# MPM 1DI Strand 2 Review 

# Measurement and Geometry 

2-1 Pythagorean Theorem<br>2-2 2-D Optimization<br>2-3 Composite Figures:<br>2-4 Surface Area \& Volume:

2-5 Angle \& Triangle Theorems:

2-6 Parallel Line Theorems:

2-7 Polygon Theorems:

## 2-1 Pythagorean Theorem

## Key Concepts

Hypotenuse: the longest side of the right triangle, opposite to the $90^{\circ}$ angle.
Pythagorean Theorem: in a right angle triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides.

$$
a^{2}+b^{2}=c^{2}
$$

1. Find the missing length to the nearest tenth.
a)

c)

b)

d)

2. A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the nearest tenth of a foot, between first base and third base?
3. Two joggers run 3 miles north and 8 miles west. What is the shortest distance, to the nearest tenth of a mile that they must travel to return to their starting point?
4. TV's are measured across the diagonal. You don't have a ruler long enough to measure the size of your TV. The only measurements you have are the two sides lengths: 48 inches and 36 inches. What size is your TV?

## 2-2 2-D Optimization

## Key Concepts

Optimization: creating the largest or smallest area or perimeter given restrictions.

Maximum area: when obtaining a maximum rectangular area:
-enclose 2 or 4 sides forming a square
-enclose 3 sides forming a rectangle where the length is double the width
Minimum perimeter: when obtaining a minimum perimeter of a rectangular area, form a square.

1. Paula plans to build a rectangular patio using 180 m of fencing. What is the maximum are of the patio that she can build?
2. Cody needs to make a rectangular pen for his pigs that will enclose a total area of 256 square feet. What is the maximum length of fencing that he will need?
3. Ella wants a new vegetable garden. She needs 144 m 2 of space for her vegetable plants. What is the least amount of fencing that she will need to purchase?
4. A doggy day care is looking to create an outdoor area for the dogs boarding for the day. There is 900 m of fencing available to enclose the area. Which shape offers the largest area: a rectangular pen or a circular pen?
5. While working at a summer camp on the lake you are asked to create a swimming enclosure for the campers. You have 900 feet of rope with buoys on it. What are the dimensions of the maximum area that can be enclosed? What area will be created?

## 2-3 Composite Figures:

## Key Concepts

Composite Figures: figures that are made up of two or more two-dimensional figures: triangles, squares, rectangles, semicircles, etc.

1. Calculate the area and perimeter, to the nearest tenth, of each of the figures.
a)

b)

2. The diagram shows a running track at a high school. It consists of two parallel line segments with a semicircle at each end. The track is 10 m wide. Kayla runs on the inside of the track and Emily runs on the outer edge. How much farther does Emily run in one lap, than Kayla?

3. Bradley is planning a garage sale. To direct customers to his house, he is painting six arrow signs.
a) Calculate the area of one sign.
b) Each can of paint can covers $10 \mathrm{~cm}^{2}$. How many cans of paint should Bradley buy for all six signs? Explain your answer.


## 2-4 Surface Area \& Volume:

## Key Concepts

Surface Area: the number of square units needed to cover the surface of a 3-D object.

Lateral Faces: the faces of a prism or pyramid that are not bases.
Volume: the amount of space that an object occupies, measured in cubic units.

Prism: a 3-D object with two parallel congruent polygonal bases.
Volume of Prism = AREA of the BASE $\times$ Height

Pyramid: a 3-D object with one polygonal base and all lateral surfaces meeting at an apex.
Volume of Pyramid = AREA of the BASE $\times$ Height

1. Calculate the volume of each pyramid, to the nearest tenth.
a)

b)

c)

2. Calculate the volume of each prism, to the nearest tenth.
a)

b)


3. Calculate the surface area of each pyramid, to the nearest tenth.
a)

b)

c)

4. Calculate the surface area of each prism, to the nearest tenth.
a)

b)

c)

5. The Great Pyramids of Khufu is located in Egypt. It is 481 feet high and has a square base with a base edge of 756 feet. What is the surface area of the Great Pyramid? Round your answers to two decimal places.
6. Lindsey buys mulch for her flower garden. How many cubic feet of mulch will fit in her truck bed that is 5 feet by 8 feet by 2 feet?
7. A water tank has been purchased for the farm. It will be used to water cattle. It is a cylindrical shaped metal container that is 2.6 feet tall. The area of the bottom of the tank is 9.3 square feet. If the cattle drink two hundred four cubic feet of water a day, how many times per day will the tank have to be filled?
8. Jen found a plant holder at a garage sale, in the shape of a square frustum. The frustum has a slant height of 40 cm , a base width of 35 cm , and a top width of 17 cm . She needs to give it a fresh coat of paint. How much surface area will she need to cover? * Include the top surface but not the bottom.


## 2-5 Angle \& Triangle Theorems:

Polygon: A closed figure made up of line segments.
Regular Polygon: A polygon where all the sides and angles are equal.
Concave Polygon: A polygon with at least one angle greater than $180^{\circ}$
Convex Polygon: A polygon with all angles less then $180^{\circ}$

Kite: Quadrilateral with two pairs of adjacent sides equal
Vertex: The point where two or more sides meet.
Interior Angle: Angle formed on the inside of a polygon by two sides meeting at a vertex.
Exterior angle: Angle formed on the outside of a geometric shape by extending one of the sides past a vertex

## Polygon Exterior Angle Theorem (PEAST) : Exterior angles of

 any polygon add to 360 degrees.Angle Sum Triangle Theorem (ASTT): Sum of Interior angles of a triangle add to 180 degrees.

Angle Sum Quadrilateral Theorem (ASQT): Sum of interior angles of a quadrilateral add to 360 degrees.

Exterior Angle Theorem (EAT): Any exterior angle of a triangle is equal to the sum of the two opposite interior angles

Angle Sum Polygon Theorem (ASPT): Total sum of all interior angles for any polygon can be determined with the formula

$$
\begin{aligned}
& S=180(n-2), \text { where } n \text { represents the number of } \\
& \text { sides and } S \text { represents the total sum of } \\
& \text { interior angles }
\end{aligned}
$$

Adjacent: Adjoining or next to
Obtuse Angle: An angle greater than 90 degrees
Acute Angle: An angle less than 90 degrees
Supplementary: Adding to 180 degrees
Complementary: Adding to 90 degrees
Transversal: A line intersecting two parallel lines

## Parallel Lines

-/Alternate angles are equal (Z pattern) (AA)
-/Corresponding angles are equal (F pattern) (CA)

- Co - Interior angles add to 180 degrees (C Pattern) (CIA)

Opposite Angles: Equal (OAT)
Midpoint: A point that divides a line segment into two equal parts

Right Bisector: A line perpendicular to a line segment passing through its midpoint

Median: The line segment joining the vertex of a triangle to the midpoint of the opposite side.

Equilateral Triangle: All sides are the same length, and therefore all angles are equal ( 60 degrees).
Isosceles Triangle: Two sides are equal in length, and therefore there are two equal angles.
Scalene Triangle: All sides are different length, and therefore all angles are different.

1. Find the value of the missing angles.

b)

2. Find the measure of each indicated angle.

3. Find the value of each variable. $A$


## 2-6 Parallel Line Theorems:

## Key Concepts

Corresponding Angles: pairs of corresponding angles associated with a transversal are equal.
Alternate Angles: pairs of alternate angles associated with a transversal are equal.

Co-interior Angle: pairs of co-interior angles associated with a transversal have a sum of 180 degrees.

1. Find the measure of each of the unknown angles indicated.


## 2-7 Polygon Theorems:

1. Find the measure of one interior angle in each regular polygon, to the nearest degree
a)

b) regular 15 -sided polygon
2. Find the measure of one exterior angle in each regular polygon.
a)

b) regular 13-sided polygon
6 sides
3. Find the value of each missing angle.


## EQAO Sample Questions:

1. A garden is in the shape of a rectangle and a semicircle as shown below.

Which of the following is closest to the amount of fencing needed to enclose the garden?
a 60 cm
b 70 cm
c 75 cm

d 85 cm
See 2-2
2. Ella wants a rectangle with:
-a perimeter of 100 cm and -the largest possible area What are the dimensions of the rectangle that satisfies her conditions?
a $10 \mathrm{~cm} \times 10 \mathrm{~cm}$
b $20 \mathrm{~cm} \times 30 \mathrm{~cm}$
c $25 \mathrm{~cm} \times 25 \mathrm{~cm}$
d $40 \mathrm{~cm} \times 60 \mathrm{~cm}$
See 2-2
4. Consider the parallelogram shown below. What is the perimeter of WXYZ?
a 28 cm
b 30 cm
c 31 cm
d 34 cm


See 2-1
5. Consider the diagram below.

See 2-4
3. Chris has a square garden with an area of $38.4 \mathrm{~m}^{2}$, as shown in the diagram


He decreases the length of each side by 1.7 m to make a smaller garden. Which is the closet to the perimeter of the smaller garden?
a 37 m
b 32 m
c 25 m
d 18 m
See Formula Sheet

a $55^{\circ}$
b $70^{\circ}$
c $125^{\circ}$
d $130^{\circ}$

|  |  |
| :--- | :--- |
| a |  |
| b | $75^{\circ}$ |
| c $125^{\circ}$ |  |
| d $130^{\circ}$ |  |

6. The playing chips of a board game are stored in cylindrical plastic cases. The plastic cases have a volume of 25 $120 \mathrm{~mm}^{3}$ and a diameter of 40 mm . Which of the following is closest to the beight of one playing chip if 50 playing chips can fittightly into the plastic case as shown above?
a 0.1 mm
b 0.4 mm
c 1.3 mm
d 2.5 mm


See 2-3
7. Consider the following diagram.


What is the value of. $x$ ?
a $80^{\circ}$
b $120^{\circ}$
c $140^{\circ}$
d $170^{\circ}$

## See 2-4 \& 2-5

9. Toy Sailboats

Emelina makes toy sailboats as shown below. Determine the total area of the shaded sails. Show your work
8. What is the sum of the interior angles of a 12 -sides regular polygon?
a $1080^{\circ}$
b $1800^{\circ}$
c $1980^{\circ}$
d $2160^{\circ}$
See 2-6

See 2-1

## 10. What's Missing?

Consider the diagram below.
Complete the table below. Justify your answers using geometric properties.

| Angle measure |  |
| :--- | :--- |
|  |  |
| $x=\ldots$ |  |
| $y=1$ |  |



## Answers

## 2-1 Pythagorean Theorem

1. a) 7.6 cm
b) 12.2 cm
c) 9.3 mm
d) 5.9 m
2. 127.3 ft
3. 8.5 miles
4. 60 inches

## 2-2 Optimization

1. 45 m by 45 m
2.64 feet
3.48 m
2. Circle
3. 225 ft by $450 \mathrm{ft} ; 101,250 \mathrm{ft}^{2}$

## 2-3 Composite Figures

1. a) Area: $66.5 \mathrm{~m}^{2}$ Perimeter: 41.7 m
b) Area: $49.5 \mathrm{~cm}^{2}$; Perimeter: 33.8 cm
2. Emily runs 62.8 m farther
3. a) $39.5 \mathrm{~cm}^{2}$
b) 24 cans of paint

## 2-4 Surface Area \& Volume

1. a) $\frac{88.0}{3} \mathrm{mi}^{3}=29.3 \mathrm{mi}^{3}$
2. a) $314.2 \mathrm{in}^{3}$
b) $484.0 \mathrm{~cm}^{3}$
b) 210.0
c) $8 \mathrm{~m}^{3}$
$\mathrm{cm}^{3}$
c) $18 \mathrm{ft}^{3}$
3. a) $56.5 \mathrm{~cm}^{2}$
b) $1082.4 \mathrm{~cm}^{2}$ (height of each pyramid
c) $2944 \mathrm{in}^{2}$
4. a) $659.7 \mathrm{~km}^{3}$
b) $144 \mathrm{~km}^{2}$
c) $238.0 \mathrm{yd}^{2}$
5. $934,515 \mathrm{ft}^{2}$
6. $80 \mathrm{ft}^{3}$
7. 9 times
$8.4449 \mathrm{~cm}^{2}$

## 2-5 Angles and Triangle Theorems

1. a) $\mathrm{a}=55^{\circ}, \mathrm{b}=85^{\circ}, \mathrm{c}=95$
2. a) $x=36 ; \angle \mathrm{NOM}=108^{\circ}, \angle \mathrm{ONM}=36^{\circ}$
b) $a=57^{\circ}, b=123^{\circ}, c=32$
b) $x=22 ; \angle \mathrm{ABC}=110^{\circ}, \angle \mathrm{BAD}=64^{\circ}, \angle \mathrm{ADB}=46^{\circ}$
3. $\angle \mathrm{AFC}=121^{\circ}, \angle \mathrm{CBE}=131^{\circ}, \angle \mathrm{EDA}=108^{\circ}, \angle \mathrm{CFD}=59^{\circ}, \angle \mathrm{ADB}=72^{\circ}, \angle \mathrm{EBG}=49^{\circ}$

## 2-6 Parallel Line Theorems

1. a) $\angle \mathrm{GEF}=53^{\circ}$
d) $\mathrm{x}=10 ; \angle \mathrm{AHF}=60^{\circ}, \angle \mathrm{GEF}=60^{\circ}$
b) $\mathrm{x}=8^{\circ} ; \angle \mathrm{PTV}=\mathrm{B}^{\circ}$
e) $x=-9 ; \angle A H F=100^{\circ}, \angle G E B=80^{\circ}$
c) $\mathrm{x}=9 ;<\mathrm{STV}=125^{\circ}, \angle \mathrm{WXY}=125^{\circ}$
f) $\mathrm{x}=18$; $\angle \mathrm{NOP}=120^{\circ}, \angle \mathrm{POS}=60^{\circ}$, $<$ QRS $=60^{\circ}$

## 2-7 Polygon Theorems

1. a) $144^{\circ}$
b) $156^{\circ}$
2. a) $60^{\circ}$
b) $27.7^{\circ}$
3. $\angle \mathrm{KLJ}=60^{\circ}{ }^{\circ} \angle \mathrm{MLO}=60^{\circ}, \angle \mathrm{LMN}=90^{\circ}$
4. $\angle \mathrm{GFE}=142^{\circ}, \angle \mathrm{IHG}=142^{\circ}$

## EQAO Multiple Choice

1. A
2. C
3. D
4. B
5. B
6. B
7. C
8. $B$
9. $41 \mathrm{~cm}^{2}$
10. $\mathrm{x}=60^{\circ}, \mathrm{y}=133^{\circ}$
