U7D3_T Geometric Sequences

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U7D3_T Geometri...

U7D3MCR3UI **Geometric Sequences**

What is similar about the following sequences?

1. 2, 6, 18, 54 . . . .

   ×3 ×3 ×3

2. 2, 10, 50, 250 . . . .

   ×5 ×5 ×5

3. 5, -10, 20, -40, 80 . . . .

   ×-2 ×-2 ×-2 ×-2

* all are exponential functions
* they all have a common ratio

All of these sequences are classified as **geometric** sequences since each term is generated

by multiplying the previous term by the same amount called the **COMMON RATIO**.

A geometric sequence looks like :

$$a, ar, ar^2, ar^3, \ldots \quad \text{or}$$

In general, $\boxed{t_n = ar^{n-1}}$

$t_n =$ general term *or* nth term

$a =$ first term

$n =$ term number *or* number of terms

$r =$ common ratio *or multiplying factor*

Examples:

1. Determine t_n and t_{10} for the following geometric sequences:

a) 5, 20, 80, 320 . . . .

$\times 4 \quad \times 4 \quad \times 4$

$a = 5 \quad r = 4$

$$\boxed{t_n = 5(4)^{n-1}}$$

$t_{10} = 5(4)^{10-1}$

$t_{10} = 5(4)^9$

$$\boxed{t_{10} = 1\,310\,720}$$

b) $2, -\frac{3}{2}, \frac{9}{8}, -\frac{27}{32}$

$r = \frac{-27}{32} \div \frac{9}{8}$

$r = \frac{-27}{32} \times \frac{8}{9}$

$r = \frac{-3}{4}$

$r = \frac{9}{8} \div \frac{-3}{2}$

$r = \frac{9}{8} \times \frac{-2}{3}$

$r = \frac{-3}{4}$

$r = \frac{-3}{2} \div \frac{2}{1}$

$r = \frac{-3}{2} \times \frac{1}{2}$

$r = \frac{-3}{4}$

$\therefore$ common ratio $-\frac{3}{4}$

$a = 2 \quad r = \frac{-3}{4}$

$$\boxed{t_n = 2\left(\frac{-3}{4}\right)^{n-1}}$$

$t_{10} = \frac{2(-3)^9}{4^9}$

$$\boxed{t_{10} = \frac{-19683}{131072}}$$

2. Determine the number of terms in the sequence 3, 6, 12, 24 . . . . 96.

$a = 3$ $r = 2$ $n = ?$ $t_n = 96$

Method: * common base

$3(2)^{n-1} = 96$ isolate power on L.S.

$2^{n-1} = 96 \div 3$

$2^{n-1} = 32$ write 32 as a power of 2

$2^{n-1} = 2^5$ set exponents equal

$n - 1 = 5$

$\boxed{n = 6}$

trial & error method.

n	$3(2)^{n-1}$

want 96

∴ there are six terms in this finite geometric sequence.

3. Determine t_{10} if for each of the following geometric sequences: $t_n = ar^{n-1}$

a) if $t_3 = 15$ and $t_6 = -405$

$ar^2 = 15$ $ar^5 = -405$

Substitution Method

① $ar^2 = 15$ ② $ar^5 = -405$

$a = \frac{15}{r^2}$

$\frac{15}{r^2}(r^5) = -405$

$\vdots$

$r^3 = -27$

do NOT need ± in front of an ODD root.

Elimination Method

* BETTER *

$\frac{ar^5}{ar^2} = \frac{-405}{15}$

$r^{5-2} = -27$

$r^3 = -27$

$r = \sqrt[3]{-27}$

$r = -3$

$ar^2 = 15$

$a(-3)^2 = 15$

$a = \frac{15}{9}$

$a = \frac{5}{3}$

$t_{10} = ar^{10-1}$

$t_{10} = \frac{5}{3}(-3)^9$

$\boxed{t_{10} = -32805}$

3. continued... Determine t_{10} if for each of the following geometric sequences:

$ar^{n-1} = t_n$

b) if $t_3 = 60$ and $t_7 = 960$.

$\frac{ar^6}{ar^2} = \frac{960}{60}$

$r^4 = 16$

$r = \pm 2$

$(-2)^4 = 16$

$(2)^4 = 16$

$ar^2 = 60$

$a(2)^2 = 60$

$a = 15$

*need $\pm$ since we are taking the 4th root

an EVEN root.

$t_n = 15(2)^{n-1}$

$t_{10} = 15(2)^9$

$\boxed{t_{10} = 7680}$

OR

$\boxed{t_{10} = -7680}$

4. Express the geometric sequences defined by the general term $t_n = 3\left(\frac{2}{5}\right)^{n-1}$, as a recursive sequence.

$$\boxed{t_k = (t_{k-1})\left(\frac{2}{5}\right),\ t_1 = 3.\quad k \in \mathbb{N},\ k > 1}$$

OR $t_k = \frac{2}{5} t_{k-1},\ t_1 = 3$

OR $t_k = t_{k-1} \times \left(\frac{2}{5}\right),\ t_1 = 3$

OR $t_{k+1} = t_k\left(\frac{2}{5}\right),\ t_1 = 3$

U7D3 Practice: p. 452 #1-7(eoo), 9, 12, **16**