U7D7_T Short Binomial Exp _ Pascal

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U7D7_T
Short Bin...

U7D7 MCR 3UI

Pascal's Triangle

Preamble A **binomial** is an algebraic expression containing two terms.

Ex. $3x+1$, $1-x^2$, $3x-7y$

Today we will learn how to expand binomials raised to any power without the use of tedious and lengthy calculations.

Ex. $(3x-2y)^3 = (3x-2y)(3x-2y)(3x-2y)$

$= (9x^2-12xy+4y^2)(3x-2y)$

$= 27x^3 - 18x^2y - 36x^2y + 24xy^2 + 12xy^2 - 8y^3$

$= 27x^3 - 54x^2y + 36xy^2 - 8y^3$

Part A The following powers of the binomial $(x+y)$ have been expanded and simplified.

Keep in mind that to expand $(x+y)^n$,
you must multiply $(x+y)$ by $(x+y)^{n-1}$.

In other words, the previous answer must be used to proceed.

$(x+y)^0 =$ 1

$(x+y)^1 =$ $x + y$

$(x+y)^2 = (x+y)(x+y) =$ $x^2 + 2xy + y^2$

$(x+y)^3 = (x^2 + 2xy + y^2)(x+y) =$ $x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = (x^3 + 3x^2y + 3xy^2 + y^3)(x+y) = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)^5 =$ $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Part B These patterns are found in the simplified expansions.

$$1$$
$$x + y$$
$$x^2 + 2xy + y^2$$
$$x^3 + 3x^2y + 3xy^2 + y^3$$
$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$
$$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

If n is the exponent on $(x+y)$,

1. Pattern of **x −exponents**: $n, n-1, n-2, \ldots, 1, 0$
2. Pattern of **y −exponents**: $0, 1, 2, \ldots, n-1, n-2$
3. The **sum** of the exponents in each term is n.
4. The **first** and **last** terms in each expansion have a coefficient 1.
5. The second and second-last terms in each expansion have a coefficient $\boldsymbol{n}$.
6. There are $\boldsymbol{n+1}$ terms in the expansion.

<u>Part C</u> Summary of the expansion coefficients.

n	Coefficients	Row Sum
0	1	1 $=2^0$
1	1 1	2 $=2^1$
2	1 2 1	4 $=2^2$
3	1 3 3 1	8 $=2^3$
4	1 4 6 4 1	16
5	1 5 10 10 5 1	32
6	1 6 15 20 15 6 1	64
7	1 7 21 35 35 21 7 1	128
8	1 8 28 56 70 56 28 8 1	256
n		2^n

This is Pascal's Triangle.

$\binom{8}{0}$ $\binom{8}{1}$ $\binom{8}{2}$ $\binom{8}{3}$ . . . $\binom{8}{8}$

${}_nC_r$ ${}_8C_0 = 1$ ${}_8C_1 = 8$ ${}_8C_2 = 28$ ${}_8C_8 = 1$

Part D These are characteristics of the numbers in Pascal's triangle.

1. The sum of row n is 2^n

2. Each row begins and ends with a 1, each row is symmetric

3. Each term is the **sum** of the two terms directly above.

4. Row n has $\boldsymbol{n+1}$ terms.

Part E Using Pascal's triangle and the patterns you have discovered today, expand :

$(x+y)^6$ 1 6 15 20 15 6 1

$= 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$

$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

Part F How is the expansion of $(x-y)^2$ different from $(x+y)^2$? $[x+(-y)]^2$

1 2 1

$= 1x^2(-y)^0 + 2x^1(-y)^1 + 1x^0(-y)^2$

$= x^2 - 2xy + y^2$

Part G Expand the following.

1 3 3

$(x-y)^3 = x^3 + 3x^2(-y) + 3x(-y)^2 + (-y)^3$

$= x^3 - 3x^2y + 3xy^2 - y^3$

1 5 10 10 5 1

$(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

1 5 10 10 5 1

5×16×3 10×8×9 10×4×27

$$(2x+3)^5 = (2x)^5 + 5(2x)^4(3) + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)(3)^4 + 3^5$$

10×81

$$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$$

1 6 15 20 15 6 1

$$(x^2-1)^6 = (x^2)^6 - 6(x^2)^5 + 15(x^2)^4 - 20(x^2)^3 + 15(x^2)^2 - 6(x^2) + 1$$

$$= x^{12} - 6x^{10} + 15x^8 - 20x^6 + 15x^4 - 6x^2 + 1$$

1 7 21 35 35 21 7 1

$$(2x^2-y)^7 = (2x^2)^7 - 7(2x^2)^6 y + 21(2x^2)^5 y^2 - 35(2x^2)^4 y^3 + 35(2x^2)^3 y^4 - 21(2x^2)^2 y^5 + 7(2x^2) y^6 - y^7$$

$$= 128x^{14} - 448x^{12}y + 672x^{10}y^2 - 560x^8y^3 + 280x^6y^4 - 84x^4y^5 + 14x^2y^6 - y^7$$

In the expansion of $(3x^2-y)^{10}$, what is the coefficient of the $x^{16}y^2$ term?

Given 1 8 28 56 70 ... is one line of Pascal's Δ.

U7D7 Practice: Worksheet – Pascal's Triangle

Review for Unit Test: p. 480 – 485 (Pick N Choose)

1 9 36

1 10 45

$45(3x^2)^8(-1)^2$

the coefficient is $45(3)^8(-1)^2 = 295245$

$_{10}C_2 = 45$