\times12=5\frac{2}{3}$

In September, 2033 the home would be worth $1M.

+1 for Sept.

c. Use your exponential equation to determine the expected resale value of the home on April 1, 2025. [2]

$V(6)=600000(1.036)^6$

$V(6)=741\,839.2075$

∴ the home would be worth $741839.21 on Apr 1/25.

MCR3UI-04 **Unit 4 Test: Exponents & Exponential Functions** NAME:

10. Complete the table below. Graph the parent AND transformed function for a) and b) on the grids provided. Use proper form. Intermediate graphs or tables may be used to show work. Be sure to show the y-intercept and any asymptotes.

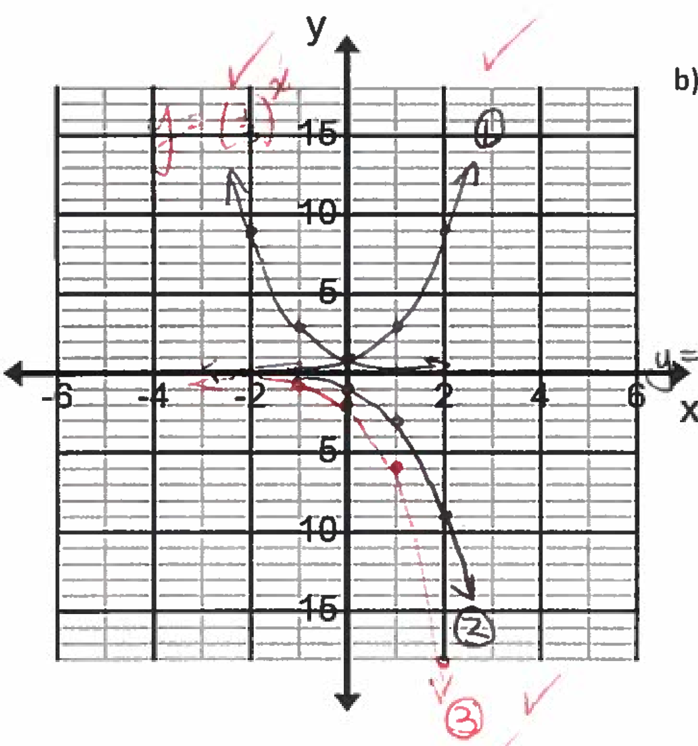
[19]

	a. $y = -2\left(\frac{1}{3}\right)^{-x}$	b. $y = (2)^{\frac{1}{2}x+1} - 4 = (2)^{\frac{1}{2}(x+2)} - 4$
Parent Function	$f(x) = \left(\frac{1}{3}\right)^x$	$g(x) = 2^x$
List the Transformations (in order)	reflection in ② x-axis ① & y-axis; Vert. stretch factor 2	① Hor. stretch factor 2; ② Shift Left 2; ③ Down 4
y-intercept	$y = -2$ or $(0,-2)$	$y = -2$ or $(0,-2)$
x-intercept	—	$x = 2$ or $(2,0)$
Equation of Asymptote	$y = 0$	$y = -4$
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$
Range	$\{y < 0\}$	$\{y > -4\}$
Increasing or Decreasing for $x \in R$?	Dec.	Inc.

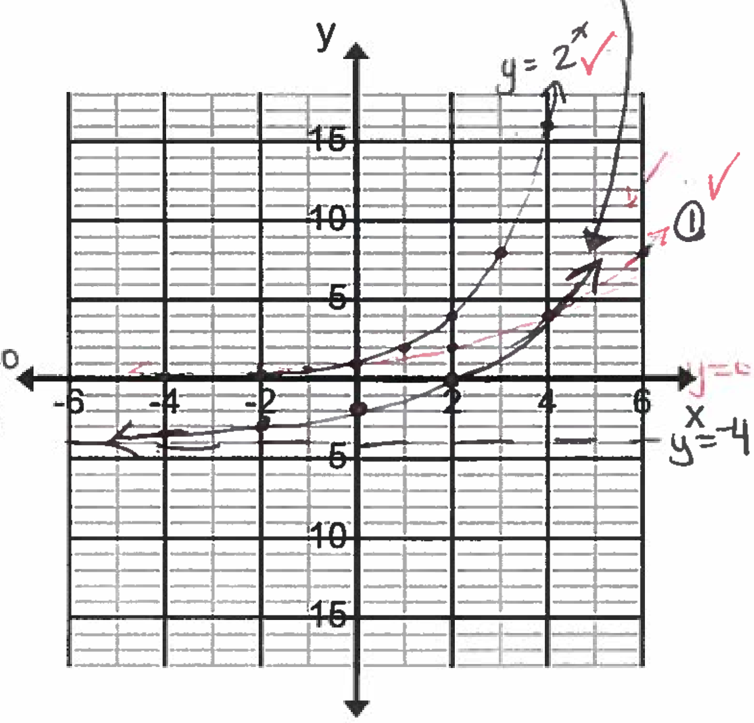
If Left 1 (-1); x=3 graph

$y = 2^{\frac{1}{2}x+1} - 4$ ②③

a)



b)



11. In each case, write an equation that models the situation described.

[2]

Situation	Equation
A bacteria colony has a population of 800 at 10:00a.m. The population quadruples every 10 hours. Let P(t) represent the population of the bacteria colony, t hours after 10:00a.m.	$P(t) = 800(4)^{t/10}$
The value of a $60 000 truck depreciates at 28% per year for tax purposes. Let V(t) represent the value of the truck after t years.	$V(t) = 60000(0.72)^t$

[+1] BONUS: Solve. $\sqrt{9x^6 + 30x^5 + 25x^4} = 0$

$\left[(3x^3+5x^2)^2\right]^{\frac{1}{2}} = 0$

$3x^3 + 5x^2 = 0$

$x^2(3x+5) = 0$

$x = 0$ or $x = -\frac{5}{3}$

MCR3UI-05 **Unit 4 Test: Exponents & Exponential Functions** **NAME:**________________________

For each of the following questions, assume we are working in the set of Real Numbers, unless otherwise specified. Thursday, April 18, 2019 Period 4

Part A: Exponents [19 marks] /60

1. Simplify using exponent laws then evaluate when possible. Ensure all final answers are rewritten with positive exponents. Do not convert fraction answers to decimals. You must show steps for full marks.

[13]

(a) $x \div (x^4) \times (x^2)^3$ (b) $\left(\frac{-20x^4y^{-7}}{15x^{-4}y^7}\right)^{-4}$ (c) $(27x^4)^{\frac{1}{3}}(64x^{10})^{\frac{1}{6}}$

(d) $\frac{\sqrt[3]{250x^4y^2}}{\sqrt[3]{2xy^{-10}}}$ (e) $\frac{11^{101}-11^{100}}{11^{102}+11^{101}}$

2. Rewrite in radical form then evaluate. You must show steps for full marks. Fraction answers must be expressed in fraction form.

$\left(\frac{256}{14641}\right)^{-\frac{3}{4}}$

[3]
[2]

3. Express $\sqrt[8]{64^9}$ as a power of 2.

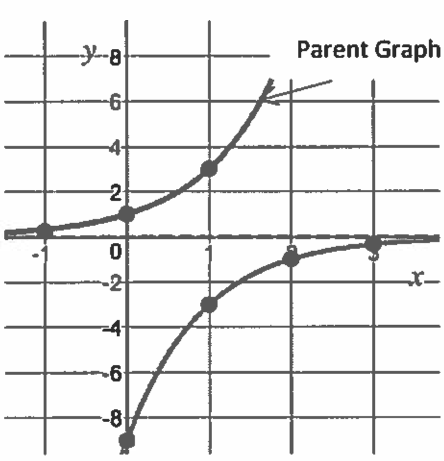
4. Solve.

(a) $3^x = \frac{1}{2187}$ (b) $\frac{2^{3x-2}}{5} = \frac{16^{2x-4}}{10}$

[3]

5. Given the parent graph and a transformed image of the original, describe each of the transformations, write the equation of the parent graph and write the new image equation.

[4]



v = 0

Equation of parent graph:

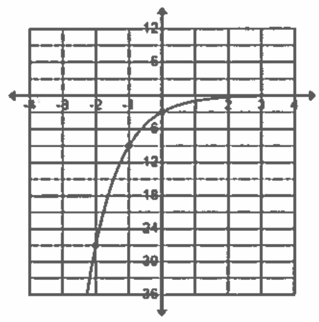
Description of transformations:

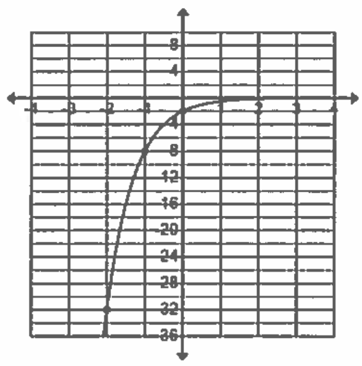
1.
2.
3.

Image Equation:

Part B: Exponential Functions [40 marks]
Match each equation with its graph below.

[4] 6.





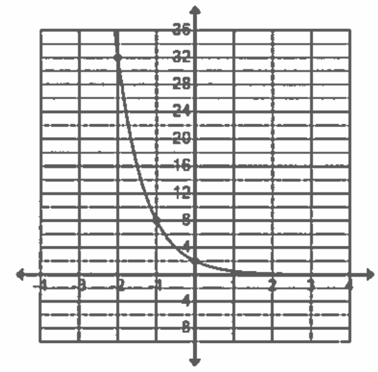
EQUATIONS

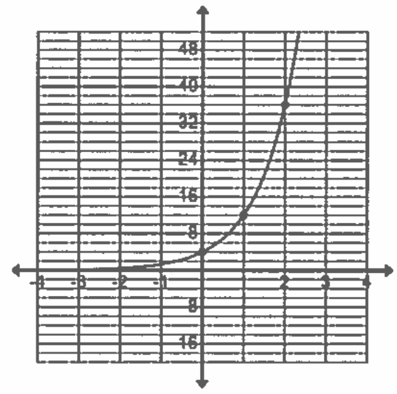
$y = 4\left(\frac{1}{3}\right)^{-x}$

$y = -(3)^{-(x-1)}$

$y = 2(4)^{-x}$

$y = -2\left(\frac{1}{4}\right)^{x}$





7. Given the base function of $y = \left(\frac{1}{8}\right)^{x}$, write equation of the transformed function given it is compressed horizontally by 3, stretched vertically by factor 4, and shifted left 1 units. [2]

8. A bacteria colony had a population of 50 at 9:00 a.m. on Tuesday, April 16, 2019. The population quadruples every 15 hours and is represented by this formula: $P(t) = 50(4)^{\frac{t}{15}}$, where $P(t)$ represents the population of the bacteria colony, t hours after 9:00 a.m. on Tuesday, April 16, 2019 [4]
   a. Determine the number of bacteria at 9:00 p.m. on Wednesday, April 17, 2019. [2]

   b. At this rate, determine the date and time, the original population would triple (accurate to the nearest tenth of an hour). [2]

9. The population of Hicksville, Montana had a population of 42150 in April of 2000. Since then, the population has been **declining** at a rate of 4.5% each year. [6]
   a. Assuming this trend continues, write an exponential equation that models the population of Hicksville, Montana over time. Include let statements. [2]

   b. Use your exponential equation to determine the current population (in April, 2019). [2]

   c. At this rate, determine the date the original population would first fall below 21 000 (accurate to the nearest month). [2]

MCR3UI-05 **Unit 4 Test: Exponents & Exponential Functions** NAME:____________________

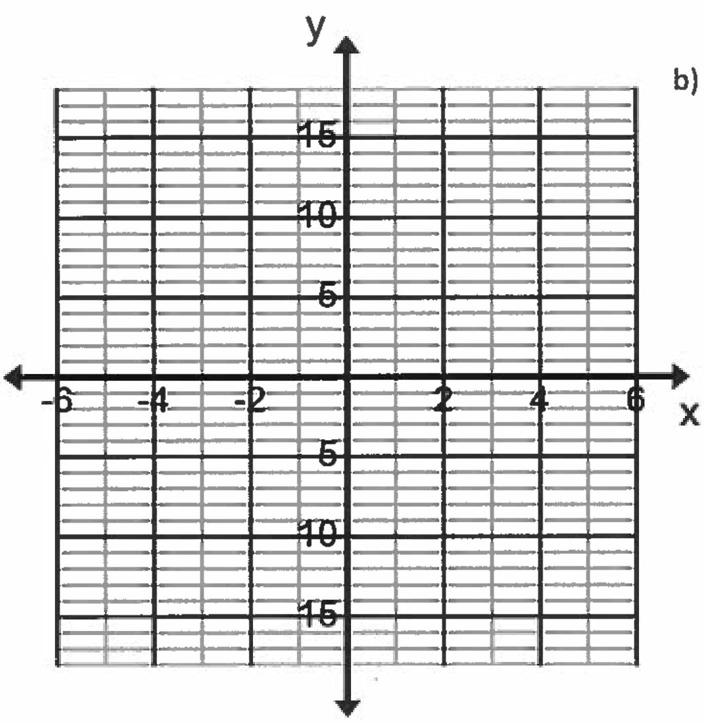
10. Complete the table below. Graph the parent AND transformed function for a) and b) on the grids provided. Use proper form. Intermediate graphs or tables may be used to show work. Be sure to show the y-intercept and any asymptotes.

[19]

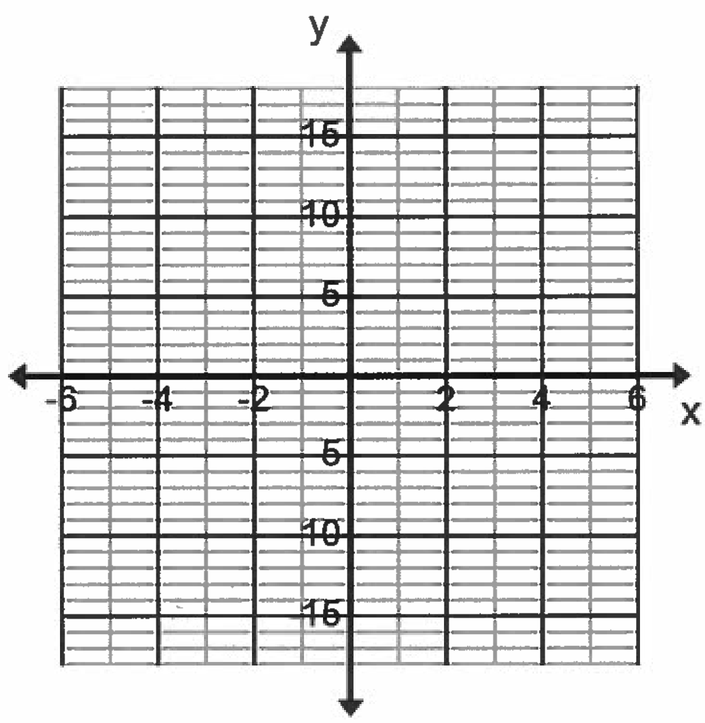
	a. $y = -\left(\frac{1}{3}\right)^{\frac{1}{2}x}$	b. $y = 4(2)^{-x+2} - 8$
Parent Function	$f(x)$	$g(x)$
List the Transformations (in order)		
y-intercept		
x-intercept		
Equation of Asymptote		
Domain		
Range		
Increasing or Decreasing for $x \in R$?		

a)

[2]



b)



11. In each case, write an equation that models the situation described.

Situation	Equation
The temperature of a hot liquid is initially 100 degrees Celsius. The temperature drops 42% every 25 minutes. Let T(x) represent the temperature, in degrees Celsius, and let x represent the elapsed time, in minutes.	
The value of a $10 000 investment grows at 3.2% per year. Let V(t) represent the value of the investment after t years.	

[+1] BONUS: If $\left(\frac{2}{x} - \frac{x}{2}\right)^2 = 0$, find the value of x^6.

MCR3UI-05 **Unit 4 Test: Exponents & Exponential Functions** NAME:

For each of the following questions, assume we are working in the set of Real Numbers, unless otherwise specified. Thursday, April 18, 2019 Period 4

Part A: Exponents [19 marks]

1. Simplify using exponent laws then evaluate when possible. Ensure all final answers are rewritten with positive exponents. Do not convert fraction answers to decimals. You must show steps for full marks.

[13]

(a) $x \div (x^4) \times (x^2)^3$

$= x^{-3+6}$

$= x^3$

(b) $\left(\frac{-20x^4y^{-7}}{15x^{-4}y^7}\right)^{-4}$

$= \left(\frac{-4x^8}{3y^{14}}\right)^{-4}$

$= \frac{(3)^4 y^{14\times4}}{(-4)^4(x^{8\times4})}$

$= \frac{81y^{56}}{256x^{32}}$

$\frac{(-20)^{-4}x^{-16}y^{28}}{(15)^{-4}x^{+16}y^{-28}}$

$= \left(\frac{-4}{3}\right)^{-4} x^{-32} y^{56}$

$= \frac{81y^{56}}{256x^{32}}$

(c) $(27x^4)^{\frac{1}{3}}(64x^{10})^{\frac{1}{6}}$

$= \sqrt[3]{27}\sqrt[6]{64}\, x^{\frac{4}{3}} x^{\frac{10}{6}}$

$= 3(2)x^{\frac{4}{3}+\frac{5}{3}}$

$= 6x^3$

(d) $\frac{\sqrt[3]{250x^4y^2}}{\sqrt[3]{2xy^{-10}}}$

$= \sqrt[3]{125}\, x^{3/3} y^{12/3}$

$= 5xy^4$

(e) $\frac{11^{101}-11^{100}}{11^{102}+11^{101}} \times \frac{11^{-100}}{11^{-100}}$

$= \frac{11^1-11^0}{11^2+11^1}$

$= \frac{11-1}{121+11}$

$= \frac{10}{132} = \frac{5}{66}$

OR

$\frac{11^{100}(11^1-11^0)}{11^{101}(11^1+11^0)}$

$= \frac{1}{11}\left(\frac{11-1}{11+1}\right)$

$= \frac{1(10)}{11(12)} = \frac{5}{66}$

2. Rewrite in radical form then evaluate.
You must show steps for full marks.
Fraction answers must be expressed in fraction form.

[3]
[2]

$\left(\frac{256}{14641}\right)^{-\frac{3}{4}}$

$= \frac{(\sqrt[4]{14641})^3}{(\sqrt[4]{256})^3}$

$= \frac{11^3}{4^3} = \frac{1331}{64}$

3. Express $\sqrt[8]{64^9}$ as a power of 2.

$(2^6)^{9/8}$

$= 2^{\frac{27}{4}}$

4. Solve.

[3]

(a) $3^x = \frac{1}{2187}$

$3^x = 3^{-7}$

$x = -7$

(b) $\frac{2^{3x-2}}{5} = \frac{16^{2x-4}}{10}$

$10\frac{(2)^{3x-2}}{5} = \frac{10(16)^{2x-4}}{10}$

$2^{3x-1} = 2^{8x-16}$

$3x-1 = 8x-16$

$15 = 5x$

$x = 3$

check LS $\frac{2^7}{5} = \frac{128}{5}$, RS $\frac{16^2}{10} = \frac{128}{5}$

OR

$\frac{10}{5} = \frac{(2^4)^{2x-4}}{2^{3x-2}}$

$2^1 = 2^{8x-16-3x+2}$

$5x-14 = 1$

$5x = 15$

$x = 3$

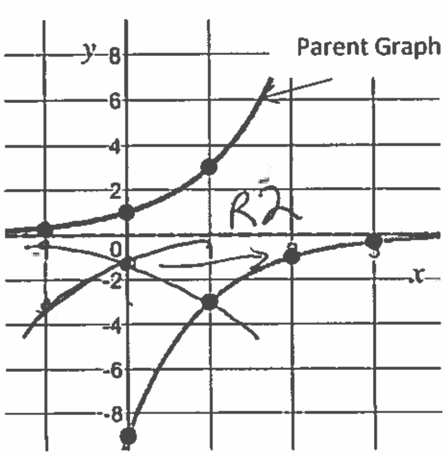
OR

$\frac{2^{3x-2}}{5} = \frac{(2^4)^{2x-4}}{5 \cdot 2^1} \Rightarrow 2^{3x-2} = 2^{8x-16-1}$

$3x-2 = 8x-17$

5. Given the parent graph and a transformed image of the original, describe each of the transformations, write the equation of the parent graph and write the new image equation.

[4]



v = 0

Equation of parent graph:

$f(x) = 3^x$

Description of transformations:

1. reflections in x-axis and y-axis
2.
3. shift right 2.

Image Equation:

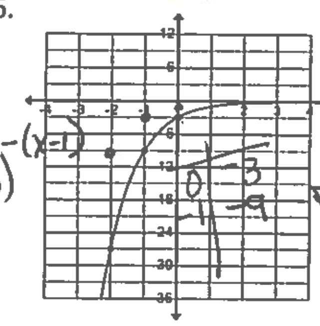
$y = -3^{-(x-2)}$ OR $y = -\left(\frac{1}{3}\right)^{x-2}$

25

Part B: Exponential Functions [40 marks]

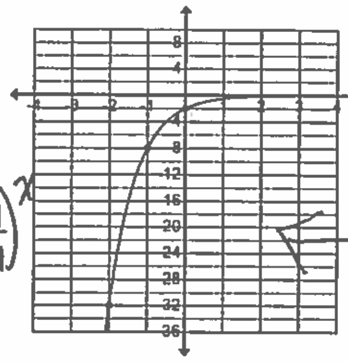
Match each equation with its graph below.

[4] 6.



$y = -(3)^{-(x-1)}$

y = 0



$y = -2\left(\frac{1}{4}\right)^{x}$

y = 0

EQUATIONS

$y = 4\left(\frac{1}{3}\right)^{-x}$

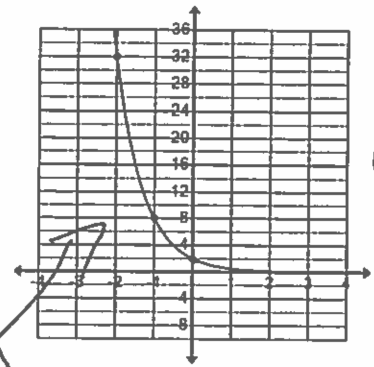
$y = 4(3)^{x}$

$y = -(3)^{-(x-1)}$

$= -\left(\frac{1}{3}\right)^{x-1}$

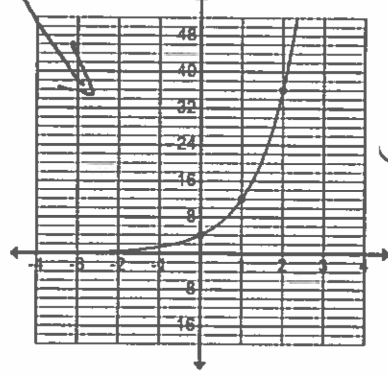
$y = 2(4)^{-x}$

$y = -2\left(\frac{1}{4}\right)^{x}$



$y = 2(4)^{-x}$

y = 0



$y = 4\left(\frac{1}{13}\right)^{-x}$

y = 0

[2] 7. Given the base function of $y = \left(\frac{1}{8}\right)^{x}$, write equation of the transformed function given it is compressed horizontally by 3, stretched vertically by factor 4, and shifted left 1 units.

$y = 4\left(\frac{1}{8}\right)^{3(x+1)}$

[4] 8. A bacteria colony had a population of 50 at 9:00 a.m. on Tuesday, April 16, 2019. The population quadruples every 15 hours and is represented by this formula: $P(t) = 50(4)^{\frac{t}{15}}$, where $P(t)$ represents the population of the bacteria colony, t hours after 9:00 a.m. on Tuesday, April 16, 2019

a. Determine the number of bacteria at 9:00 p.m. on Wednesday, April 17, 2019. [2]

$t = 36$ hours ✓ $P(36) = 50(4)^{(36/15)}$

24 + 12 = 36 hours

$P(36) = 1392.88$ ✓

$P(24) = 459.479...$
$P(12) = 151.57...$

b. At this rate, determine the date and time, the original population would triple (accurate to the nearest tenth of an hour). [2]

$50(4)^{t/15} = 150$ ✓

t	P(t)
12	151.57
11.9	150.177
11.8	148.7957

11.9 hours after 9 am Tues. Apr 16th.

11 hrs, 54 mins after 9 a.m.

8:54 p.m. Tues. Apr. 16 ✓

full marks for 11.9 hrs (+1 for)

9. The population of Hicksville, Montana had a population of 42150 in April of 2000. Since then, the population has been **declining** at a rate of 4.5% each year.

[6] a. Assuming this trend continues, write an exponential equation that models the population of Hicksville, Montana over time. Include let statements. [2]

$P(t) = 42150(0.955)^{t}$ where $P(t)$ is the population after t years.

−1 if missing lets

b. Use your exponential equation to determine the current population (in April, 2019). [2]

$t = 19$ ✓

$P(19) = 17573.6365$ ✓

c. At this rate, determine the date the original population would first fall below 21 000 (accurate to the nearest month). [2]

$42150(0.955)^{t} < 21000$ (½)

t	P(t)
15.1	21 030
15.2	20 933

0.2 × 12 = 2.4 months.

✓ for 15.2

In June, 2015, the population would first fall below 21000

if original was April! July if started at Apr 3

15.137 May or June

(+1)

MCR3UI-05 **Unit 4 Test: Exponents & Exponential Functions** NAME:

10. Complete the table below. Graph the parent AND transformed function for a) and b) on the grids provided. Use proper form. Intermediate graphs or tables may be used to show work. Be sure to show the y-intercept and any asymptotes.

[19]

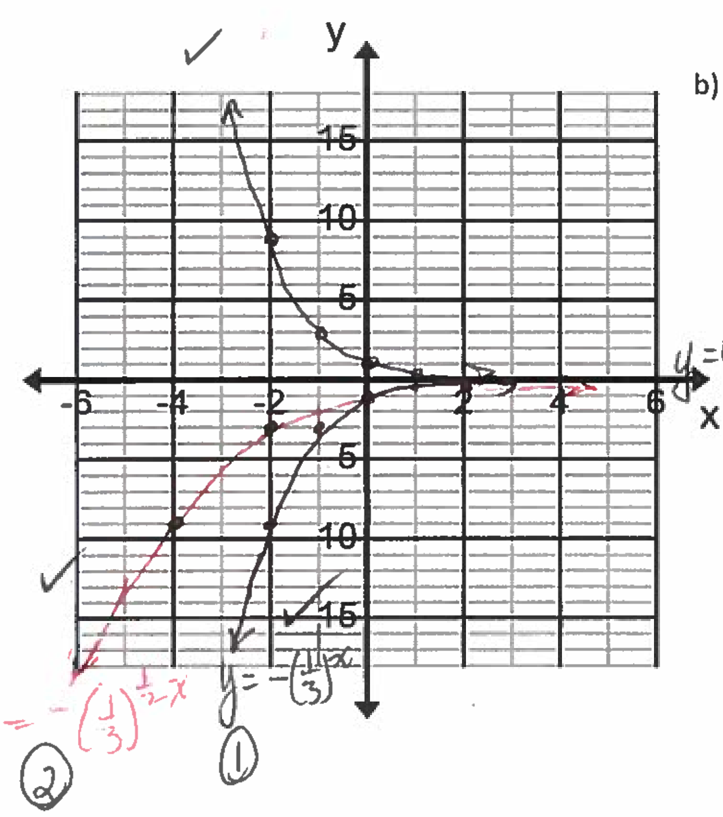
$4(2)^{-(x-2)}-8$

	a. $y=-\left(\frac{1}{3}\right)^{\frac{1}{2}x}$ $8\frac{1}{2}$	b. $y=4(2)^{-x+2}-8$ $10\frac{1}{2}$
Parent Function	$f(x)=\left(\frac{1}{3}\right)^x$	$g(x)=2^x$ / or may use $g(x)=\left(\frac{1}{2}\right)^x$
List the Transformations (in order)	① reflection in x-axis ② hor. stretch factor 2	② vert. stretch factor 4 ① reflection in y-axis ③ shift right 2 ④ shift down 8 then not this; didn't factor left 2
y-intercept	$y=-1$ (0,−1)	$y=8$ (0,8) $y=-7$
x-intercept	—	$x=1$ (1,0). $x=-3$
Equation of Asymptote	$y=0$	$y=-8$
Domain	$\{x\in R\}$	$\{x\in R\}$
Range	$\{y<0\}$	$\{y>-8\}$
Increasing or Decreasing for $x \in R$?	inc.	dec.

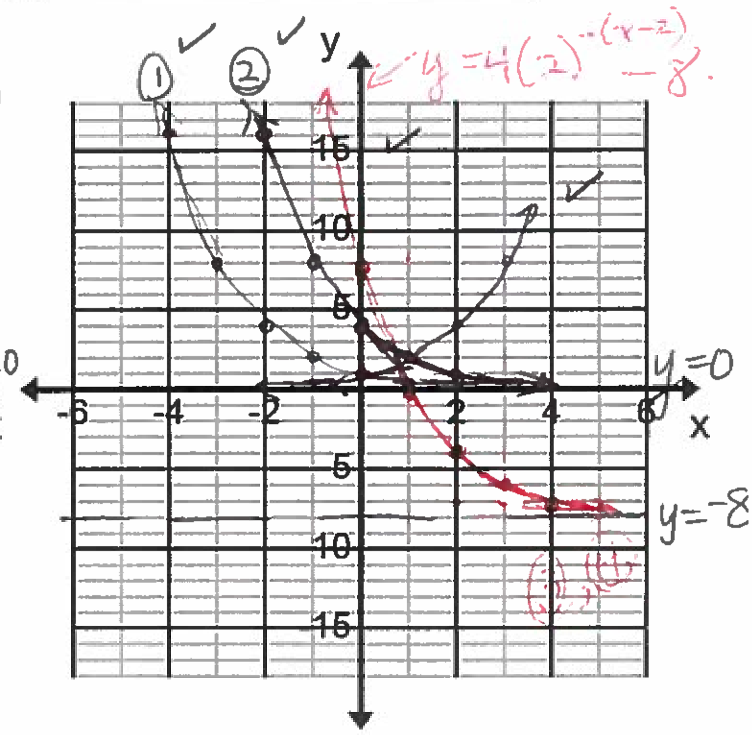
$4(2)^{-x+2}-8=0$
$2^{-x+2}=2$
$-x+2=1$
$-x=-1$
$x=1$

to rt −1 if left 2

a) [2]



b)



11. In each case, write an equation that models the situation described.

Situation	Equation
The temperature of a hot liquid is initially 100 degrees Celsius. The temperature drops 42% every 25 minutes. Let T(x) represent the temperature, in degrees Celsius, and let x represent the elapsed time, in minutes.	$T(x)=100(0.58)^{x/25}$
The value of a $10 000 investment grows at 3.2% per year. Let V(t) represent the value of the investment after t years.	$V(t)=10\,000(1.032)^t$

[+1] BONUS: If $\left(\frac{2}{x}-\frac{x}{2}\right)^2=0$, find the value of x^6.

note $x\neq0$.

$\left(\frac{4-x^2}{2x}\right)^2=0$ → $(4-x^2)^2=0$, $4-x^2=0$ → $x^2=4$, ~~$x=\pm2$~~

$(x^2)^3=x^6$ so, $x^6=(4)^3$, $x^6=64$