U8D2_T_Ordinary Annuities

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U8D2_T_Ordinary An...

U8D3 MCR 3UI **<u>Ordinary Annuities</u>**

Scenario: Frank deposits $1000 into a savings account (5%/a compounded monthly) every year for 10 years. How much will his investment be worth at the end of 10 years?

What makes the above scenario difficult to calculate? multiple 'deposits'

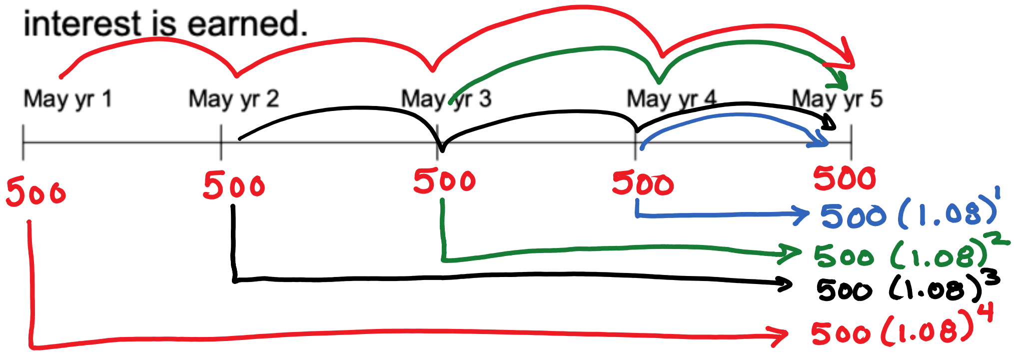
Annuity: a series of equal payments/deposits made at regular intervals of time. Each payment/deposit is made at the end of each payment period.

Example 1: Nigel deposits $500 on May 1st, every year for 5 years. The investment earns 8%/a compounded annually. How much will be in the account after he makes his final deposit?

$i = \frac{0.08}{1}$

$i = 0.08$

Solution: Use the timeline to visualize how/when the interest is earned.



Total Amount = $500 + 500(1.08) + 500(1.08)^2 + 500(1.08)^3 + 500(1.08)^4$

~~reverse the sum:~~

Wow!! This looks like a

Geometric Series with a = 500 (t_1 (first term))

and r = 1.08 and n = 5.

Therefore use the formula: $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_5 = \frac{500(1.08^5 - 1)}{1.08 - 1}$

$S_5 = 2933.30$

∴ Nigel's investment is worth $2933.30.

Ordinary Annuity Formula

$$A = \frac{R[(1+i)^n - 1]}{i}$$

R = payment made at each interval "regular payment"
i = interest rate per compounding period
n = total number of payments/deposits

Example 2: Jane deposits $100 on March 31, June 30, September 30 and December 31 every year for 20 years. The investment pays 4%/a compounded quarterly. How much is in the account when the last payment is made?

$A = ?$

$R = 100$

$i = \frac{0.04}{4}$

$i = 0.01$

$n = 20 \times 4$

$n = 80$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$A = \frac{100[1.01^{80} - 1]}{0.01}$$

$$A = 12167.15$$

∴ the investment is worth $12167.15

Example 3: You want to retire with $1000000. What equal monthly payment will achieve this goal? (Assume 35 years of regular monthly deposits). The account pays 10%/a compounded monthly.

$A = 1\,000\,000$

$R = ?$

$i = \frac{0.10}{12}$

$n = 35 \times 12$
$= 420$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$R = \frac{Ai}{[(1+i)^n - 1]}$$

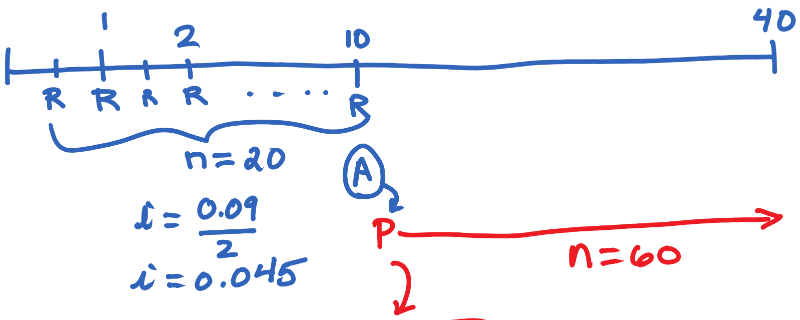
$$R = \frac{1000\,000 \times 0.10 \div 12}{[(1+0.1 \div 12)^{420} - 1]}$$

$$R = 263.39$$

** How does this change if you make regular deposits for 40 years?

<u>Example 4:</u> Suppose you deposit $1000 into an investment account every 6 months for 10 years, then leave the amount on deposit for another 30 years. The money earns an average return of 9% compounded semi-annually. How much will be in the account after 40 years?

Tip: there are 2 different investments



$i = \frac{0.09}{2}$

$i = 0.045$

$$A = \frac{1000\left[(1.045)^{20} - 1\right]}{0.045} \times (1.045)^{60}$$

$A = 440\ 059.74$

∴ there will be $440 059.74 in the account after 40 years.

U8D3 Practice: p. 532 #4, 5, 6, 8, 10, 12, **14**