## Pascal's Triangle

**<u>Preamble</u>** A \_\_\_\_\_\_ is an algebraic expression containing two terms.

Ex. 3x + 1,  $1 - x^2$ , 3x - 7y

Today we will learn how to expand binomials raised to any power without the use of tedious and lengthy calculations.

Ex.  $(3x - 2y)^3$ 

**<u>Part A</u>** The following powers of the binomial (x + y) have been expanded and simplified. Keep in mind that to expand  $(x + y)^n$ , you must multiply (x + y) by  $(x + y)^{n-1}$ . In other words, the previous answer must be used to proceed.

$$(x + y)^{0} = 1$$

$$(x + y)^{1} = x + y$$

$$(x + y)^{2} = (x + y)(x + y) = x^{2} + 2xy + y^{2}$$

$$(x + y)^{3} = (x^{2} + 2xy + y^{2})(x + y) = x^{3} + 3x^{2}y + 3xy^{2} + y^{3}$$

$$(x + y)^{4} = (x^{3} + 3x^{2}y + 3xy^{2} + y^{3})(x + y) = x^{4} + 4x^{3}y + 6x^{2}y^{2} + 4xy^{3} + y^{4}$$

$$(x + y)^{5} = x^{5} + 5x^{4}y + 10x^{3}y^{2} + 10x^{2}y^{3} + 5xy^{4} + y^{5}$$

**<u>Part B</u>** These patterns are found in the simplified expansions.

## If *n* is the exponent on (x + y),

- 1. Pattern of : n, n 1, n 2, ..., 1, 0
- 2. Pattern of : 0, 1, 2, ..., n 1, n 2
- 3. The of the exponents in each term is
- 4. The and terms in each expansion have a coefficient 1.
- 5. The second and second-last terms in each expansion have a coefficient
- 6. <u>There are terms in the expansion</u>.

## **<u>Part C</u>** Summary of the expansion coefficients.

n	Coefficients	Row Sum
0	1	1
1	1 1	2
2	1 2 1	4
3	1 3 3 1	8
4	1 4 6 4 1	
5	1 5 10 10 5 1	
6		
7		
8		

This is Pascal's Triangle.

**<u>Part D</u>** These are characteristics of the numbers in Pascal's triangle.

- 1. The sum of row *n* is
- 2. Each row begins and ends with a , each row is
- 3. Each term is the of the two terms directly above.
- 4. <u>Row *n* has terms</u>.
- **<u>Part E</u>** Using Pascal's triangle and the patterns you have discovered today, expand :  $(x + y)^6$
- **<u>Part F</u>** How is the expansion of  $(x y)^2$  different from  $(x + y)^2$ ?
- **<u>Part G</u>** Expand the following.

 $(x - y)^3 =$ 

 $(x - y)^5 =$ 

 $(2x+3)^5 =$ 

 $(x^2 - 1)^6 =$ 

$$(2x^2 - y)^7 =$$

U7D7 Practice: Worksheet – Pascal's Triangle Review for Unit Test: p. 480 – 485 (Pick N Choose)