U9D7_T_Optimization of a Square Based Prism

Wednesday, May 30, 2018 12:47 PM

U9D7_T_Optimizatio...

MPM1DI U9D7 (9.3/9.4)

Optimization of a Square Based Prism

Investigation A: How can you compare the surface areas of square-based prisms with the same volume?

1. Use 16 interlocking cubes to build as many different square-based prisms as possible with a volume of 16 cubic units.
2. Calculate the surface area of each prism. Record your results in a table.

Length	Width	Height	Volume	Surface Area
1	1	16	16 u^3	66 u^2
2	2	4	16 u^3	40 u^2
4	4	1	16 u^3	48 u^2

$16 \times 4 + 1 \times 2$

← $8 \times 4 + 4 \times 2$

$16 \times 2 + 4 \times 4$

3. What are the dimensions of the square-based prism that has the minimum, or optimal, surface area?

2 units x 2 units x 4 units

4. Describe the shape of this prism compared to the other prisms.

Closest to a cube.

5. Predict the dimensions of the square-based prism with minimum surface area if you use:

a) 27 cubes

$3 \times 3 \times 3$

b) 64 cubes

$4 \times 4 \times 4$

c) 125 cubes

$5 \times 5 \times 5$

$\boxed{\sqrt[3]{\ }}$ $\sqrt[x]{y}$

6. **REFLECT:** Summarize your findings.

a) Do any relationships exist between the length, width, and height of a square-based prism with minimum surface area for a given volume?

length = width = height

b) What is the ideal shape for minimizing the surface area of a square-based prism when given a fixed volume? A cube.

c) How can you predict the dimensions of a square-based prism with minimum surface area if you know the volume?

Take the cubed root. $V = x^3$ So, $x = \sqrt[3]{V}$

x is the cubed root of the volume.

EX. 1. Cardboard Box Dimensions.

a) The Pop-a-Lot popcorn company ships kernels of popcorn to movie theatres in large cardboard boxes with a volume of 500,000 cm^3. Determine the dimensions of the square-based prism box, to the nearest tenth of a centimeter, the will require the least amount of cardboard.

$x = \sqrt[3]{500000}$

So, $x = 79.37\ldots$

$x \doteq 79.4$

$\therefore$ a cube with all dimensions 79.4 cm minimizes the surface area.

b) Find the amount of cardboard required to make this box, to the nearest tenth of a square metre. Describe any assumptions you have made.

A_{total} = 6 square sides

$= 6x^2$

$= 6(79.4)^2$

$= 37826.16\ldots$

$\doteq 37826.2$

6 sides all sides are x by x

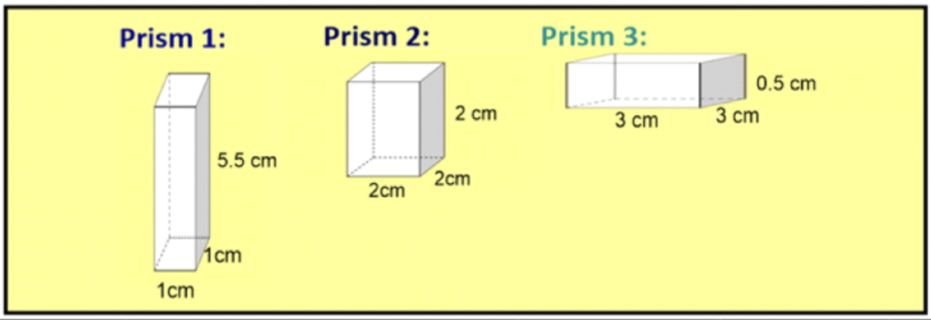
$\therefore$ the minimum amount of cardboard is $37826.2 cm^2$.

Assumption: no extra material needed for seams.

Pg 495 #2, 3, 5a, 7 & Pg 501 #2, 3, 6, 7

Investigation B: How can you compare the volumes of square-based prisms with the same surface area?

1. Each of the square-based prisms below has a surface area of 24 cm^2. Calculate the area of the base and the volume of each prism. Record your data in the table.



Prism Number	Side length of base (cm) b	Area of base (cm^2) b^2	Surface area (cm^2) $4bh + 2b^2$	Height (cm) h	Volume (cm^3) b^2h
1	1	1	24	5.5	5.5
2	2	4	24	2	8 ←
3	3	9	24	0.5	4.5

2. What are the dimensions of the square-based prism that has the maximum, or optimal, volume?

2 cm x 2 cm x 2cm

3. Describe the shape of this prism compared to the other prisms.

A cube

Pg 495 #2, 3, 5a, 7 & Pg 501 #2, 3, 6, 7

4. Predict the dimensions of the square-based prism with maximum volume if the surface area is 54 cm^2.

$A = 6x^2$

$6x^2 = 54$

$x^2 = 9$

$x = \sqrt{9}$

$\boxed{x = 3}$ OR $\cancel{-3}$

$2 \div 6$

$2 \sqrt{}$

∴ the optimal prism is 2cm × 2cm × 2cm

5. **REFLECT:** Summarize your findings.

a) Do any relationships exist between the length, width, and height of a square-based prism with maximum volume for a given surface area?

length = width = height

b) What is the ideal shape for maximizing the volume of a square-based prism when given a fixed surface area?

A CUBE

c) How can you predict the dimensions of a square-based prism with maximum volume if you know the surface area?

* $\boxed{6x^2 = A_{TOTAL}, \text{ solve for } x}$ *

$x^2 = \dfrac{A_{TOTAL}}{6}$

$x = \sqrt{\dfrac{A_{TOTAL}}{6}}$ OR $\cancel{x = -\sqrt{\dfrac{A_{TOTAL}}{6}}}$

Pg 495 #2, 3, 5a, 7 & Pg 501 #2, 3, 6, 7

EX. 2. Maximize the Volume of a Square-Based Prism

a) Determine the dimensions of the square-based prism with maximum volume that can be formed using 5400 cm^2 of cardboard.

$6x^2 = 5400$

$\div 6$

$x^2 = 900$

$\sqrt{\ }$

$x = 30$ OR $\cancel{-30}$

$\therefore$ the optimal box is 30 cm x 30 cm x 30 cm.

b) What is the volume of the prism?

$V = x^3$

$V = 30^3$

$V = 27000$

$\therefore$ the volume is 27000 cm^3.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$72 = 2^3 3^2$ Prime Factorization

Pg 495 #2, 3, 5a, 7 & Pg 501 #2, 3, 6, 7

$6 \times 6 \times 2$ $\quad$ $2 \times 2 \times 18$