## Pascal's Triangle

## Preamble A

$\qquad$ is an algebraic expression containing two terms.
Ex. $\quad 3 x+1$,
$1-x^{2}$,
$3 x-7 y$

Today we will learn how to expand binomials raised to any power without the use of tedious and lengthy calculations.
Ex. $(3 x-2 y)^{3}$
Part A The following powers of the binomial $(x+y)$ have been expanded and simplified.
Keep in mind that to expand $(x+y)^{n}$, you must multiply $(x+y)$ by $(x+y)^{n-1}$.
In other words, the previous answer must be used to proceed.
$(x+y)^{0}=$
1
$(x+y)^{1}=$
$x+y$
$(x+y)^{2}=(x+y)(x+y)=\quad x^{2}+2 x y+y^{2}$
$(x+y)^{3}=\left(x^{2}+2 x y+y^{2}\right)(x+y)=\quad x^{3}+3 x^{2} y+3 x y^{2}+y^{3}$
$(x+y)^{4}=\left(x^{3}+3 x^{2} y+3 x y^{2}+y^{3}\right)(x+y)=x^{4}+4 x^{3} y+6 x^{2} y^{2}+4 x y^{3}+y^{4}$
$(x+y)^{5}=\quad x^{5}+5 x^{4} y+10 x^{3} y^{2}+10 x^{2} y^{3}+5 x y^{4}+y^{5}$
Part B These patterns are found in the simplified expansions.
If $\boldsymbol{n}$ is the exponent on $(x+y)$,

1. Pattern of $: n, n-1, n-2, \ldots, 1,0$
2. Pattern of $: 0,1,2, \ldots, n-1, n-2$
3. The of the exponents in each term is
4. The and terms in each expansion have a coefficient 1.
5. The second and second-last terms in each expansion have a coefficient
6. There are terms in the expansion.

Part C Summary of the expansion coefficients.

| $\boldsymbol{n}$ | Coefficients | Row Sum |
| :---: | :---: | :---: |
| 0 | 1 | 1 |
| 1 | 11 | 2 |
| 2 | 121 | 4 |
| 3 | 1331 | 8 |
| 4 | $\begin{array}{lllll}1 & 4 & 6 & 1\end{array}$ |  |
| 5 | $\begin{array}{llllll}1 & 5 & 10 & 10 & 5\end{array}$ |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |

This is Pascal's Triangle.

Part D These are characteristics of the numbers in Pascal's triangle.

1. The sum of row $n$ is
2. Each row begins and ends with a , each row is
3. Each term is the of the two terms directly above.
4. Row $n$ has
terms.
Part E Using Pascal's triangle and the patterns you have discovered today, expand :

$$
(x+y)^{6}
$$

Part F How is the expansion of $(x-y)^{2}$ different from $(x+y)^{2}$ ?

Part G Expand the following.

$$
\begin{aligned}
& (x-y)^{3}= \\
& (x-y)^{5}= \\
& (2 x+3)^{5}= \\
& \left(x^{2}-1\right)^{6}= \\
& \left(2 x^{2}-y\right)^{7}=
\end{aligned}
$$

U7D7 Practice: Worksheet - Pascal's Triangle
Review for Unit Test: p. 480-485 (Pick N Choose)

