U8D3_T_Present Value Annuities

Monday, June 3, 2019 9:23 AM

U8D3_T_Present Val...

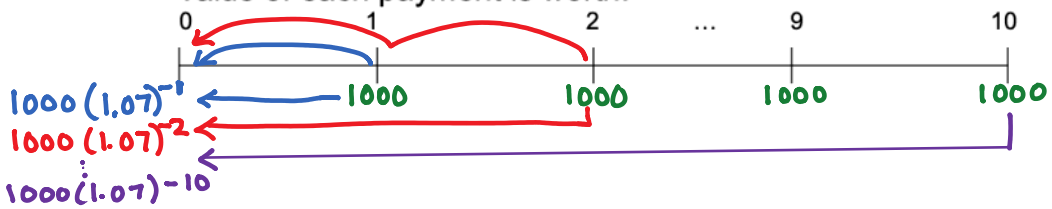
U8D3 MCR 3UI **Present Value of an Annuity**

Scenario: Suppose you wanted to go on a trip once a year for 10 years. For each trip you will need $1000. You open an account that pays 7%/a compounded annually. You deposit enough money today so that you can withdraw $1000 at the end of each year for the next 10 years. How much money do you need to deposit today?

Present Value of an Annuity: an amount invested today that will create a series of equal payments in the future.

Example 1: Determine the amount needed for the 10 trips above.

Solution: Use the timeline to visualize what the present value of each payment is worth.



Total Amount you need to deposit now $= 1000(1.07)^{-1} + 1000(1.07)^{-2} + \ldots + 1000(1.07)^{-10}$

reverse the sum: $1000(1.07)^{-10} + \ldots + 1000(1.07)^{-2} + 1000(1.07)^{-1}$

This looks like a Geometric Series with a first term of $= 1000(1.07)^{-10}$ and a common ratio = 1.07 and n = 10.

Therefore use the formula: $S_n = \dfrac{1000(1.07)^{-10}[1.07^{10} - 1]}{(1.07 - 1)}$

Or we can use the formula: $= 7023.58$

Present Value of an Annuity Formula

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$

R = payment withdrawn at each interval
i = interest rate per compounding period
n = total number of payments/deposits

$(1+i)^{-n}[(1+i)^{n} - 1]$
$= [1 - (1+i)^{-n}]$

Example 2: You just signed a 2-year lease for a new apartment where rent is $600/month, with the first payment due in a month. How much do you need in an account today at 6%/a compounded monthly to cover all the payments?

$P = ?$

$R = 600$

$i = \frac{0.06}{12}$

$i = 0.005$

$n = 2 \times 12$

$n = 24$

$$P = \frac{600\left[1-(1.005)^{-24}\right]}{0.005}$$

$P = 13537.72$

∴ she needs $13537.72 in the account today.

Example 3: Sue needs to borrow $7500 to purchase a used car. The car dealer arranges with a finance company to lend Sue the money at 2.9%/a compounded monthly for 3 years. What will Sue's monthly payment be?

$P = 7500$

$R = ?$

$i = \frac{0.029}{12}$

$n = 3 \times 12$

$= 36$

$$P = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$R = \frac{Pi}{\left[1-(1+i)^{-n}\right]}$$

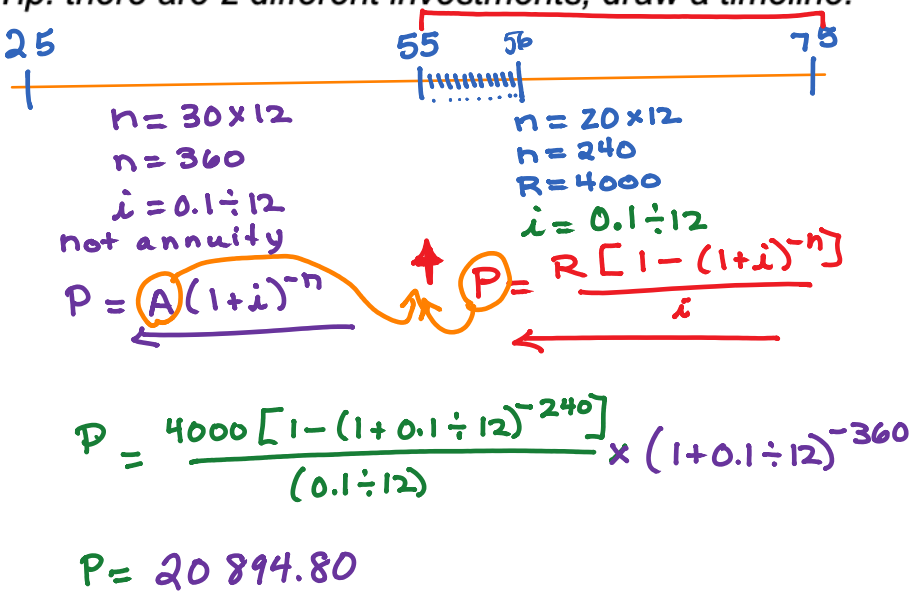
$$R = \frac{7500 \times 0.029 \div 12}{\left[1-(1+0.029 \div 12)^{-36}\right]}$$

$R = 217.78$

∴ her payment will be $217.78 per month.

Example 4: Jane is 25 years old now. She wants to be able to withdraw $4000 on a monthly basis for 20 years when she retires at age 55. She found an account that pays 10%/a compounded monthly. How much should she put into the account today?

Tip: there are 2 different investments, draw a timeline.



$$P = \frac{4000\left[1-(1+0.1\div 12)^{-240}\right]}{(0.1\div 12)} \times (1+0.1\div 12)^{-360}$$

$$P = 20\,894.80$$

U8D3 Practice: p. 541 #4, 5, 8, 10, 12, **17** **Note: If you choose to do Number 9, reword so it is compounded monthly with monthly withdrawals beginning in one month to get answer in back of text.**